Tractable Query Answering for Expressive Ontologies and Rules*

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Disjunctive Existential Rules

Disjunctive Existential Rules

Facts

Disjunctive Existential Rules

$$\mathsf{HasParent}(x, y) \wedge \mathsf{HasSister}(y, z) \rightarrow \mathsf{HasAunt}(x, z)$$

$$\mathsf{Human}(x) \rightarrow \exists y \,.\, \mathsf{HasParent}(x, y) \wedge \mathsf{Human}(y)$$

$$\mathsf{Animal}(x) \rightarrow \mathsf{Herbivore}(x) \vee \mathsf{Carnivore}(x) \vee \mathsf{Omnivore}(x)$$

$$\mathsf{P}(x, \mathsf{a}, y) \wedge \mathsf{R}(y, w) \wedge \mathsf{S}(w, x) \rightarrow \exists v \,.\, (\mathsf{R}(w, v) \wedge \mathsf{A}(v)) \vee \mathsf{D}(x)$$

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Facts

$$\mathsf{HasFriend}(\mathsf{stan}, \mathsf{kyle})$$

$$\mathsf{Dead}(\mathsf{kenny})$$

$$\mathsf{P}(\mathsf{a}, \mathsf{c}, \mathsf{d})$$

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Remark. If normalised, SROIQ ontologies can be translated into equivalent programs of disjunctive existential rules.

The Disjunctive Chase

The Disjunctive Chase

$\mathsf{Director}(x) \to \exists y\,.\,\mathsf{Directs}(x, y) \wedge \mathsf{Film}(y)$

$\mathsf{Film}(x) \to \exists z\,.\,\mathsf{IsDirectedBy}(x, z) \wedge \mathsf{Director}(z)$

$\mathsf{Film}(\mathsf{ai})$

The Skolem Disjunctive Chase

$\mathsf{Director}(x) \rightarrow \exists y\,.\,\mathsf{Directs}(x, y) \wedge \mathsf{Film}(y)$

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The Skolem Disjunctive Chase

$\mathsf{Director}(x) \rightarrow \mathsf{Directs}(x, f_y(x)) \wedge \mathsf{Film}(f_y(x))$

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$\text{Director}(x) \rightarrow \text{Directs}(x, y(x)) \wedge \text{Film}(y(x))$

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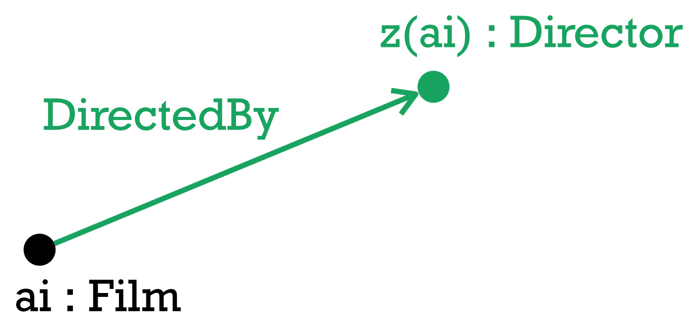

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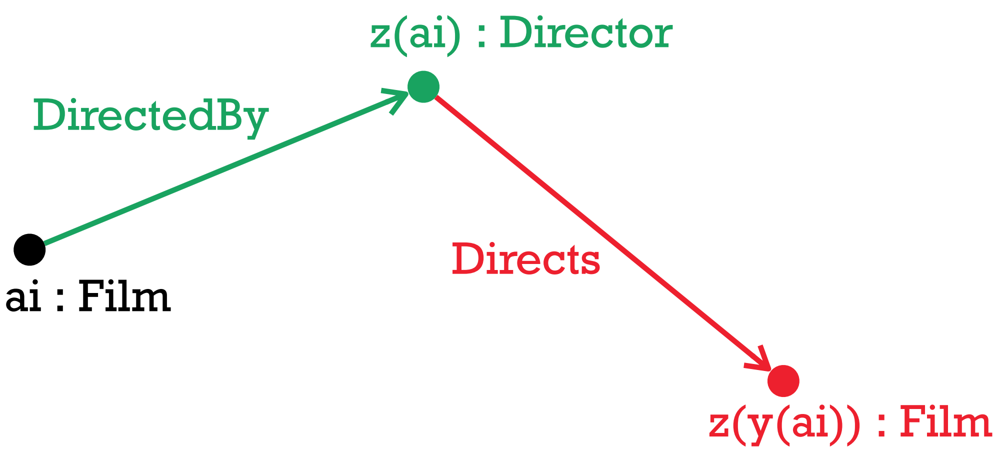

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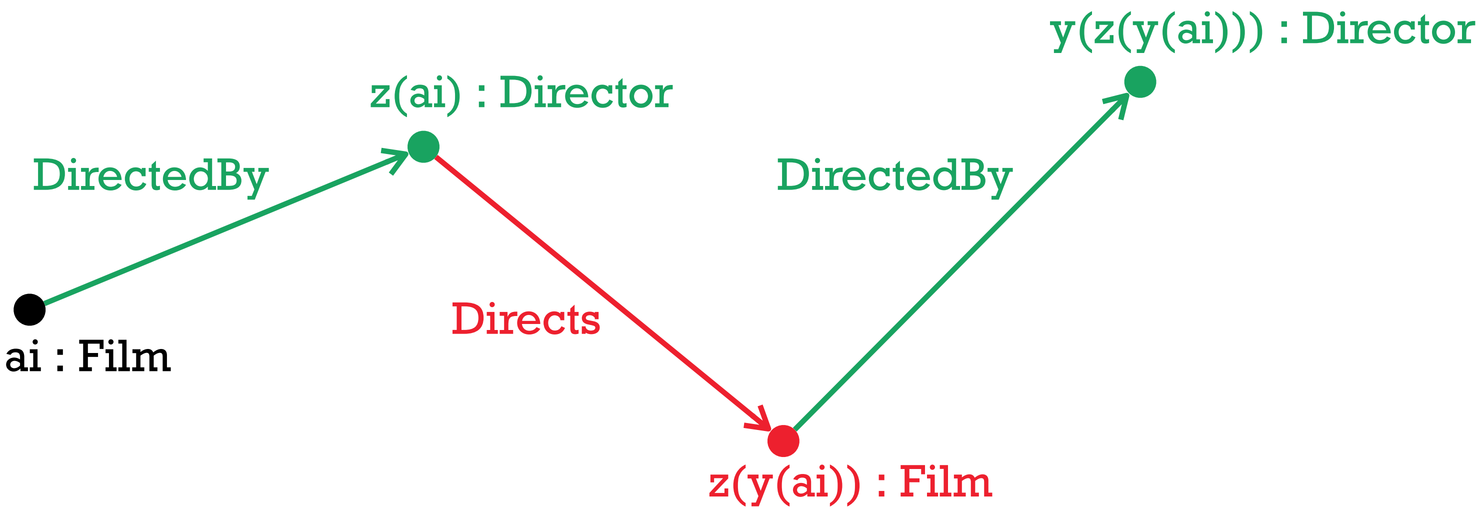

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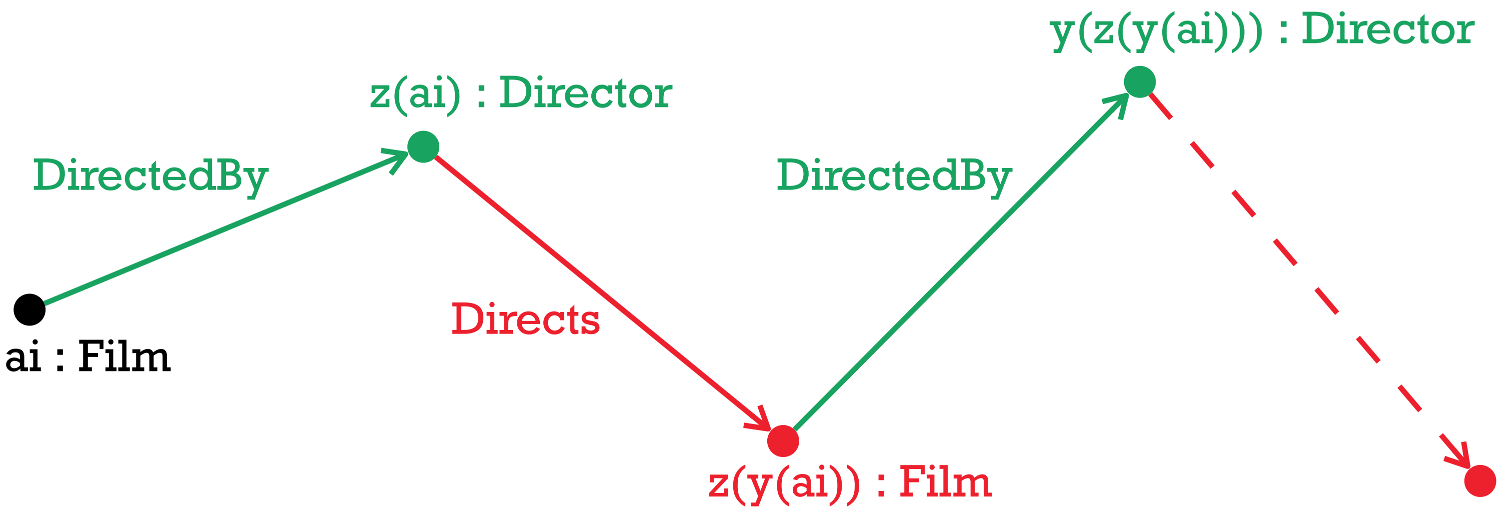

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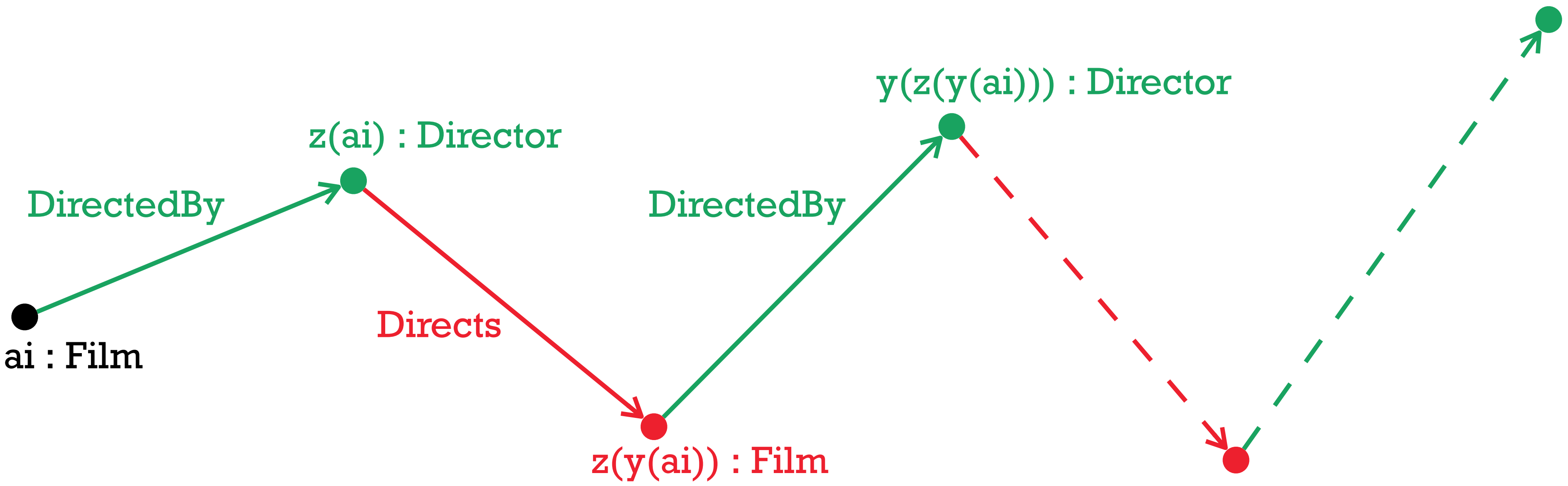

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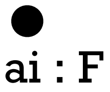

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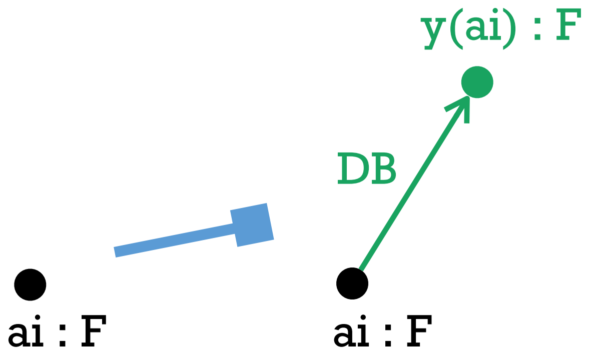

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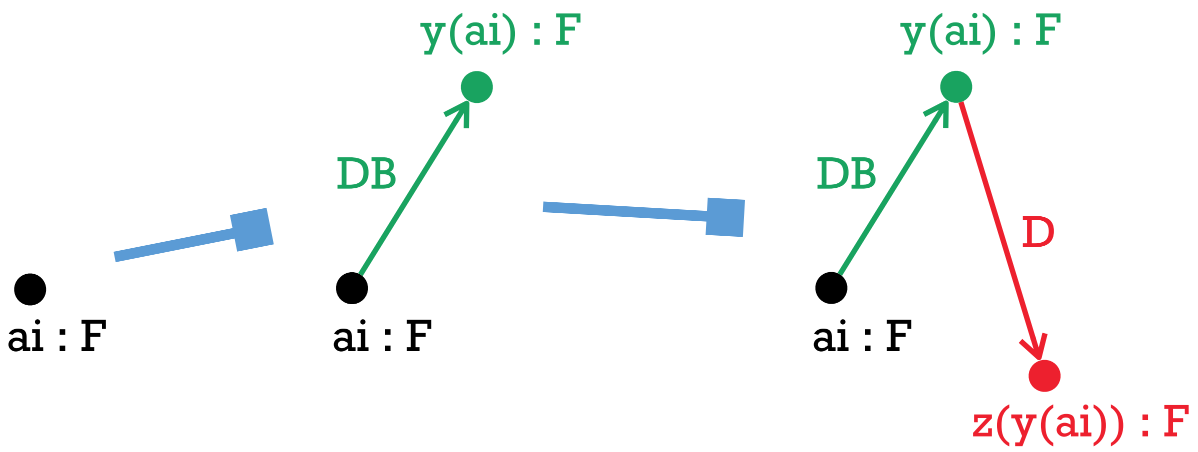

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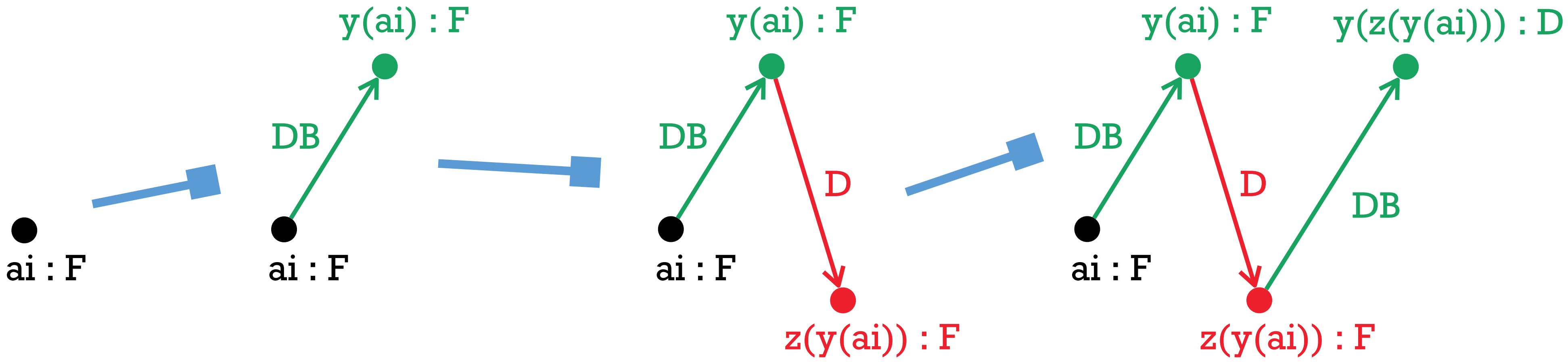

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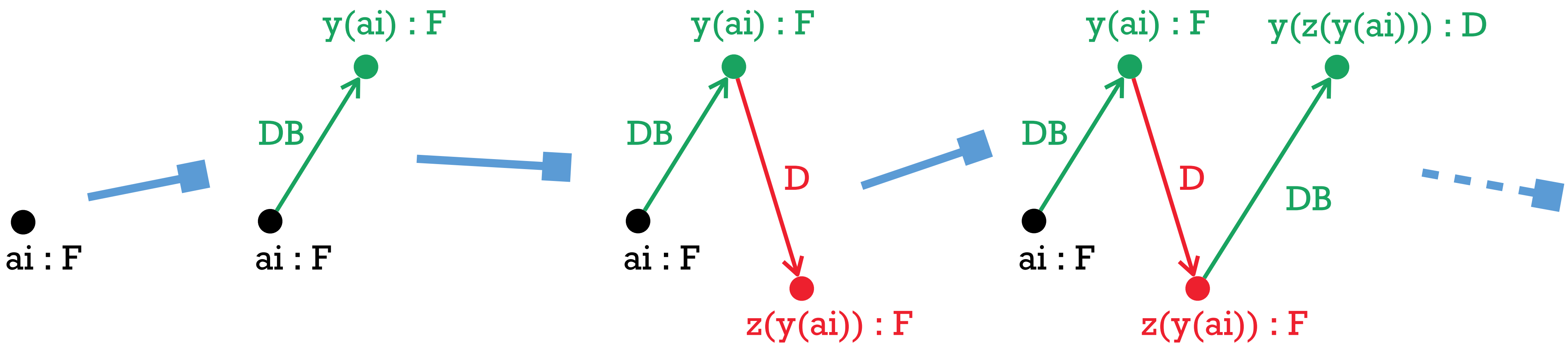


The Skolem Disjunctive Chase

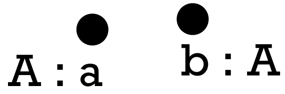
$\text{Animal}(x) \rightarrow \text{Vertebrate}(x) \vee \text{Invertebrate}(x)$

$\text{Animal}(a) \quad \text{Animal}(b)$

The Skolem Disjunctive Chase

$\text{Animal}(x) \rightarrow \text{Vertebrate}(x) \lor \text{Invertebrate}(x)$

$\text{Animal}(\text{a}) \quad \text{Animal}(\text{b})$



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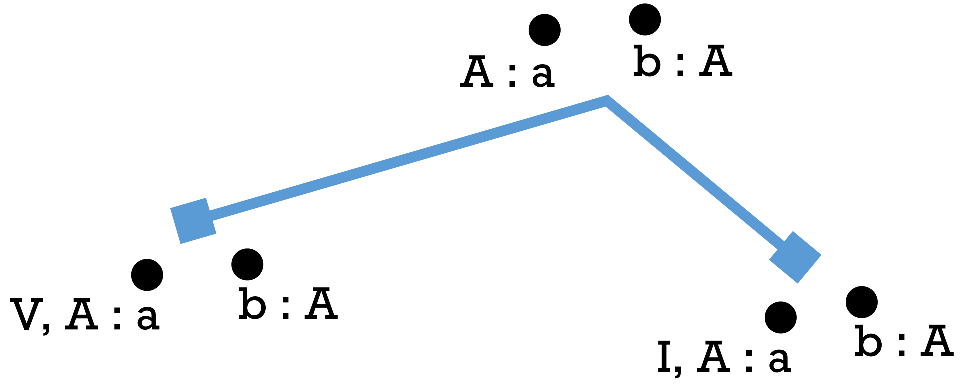
Animal(a) Animal(b)



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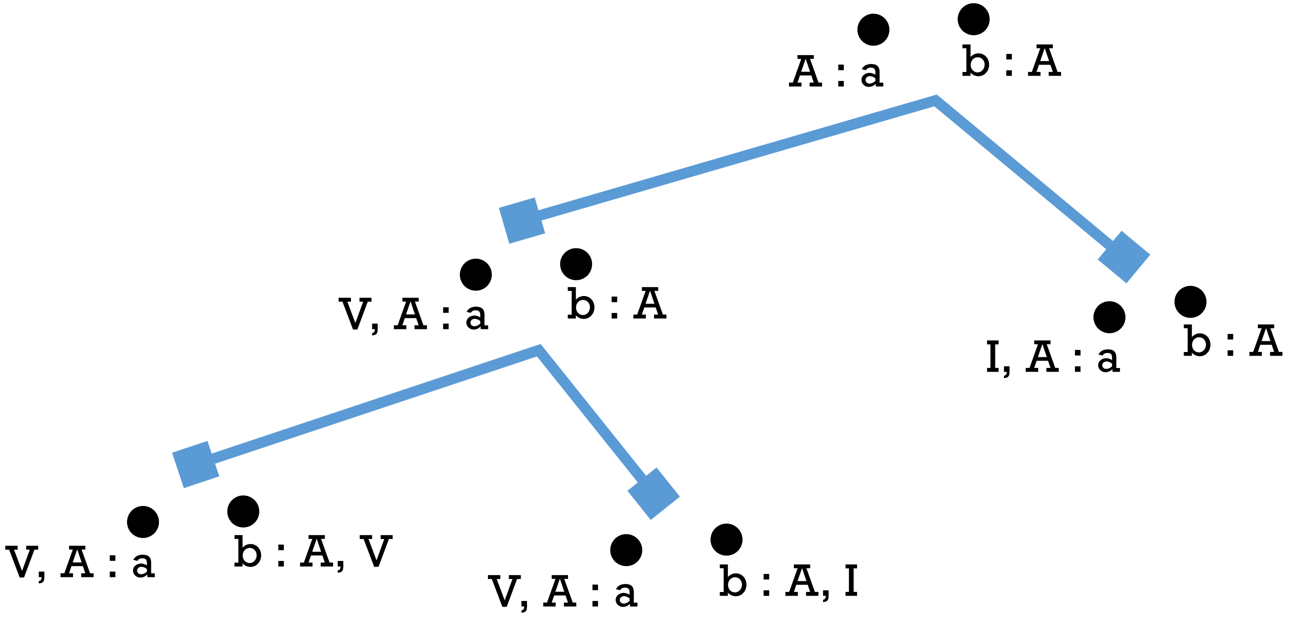

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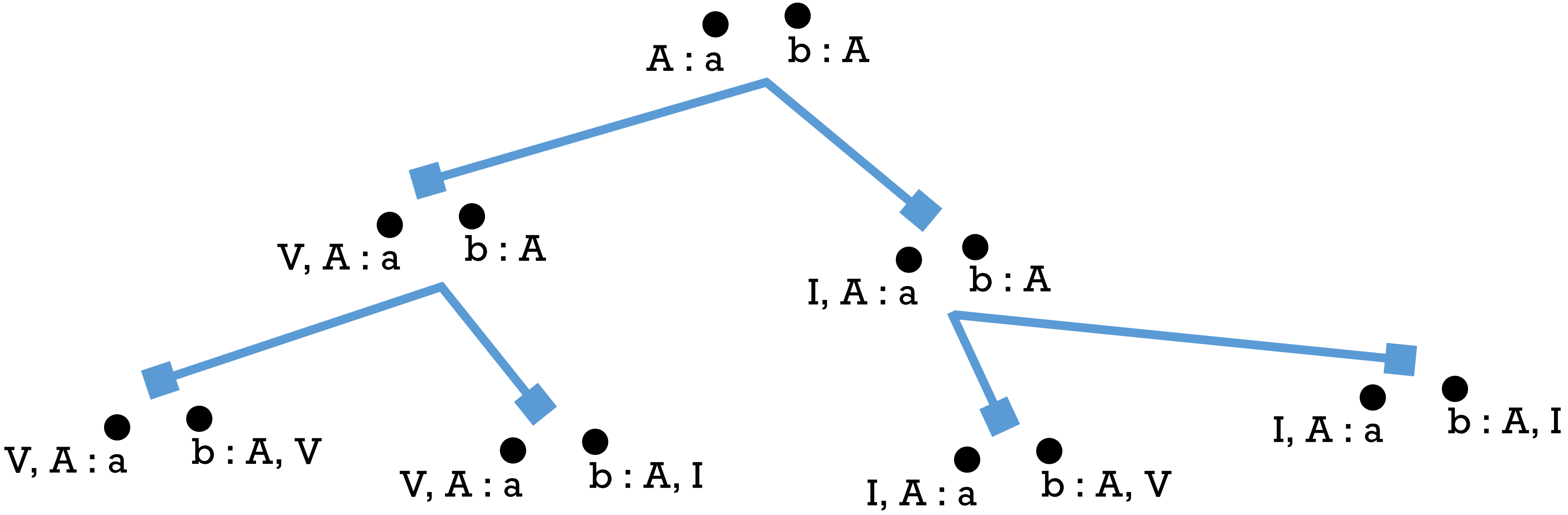

Ensuring Tractability of the Disjunctive Chase

Existential Dependency Graph

$$A(x) \to \exists y \,.\, S(x, y) \land B(y)$$

$$B(x) \to \exists z \,.\, R(x, z) \land D(z)$$

$$D(x) \to E(x)$$

$$E(x) \to \exists w \,.\, R(x, w)$$

$$B(x) \land C(x) \to E(x)$$

$$S(x, y) \to C(x)$$

Existential Dependency Graph

$$\mathsf{A}(x) \to \mathsf{S}(x, y(x)) \land \mathsf{B}(y(x))$$

$$\mathsf{B}(x) \to \mathsf{R}(x, z(x)) \land \mathsf{D}(z(x))$$

$$\mathsf{D}(x) \to \mathsf{E}(x)$$

$$\mathsf{E}(x) \to \mathsf{R}(x, w(x))$$

$$\mathsf{B}(x) \land \mathsf{C}(x) \to \mathsf{E}(x)$$

$$\mathsf{S}(x, y) \to \mathsf{C}(x)$$

Existential Dependency Graph

$$\mathsf{A}(x) \to \mathsf{S}(x, y(x)) \wedge \mathsf{B}(y(x))$$

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$$\mathsf{E}(x) \rightarrow \mathsf{R}(x, w(x))$$

$$\mathsf{B}(x) \wedge \mathsf{C}(x) \rightarrow \mathsf{E}(x)$$

$$\mathsf{S}(x, y) \rightarrow \mathsf{C}(x)$$

$\mathsf{A}(c)$

$\mathsf{S}(c, y(c)), B(y(c))$

$\mathsf{R}(y(c), z(y(c))), D(z(y(c)))$

Existential Dependency Graph

$$\mathsf{A}(x) \to \mathsf{S}(x, y(x)) \wedge \mathsf{B}(y(x))$$
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$$\mathsf{S}(x, y) \to \mathsf{C}(x)$$

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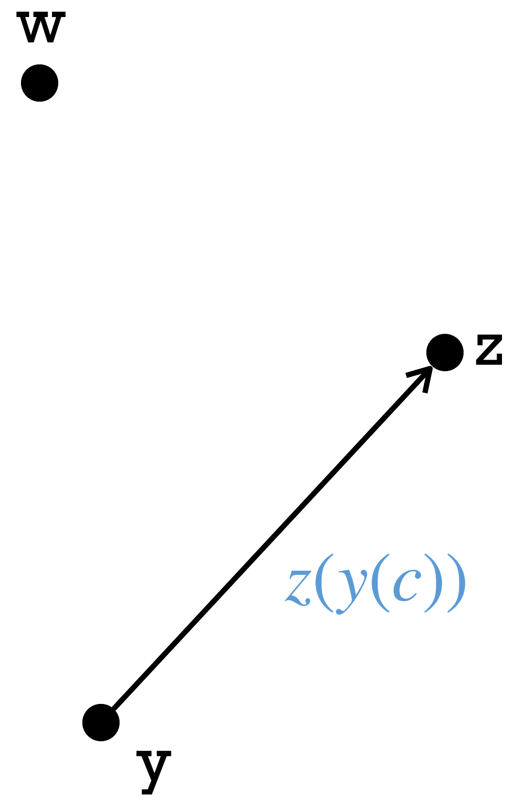
$\mathsf{R}(y(c), z(y(c))), D(z(y(c)))$



Existential Dependency Graph

$$A(x) \rightarrow S(x, y(x)) \land B(y(x))$$
$$B(x) \rightarrow R(x, z(x)) \land D(z(x))$$
$$D(x) \rightarrow E(x)$$
$$E(x) \rightarrow R(x, w(x))$$
$$B(x) \land C(x) \rightarrow E(x)$$
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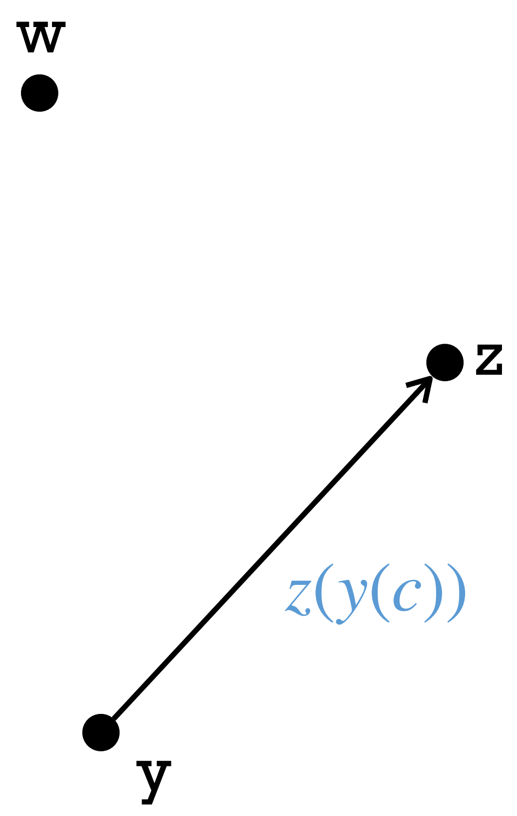

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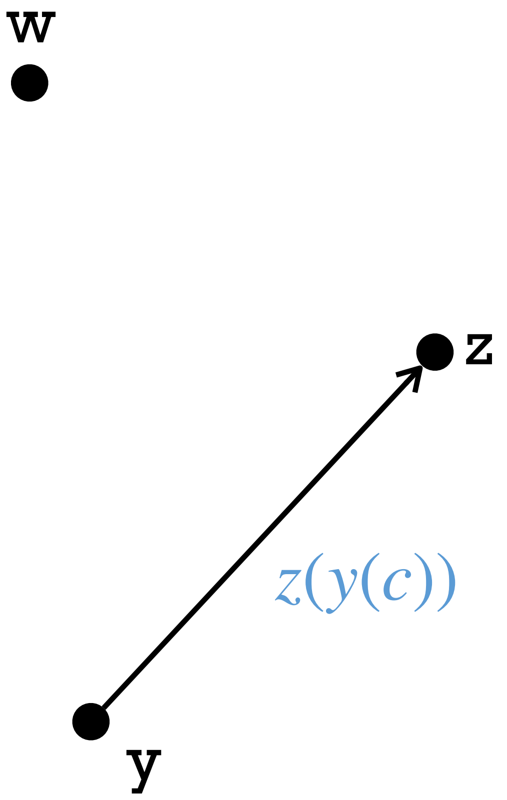

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$$E(x) \to R(x, w(x))$$

$$B(x) \wedge C(x) \to E(x)$$

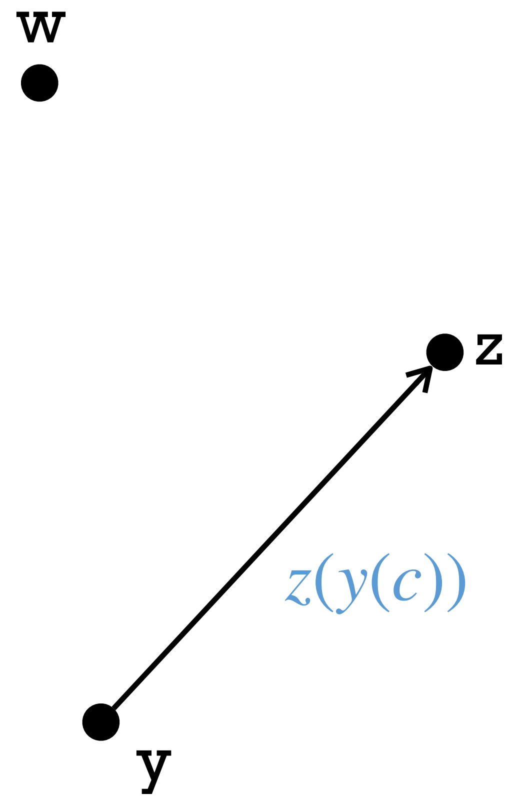
$$S(x, y) \to C(x)$$

$B(c)$

$R(c, z(c)), D(z(c)),$

$E(z(c)),$

$R(z(c), w(z(c)))$



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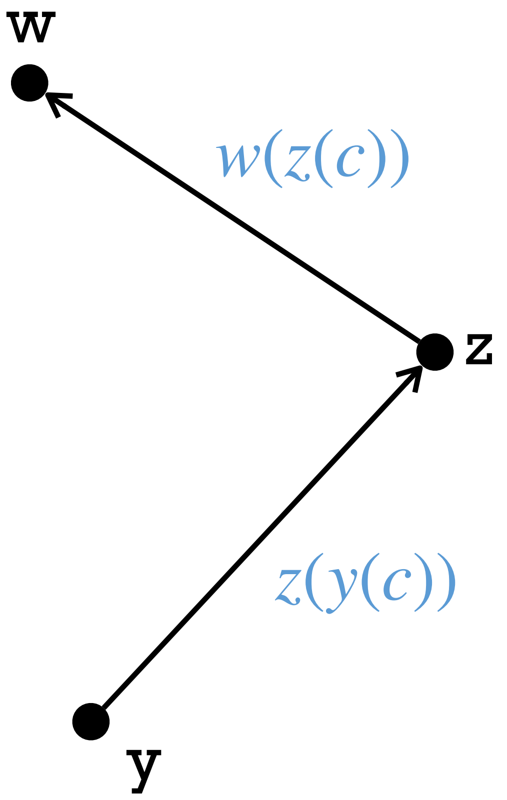

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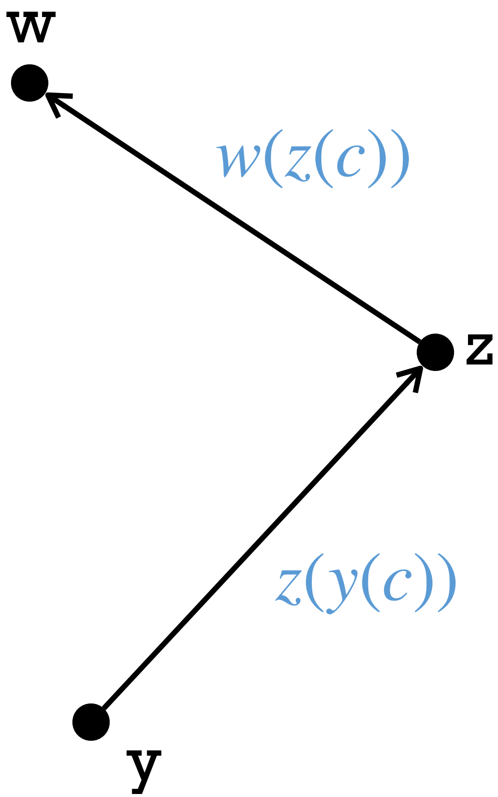

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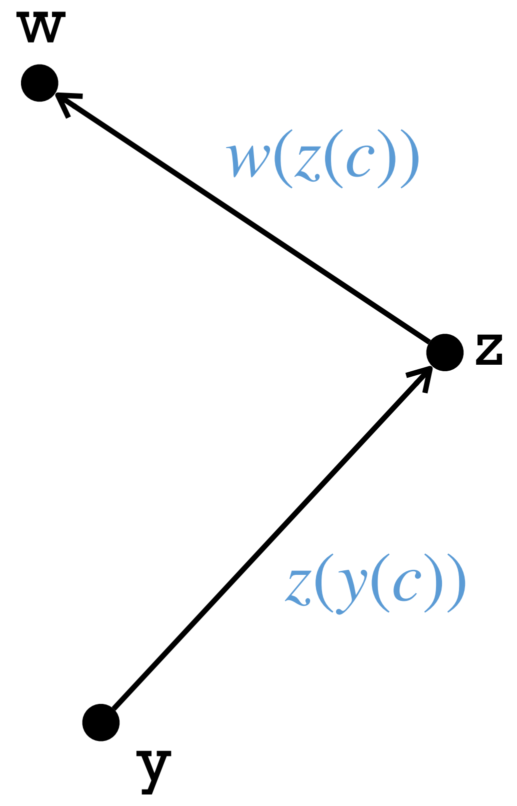

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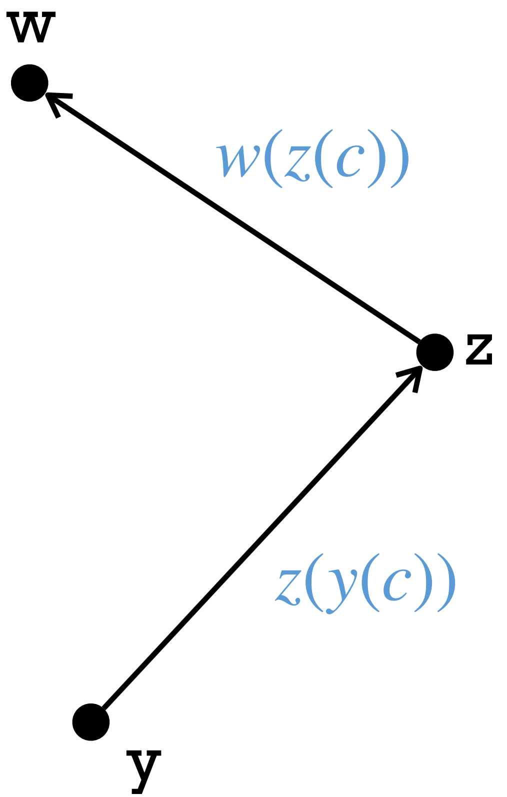

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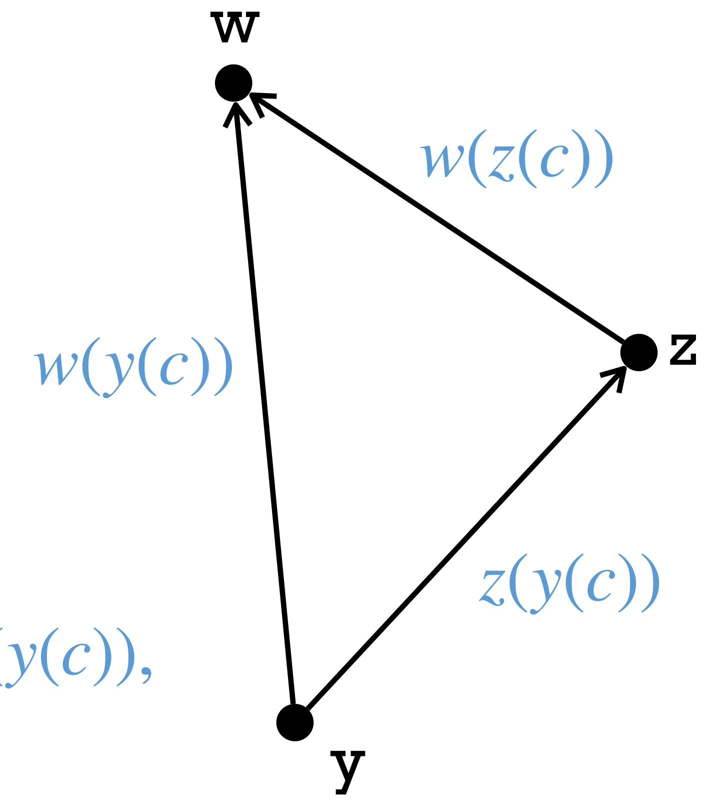

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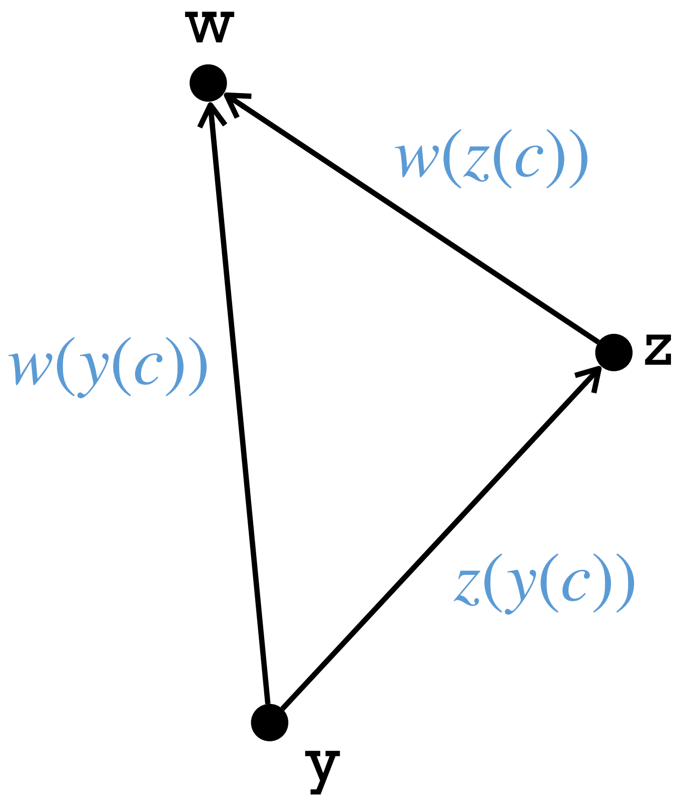

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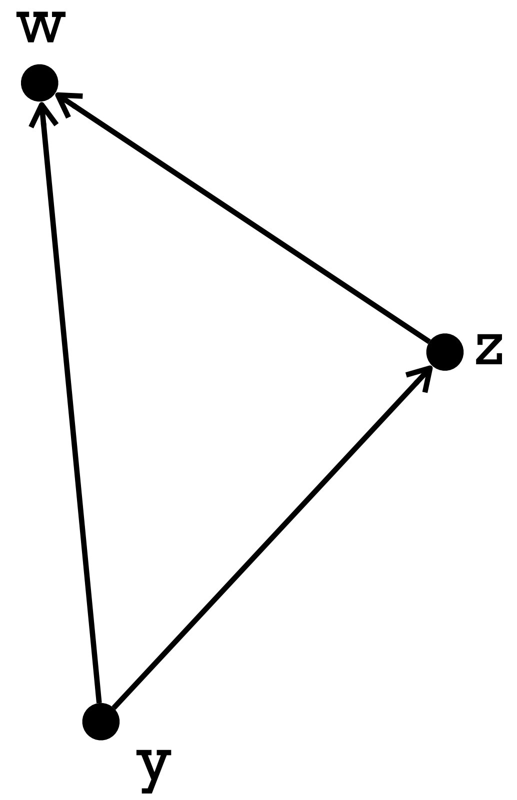
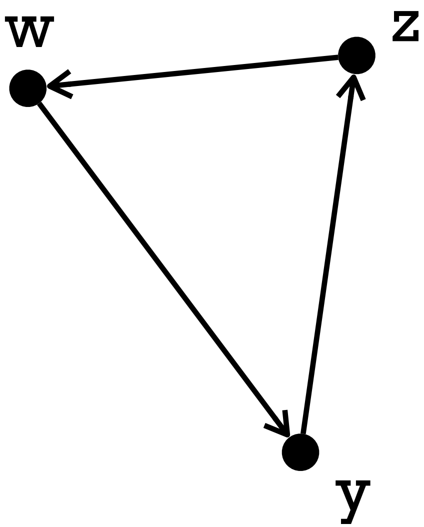
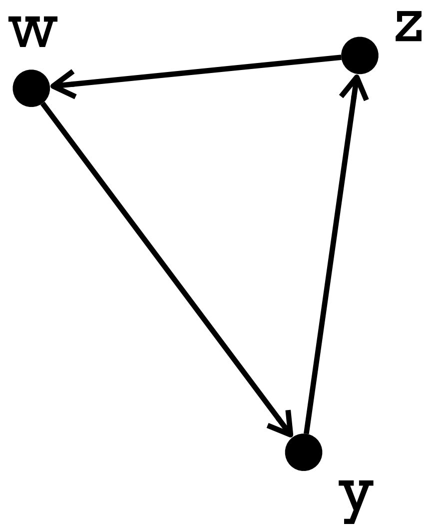


(a) Acyclicity

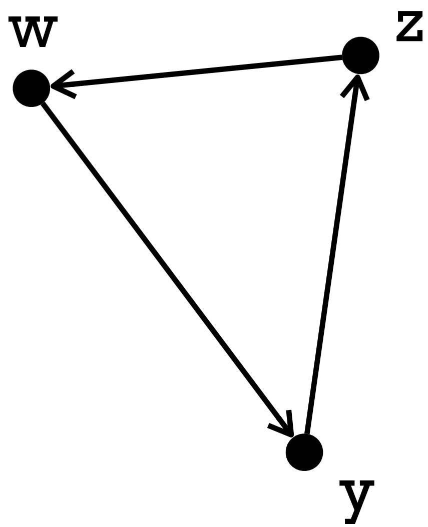


(a) Acyclicity



$z(c)$

(a) Acyclicity



$z(c)$

$w(z(c))$

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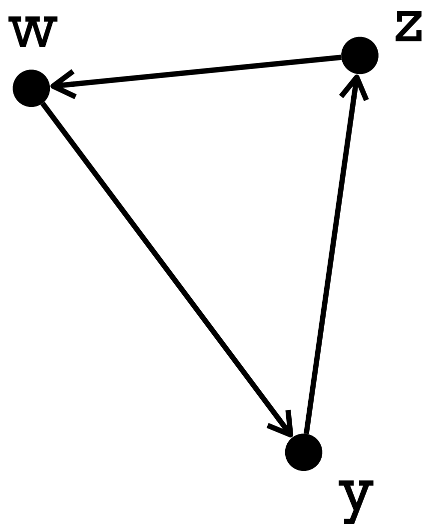

$z(c)$

$y(w(z(c)))$

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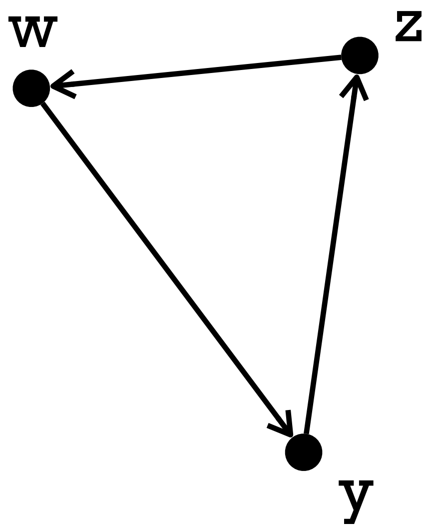

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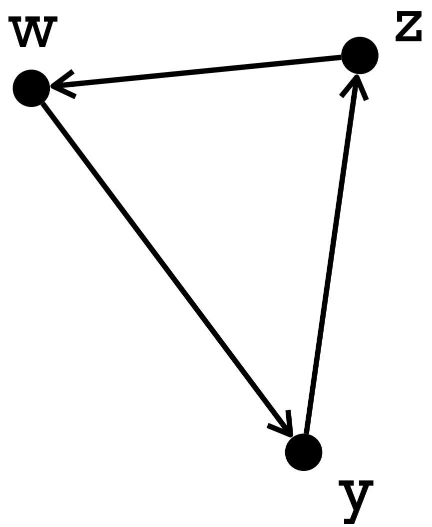

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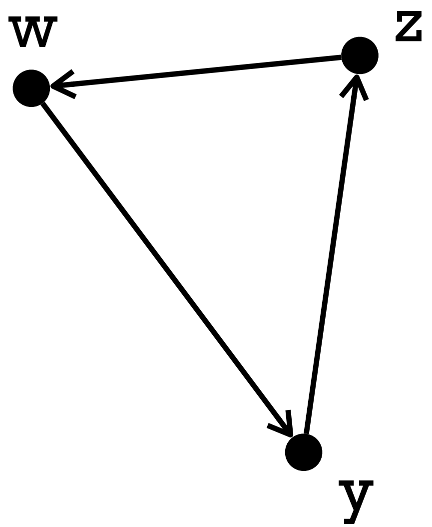

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....

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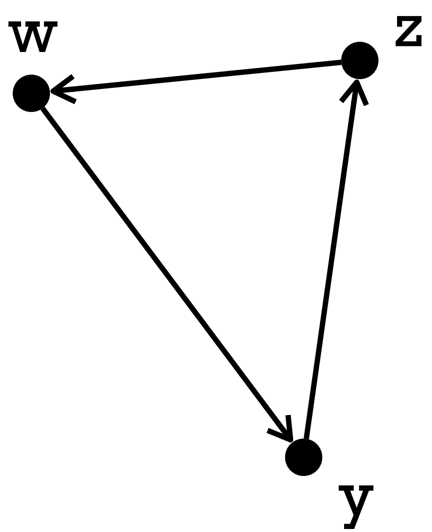

Remark. If the existential dependency graph of a given set of rules is acyclic, then the set of terms introduced during the computation of the chase is finite.

(f) Arity at Most 1

$$\mathsf{Film}(x) \to \exists y\,.\,\mathsf{IsFilmDirectedBy}(x, y) \wedge \mathsf{Director}(y)$$

$$\mathsf{A}(x) \wedge \mathsf{B}(x, w) \wedge \mathsf{C}(x, z) \to \exists z\,.\,\mathsf{R}(x, w, z)$$

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Remark. *If the arity of every function symbol in the skolemisation of a program is at most 1, then every term in the chase is of the form* ***x1(...xn(c)...)*** *with* ***c*** *constant.*

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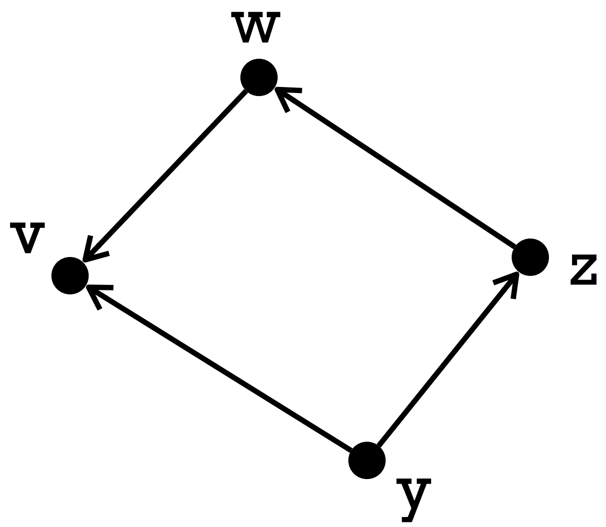

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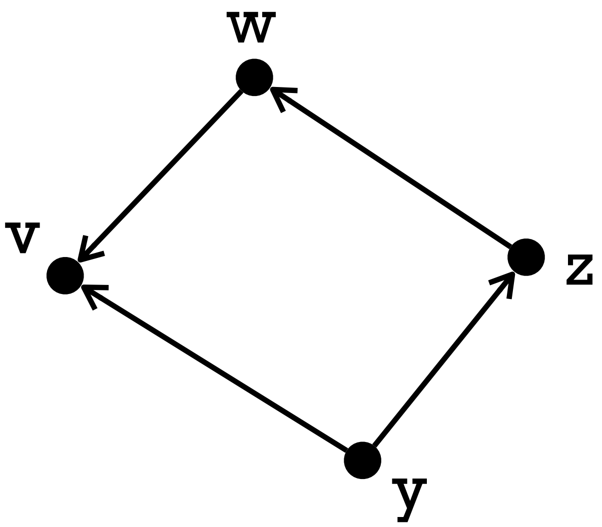

v(w(z(y(c))))
y(v(c))

Ensuring Tractability

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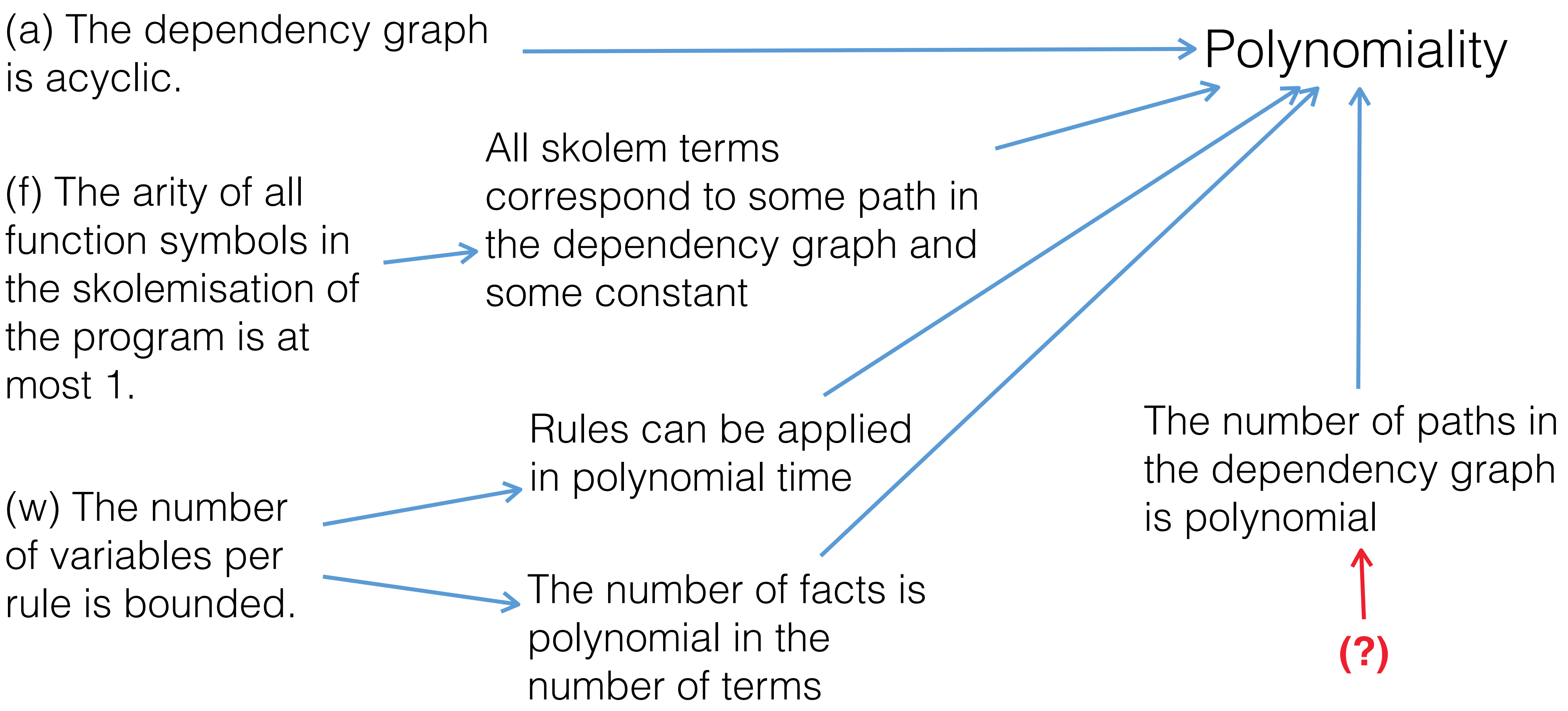
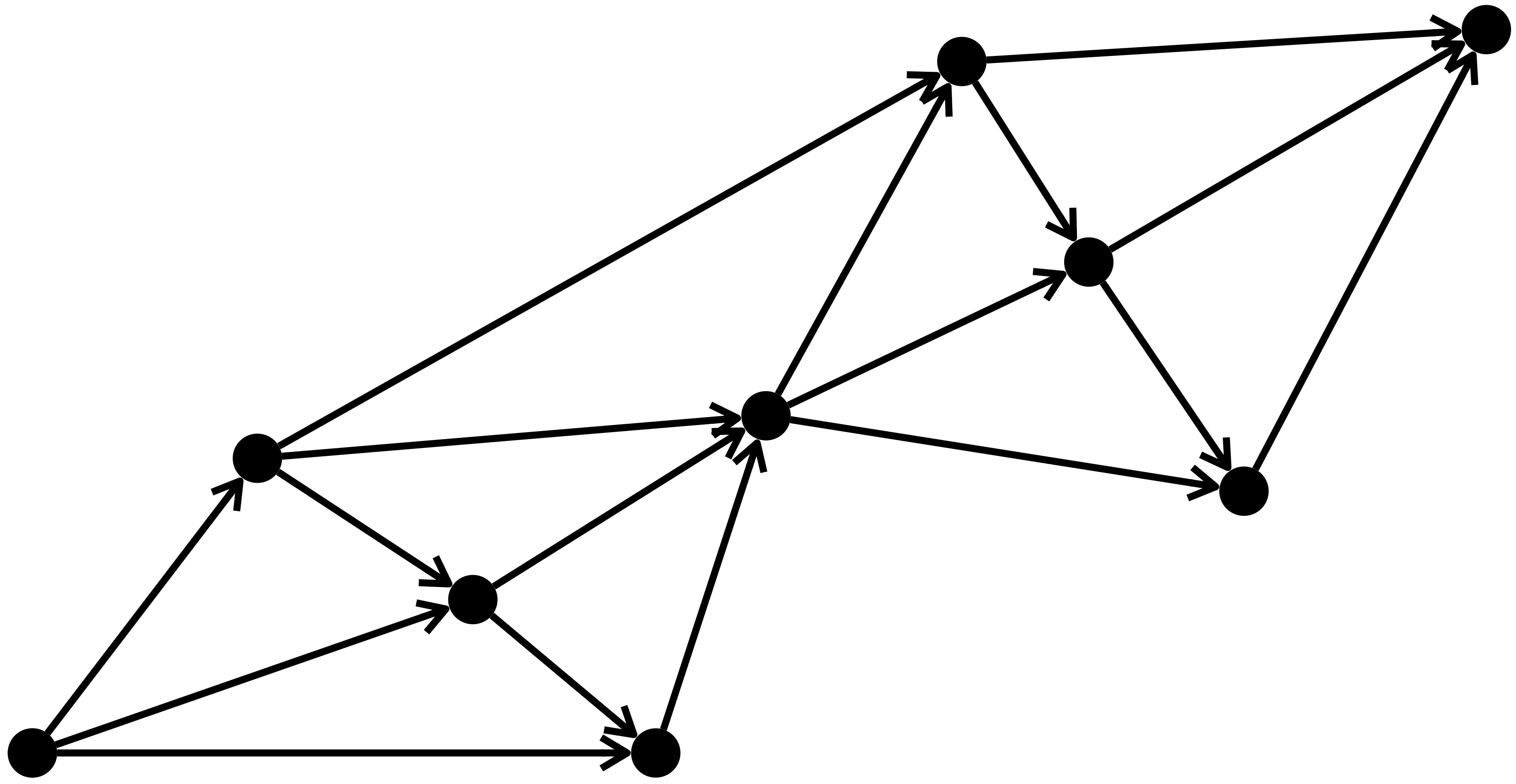


Braids



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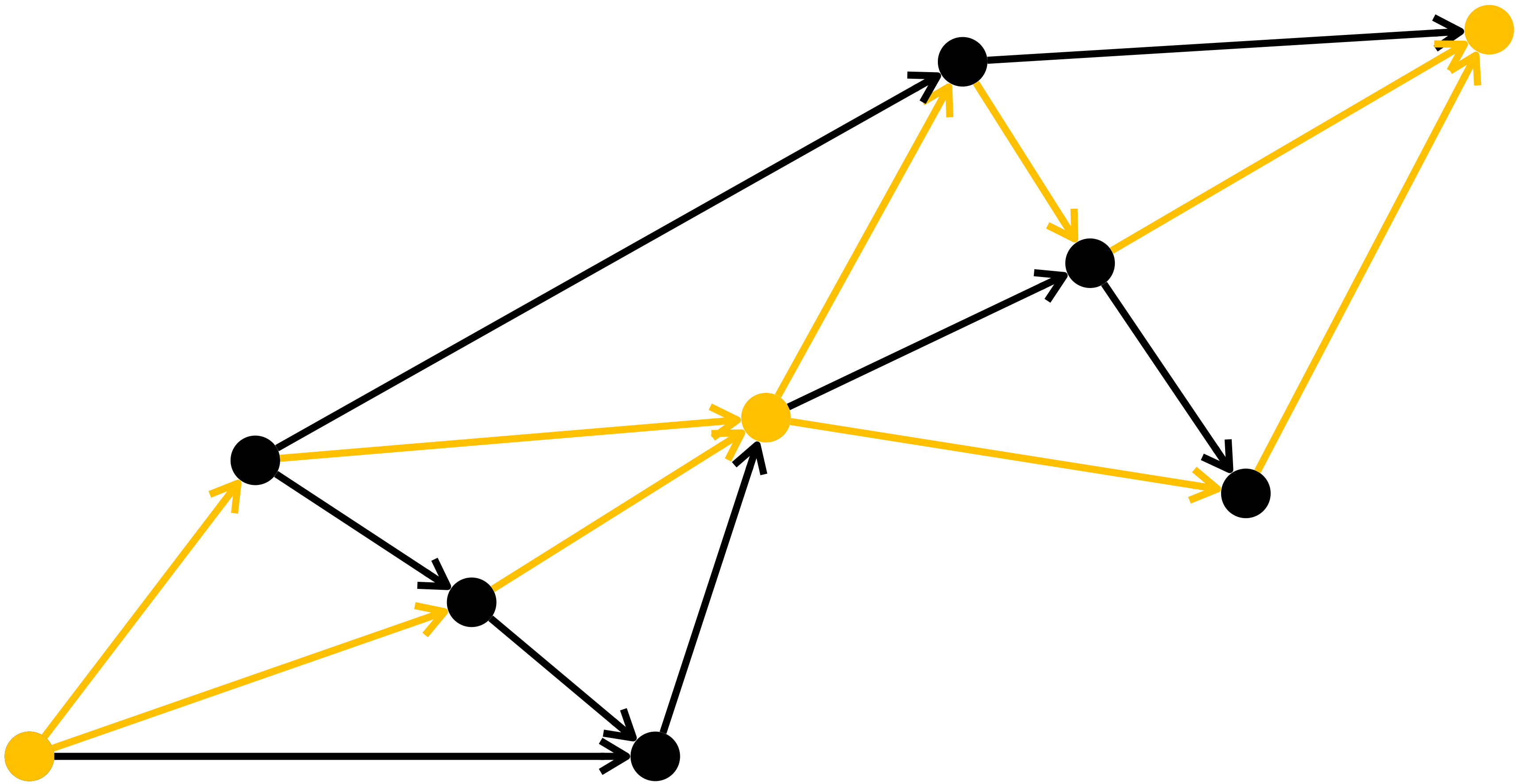

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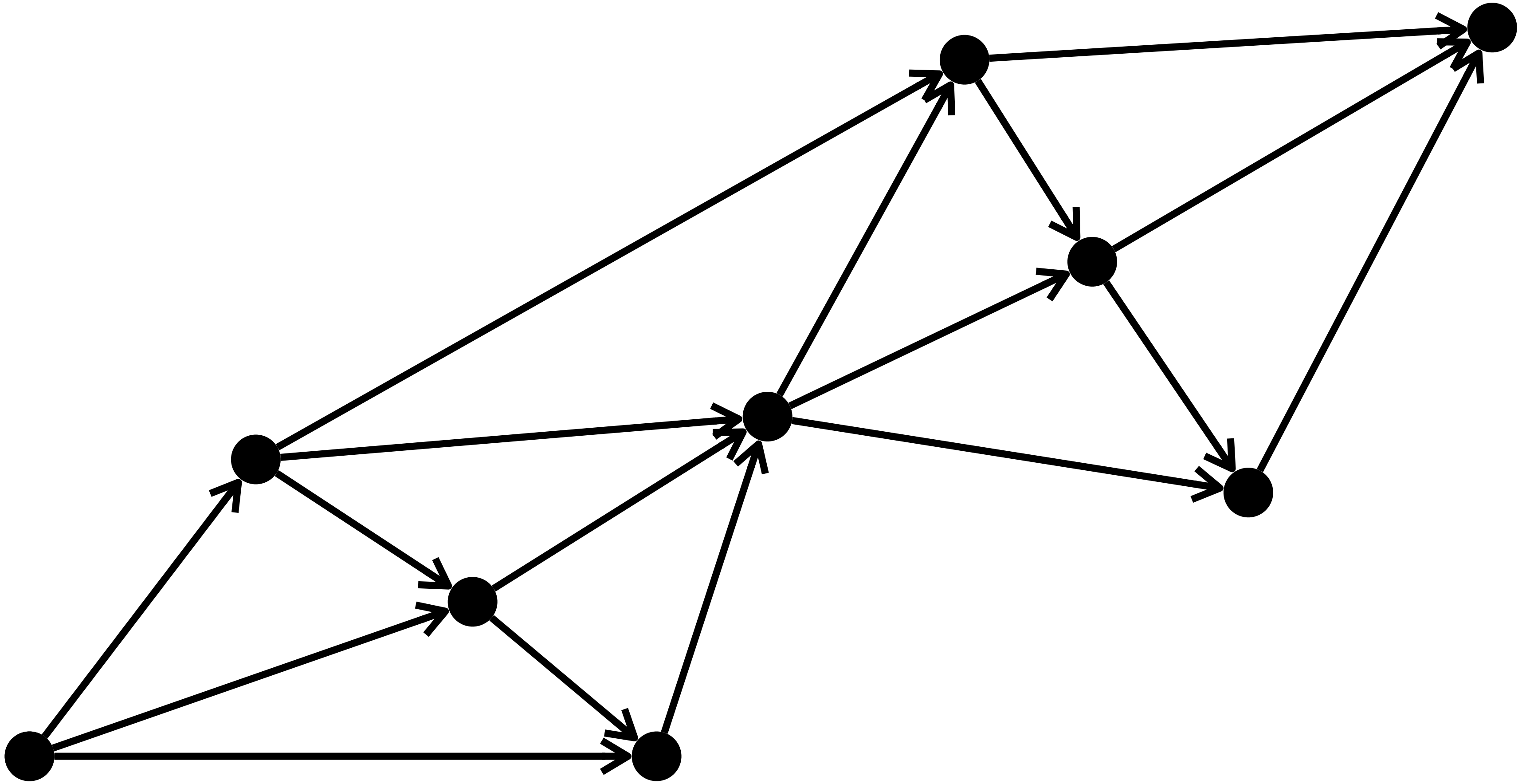

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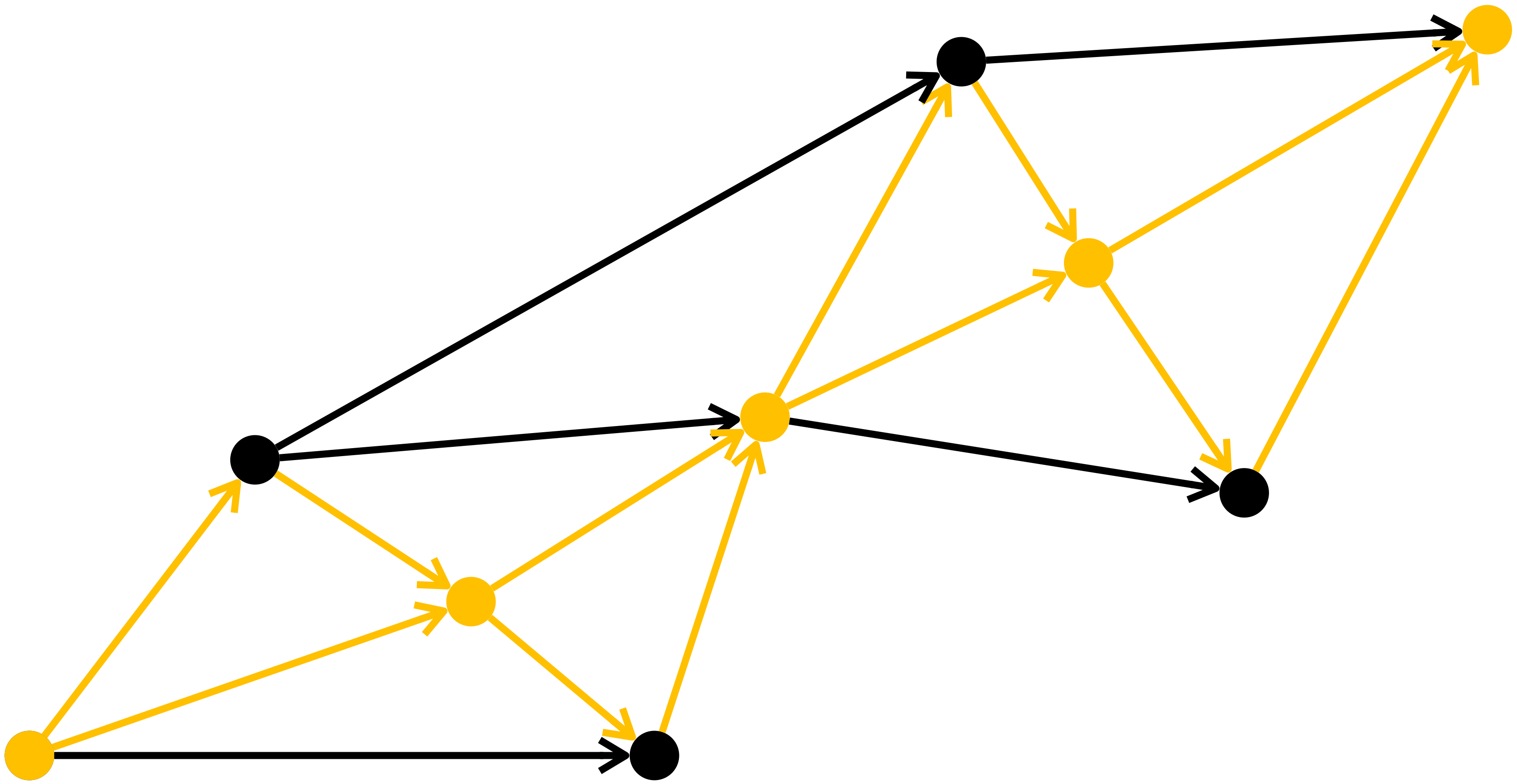

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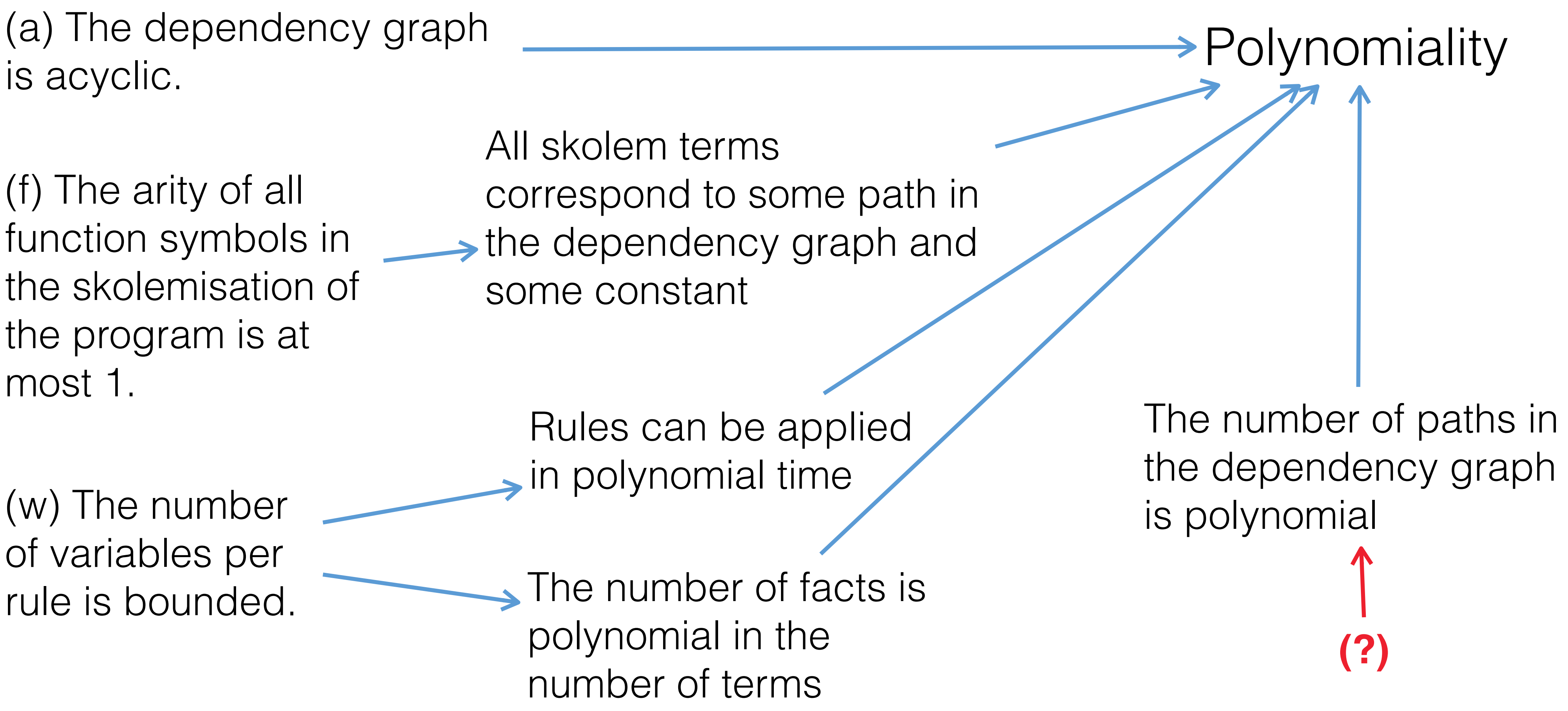

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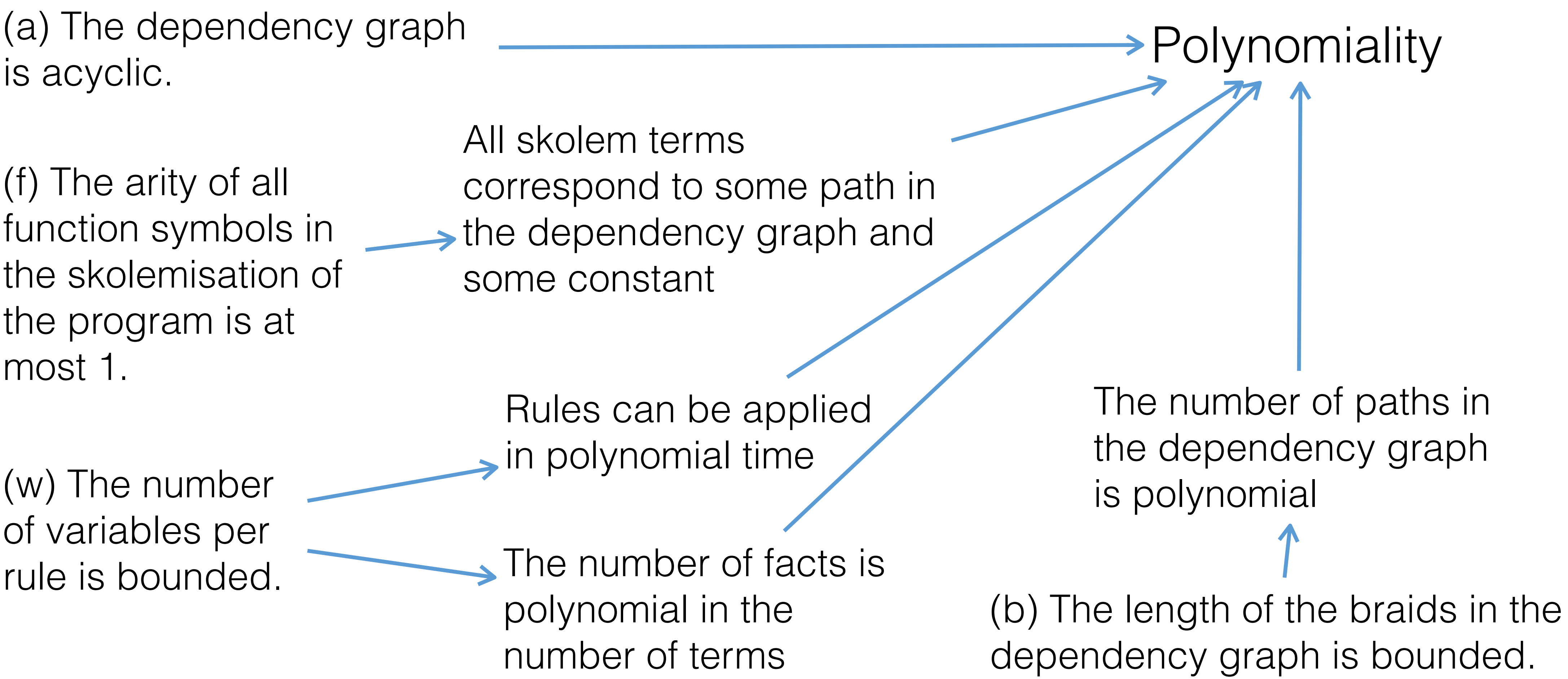

Ensuring Tractability

Caveats.
1. Fixed query size.
2. Horn rule set.

(a) The dependency graph is acyclic.

Polynomiality

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Rules can be applied in polynomial time

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The number of facts is polynomial in the number of terms

The number of paths in the dependency graph is polynomial

(b) The length of the braids in the dependency graph is bounded.

Evaluation

SRI Axioms

$$
\begin{aligned}
A_1 \sqcap \ldots \sqcap A_n \sqsubseteq B \quad &\mapsto \quad A_1(x) \land \ldots \land A_n(x) \to B(x) \\
A \sqsubseteq B_1 \sqcup \ldots \sqcup B_n \quad &\mapsto \quad A(x) \to B_1(x) \lor \ldots \lor B_n(x) \\
A \sqsubseteq \forall R\,.\,B \quad &\mapsto \quad A(y) \land R(x,y) \to B(x) \\
A \sqsubseteq \exists R\,.\,B \quad &\mapsto \quad A(x) \to \exists y\,.\,R(x,y) \land B(y) \\
R \sqsubseteq S \quad &\mapsto \quad R(x,y) \to S(x,y) \\
R \circ S \sqsubseteq V \quad &\mapsto \quad R(x,y) \land S(y,z) \to S(x,z) \\
R_1 \sqcap \ldots \sqcap R_n \sqsubseteq S \quad &\mapsto \quad R_1(x,y) \land \ldots \land R_n(x,y) \to S(x,y) \\
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SRI Axioms

$$A_1 \sqcap \ldots \sqcap A_n \sqsubseteq B \mapsto A_1(x) \wedge \ldots \wedge A_n(x) \to B(x)$$

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$$R_1 \sqcap \ldots \sqcap R_n \sqsubseteq S \mapsto R_1(x, y) \wedge \ldots \wedge R_n(x, y) \to S(x, y)$$

$$A(a) \mapsto A(a)$$

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Remark 1. Deciding CQ entailment for SRI ontologies is 2ExpTime-Hard and in 3ExpTime.

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Corollary. To guarantee that tractable reasoning over a SRI ontology is possible we only need to verify the following:
1. Acyclicity.
2. Braid length in the dependency graph is bounded.

Evaluation Results

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Acyclicity

	MOWL Corpus	Oxford Ontology Repo
Ontologies	1576	225
Acyclic	974 (61.8%)	170 (75.6%)

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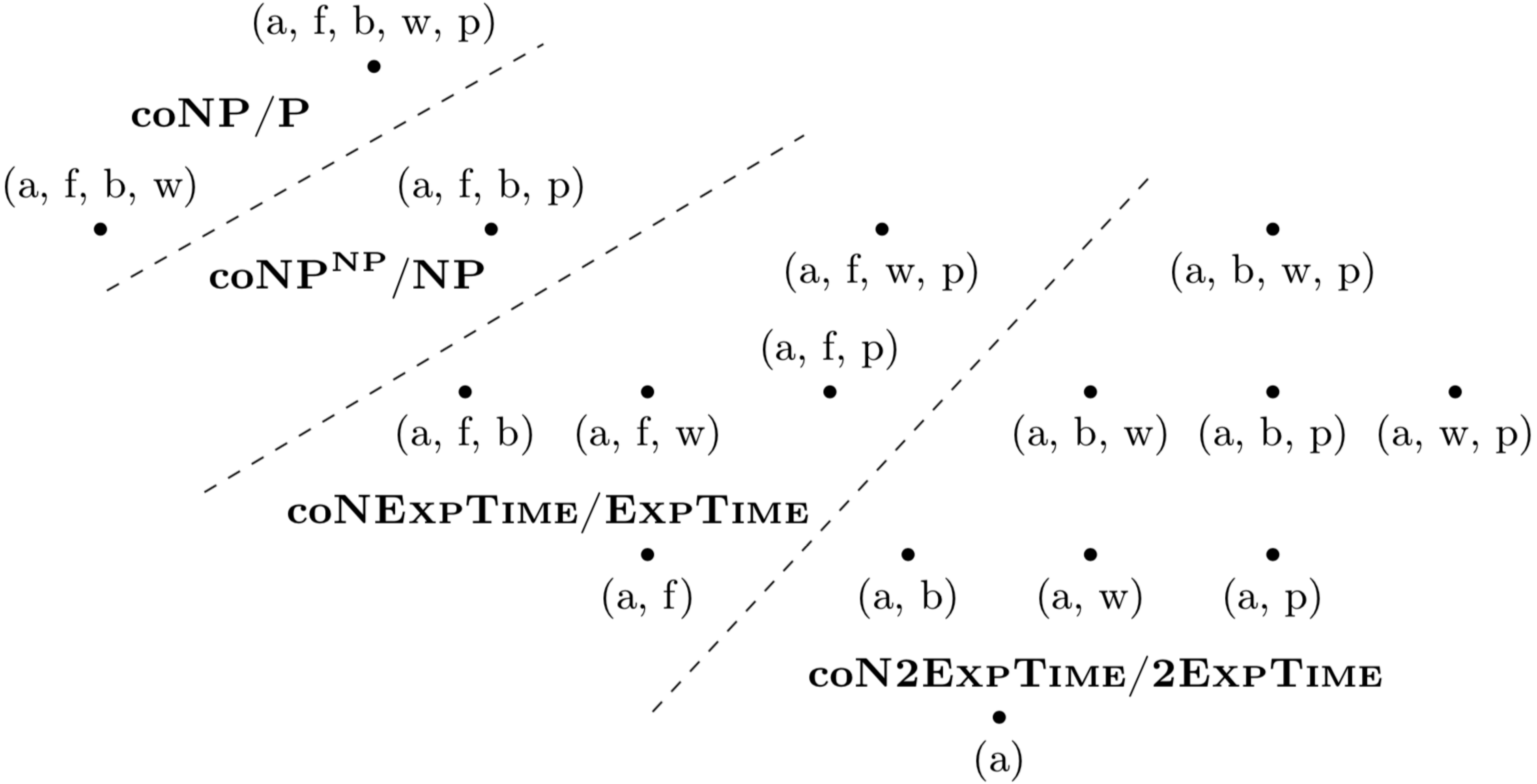
Braid Length

MOWL Corpus + Oxford

(max. length of a braid)	1	2	3	4	5	6	11	22	23	25	Total
(count)	851	153	56	61	11	1	1	2	7	1	1144
(%)	74	88	93	98	99	99	99	99.1	99.3	99.9	100

Conclusions

More Results!



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Efficient CQ Answering over a large subset of OWL 2 real-world ontologies is possible!

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Future work:
- Implementation of a chase based reasoner for these ontologies
- Refine conditions

Tractable Query Answering for Expressive Ontologies and Rules*

David Carral, Irina Dragoste, Markus Krötzsch

Knowledge-Based Systems Group at

TECHNISCHE UNIVERSITÄT DRESDEN

*Published at ISWC 2017