Experience Based Nonmonotonic Reasoning

Daniel Borchmann

TU Dresden

Abstract. Within everyday reasoning we often use argumentation patterns that employ the rather vague notion of something being *normally true*. This form of reasoning is usually captured using Reiter's Default Logic. However, in Default Logic one has to make explicit the rules which are to be used for reasoning and which are supposed to be normally true. This is a bit contrary to the everyday situation where people use *experience* to decide what normally follows from particular observations and what not, not using any kind of logical rules at all. To formalize this kind of reasoning we propose an approach which is based on *prior experiences*, using the fact that something follows normally if this is the case for "almost all" of the available experience.

1 Introduction

When we say that "the tram normally is on time," we do so because in most of our previous experiences this has been the case. Of course, when stating this fact, we very well accept the possibility that due to some road accident our tram may not come at all. Even if such a road accident occurs, we may still be of the opinion that the trams are normally on time, because road accidents are "not normal."

This kind of reasoning, which employs the very vague notion of "normality," is rather common to us, and several attempts have been made to formalize this kind of reasoning or embed it into a formal framework. Two of the most common attempts are McCarthy's *Circumscription* [3] and Reiter's *Default Logic* [4]. The former tries to restrict the usual model semantics of first order logic (or propositional logic) to models which are "as normal as possible" by minimizing the amount of abnormality they have. The latter approach adds *justifications* for inferences rules that model normality: a rule is normally applicable, but not if the justification is not valid.

These two approaches have in common that they both start with *knowledge*, expressed using logical formulas, that is assumed to be "normally true", and try to infer new knowledge based on this. However, this is not the case in the situation where we wait for our tram: we do not employ rules like "when there is no construction work nearby, then my tram is usually on time" or similar things.

In fact, what we usually do is that we *compare* our current situation to previous experiences and see what happened in these situations. If we find some occurrence often enough, then we conclude that it "normally occurs" in "situations like this" and say that it should "normally occur now" as well. This form of reasoning does not involve any kind of prior knowledge, but just makes use of previous observations and a certain kind of heuristic which decides when something happened "often enough."

However, note that we can use this comparison with previous situations to *generate* non-monotonic rules. Indeed, when we are waiting for the tram, and no construction work is nearby, then we could conclude from previous experiences that the tram should be on time. So we can extract this rule "no construction work nearby implies tram on time" from our experiences. Such rules could then be used for further reasoning.

The purpose of this work is twofold. First and foremost, we set out to formalize the notion of *normal reasoning based on prior experiences* as discussed above. To this end, we shall make use of the theory of *formal concept analysis* as a framework to model *experiences*. Furthermore, we shall make use of the notion of *confidence*, as it is employed in data mining [1], as a method to formalize the fact that some observation occurred "often enough."

Secondly, we want to foster the discussion of this approach in the non-monotonic reasoning community, and want to ask whether there are close connections to existing approaches.

2 Formal Concept Analysis

For our considerations we require very little from the theory of formal concept analysis [2]. More precisely, we shall introduce the notions of a *formal context* and *contextual derivation*.

A *formal context* is a triple $\mathbb{K} = (G, M, I)$, where G, M are sets and $I \subseteq G \times M$. A popular interpretation of formal contexts is that we think of the set G as of a set of *objects*, and of the set M as of a set of *attributes*. Then an object $g \in G$ *has* and attribute $m \in M$ if and only if $(g, m) \in I$.

Let $A \subseteq G$ be a set of objects. We can ask for the set of *common attributes* of the set A, i. e. for the set

$$A' = \{ m \in M \mid \forall g \in A \colon (g, m) \in I \}.$$

Likewise, for a set $B \subseteq M$ of attributes, we can ask for the set of *satisfying objects*, i. e. for

$$B' = \{ g \in G \mid \forall m \in B \colon (g, m) \in I \}.$$

The mappings $(\cdot)'$ are called the *derivation operators* of $\mathbb{K}$, and the sets A' and B' are called the *derivations* of A and B in $\mathbb{K}$, respectively. A set $B \subseteq M$ of attributes is called an *intent* of $\mathbb{K}$ if and only if $B = B''$.

In formal context, one can ask questions like the following: is it true that every object that has all attributes from A has also all attributes from B? To formalize this notion, we introduce the concept of *implications* as pairs (A, B) of sets $A, B \subseteq M$. To make our intention clearer, we shall often write $A \to B$ instead of (A, B). The set of all implications on a set M is denoted by $\mathrm{Imp}(M)$. The implication $A \to B$ then *holds* in $\mathbb{K}$ if and only if $A' \subseteq B'$.

3 Nonmonotonic Reasoning in Formal Contexts

We shall now make use of the notion of formal context to model non-monotonic reasoning based on prior experiences. For this, let us fix a set M of *relevant attributes*. Then we shall understand an *experience* as a subset $N \subseteq M$. Intuitively, such an experience N consists exactly of all attributes we have observed within this experience. We then collect all such experiences in a formal context $\mathbb{K} = (G, M, I)$, i. e. for each $g \in G$, the set g' is an experience.

Example 1. Suppose (again) we are waiting for our tram at a tram station. It is a sunny day, and we suspect nothing bad. In particular, we do not expect our tram to be late. However, out of the sudden we hear some sirens, which may be due to some road accident that occurred nearby. Because of this extra information, we are not that sure anymore if our tram will arrive at all!

A formal context $\mathbb{K}_{\text{tram}}$ which collects a set of such prior experiences (together with the information whether the tram arrived or not) could be given by

	sunny	sirens	tram on time
Day 1	×		×
Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

In other words, on day 1, it was sunny and the tram was on time, and on day 5, it was sunny, but there were sirens, and the tram was not on time.

The goal is now to draw conclusions from such a formal context $\mathbb{K} = (G, M, I)$ of experiences. Roughly, we suppose that we are given an *observation* $P \subseteq M$. We then ask for some $m \in M$ whether in "almost all" experiences where P occurred, m occurred as well.

We now formalize this notion of saying that a set $Q \subseteq M$ of attributes "normally follows" from our observation P. For this, we shall introduce the notion of *confidence* $\operatorname{conf}_{\mathbb{K}}(A \to B)$ for implications $(A \to B) \in \operatorname{Imp}(M)$ in the formal context $\mathbb{K}$ as

$$\operatorname{conf}_{\mathbb{K}}(A \to B) = \begin{cases} 1 & A' = \varnothing \\ \frac{|(A \cup B)'|}{|A'|} & \text{otherwise.} \end{cases}$$

In other words, the confidence of the implication $A \to B$ is the relative amount of objects in $\mathbb{K}$ satisfying all attributes in A which also satisfy all attributes in B.

Using the notion of confidence, we say, for some fixed $c \in [0, 1]$, that P *normally (with threshold c) implies* Q in $\mathbb{K}$ if and only if

$$\operatorname{conf}_{\mathbb{K}}(P \to Q) \geqslant c.$$

Example 2. Let us consider Example 1 again, and let us choose $c = 0.8$. Then on sunny days our tram is normally on time, because

$$\operatorname{conf}_{\mathbb{K}_{\text{tram}}}(\{\text{ sunny }\} \to \{\text{ tram on time }\}) = \frac{4}{5}.$$

However, if we add the extra information that we heard some sirens, then it is not true that our tram will be normally on time, even if it is a sunny day, since

$$\operatorname{conf}_{\mathbb{K}_{\text{tram}}}(\{\text{ sunny, sirens }\} \to \{\text{ tram on time }\}) = \frac{1}{2}.$$

From this example, we already see that this kind of inference is non-monotonic. More precisely, it may happen for some sets $P, Q \subseteq M$ and $p \in M$ that

$$\operatorname{conf}_{\mathbb{K}}(P \to Q) \geqslant c$$
$$\operatorname{conf}_{\mathbb{K}}(P \cup \{ p \} \to Q) < c.$$

Furthermore, it is worthwhile to note that the notion of "normally implies" is not transitive in the usual sense.

Example 3. Let us consider the formal context

	a	b	d
1	×		
2	×	×	
3	×	×	×
4		×	×
5			×

and choose $c = \frac{2}{3}$. Then $\{ a \}$ normally implies $\{ b \}$, and $\{ b \}$ normally implies $\{ d \}$, but $\{ a \}$ does not normally imply $\{ d \}$. Even $\{ a, b \}$ does not normally imply $\{ d \}$.

On the other hand, it is easy to see that

$$\operatorname{conf}_{\mathbb{K}}(A \to C) = \operatorname{conf}_{\mathbb{K}}(A \to B) \cdot \operatorname{conf}_{\mathbb{K}}(B \to C)$$

for $A \subseteq B \subseteq C \subseteq M$.

4 Non-Monotonic Rules

Our approach described so far can be used to define a certain kind of non-monotonic rules. More precisely, let us call an implication $A \to B$ a *non-monotonic rule*, and let us say that this rule is *valid* in $\mathbb{K}$ with threshold c if and only if $\operatorname{conf}_{\mathbb{K}}(A \to B) \geqslant c$. Those rules capture all the knowledge we get from non-monotonic reasoning in the formal context $\mathbb{K}$ using the threshold c. The semantics of these rules is obviously model-based.

Let us denote with $R(\mathbb{K}, c)$ the set of all rules of $\mathbb{K}$ using c as threshold. As these rules enjoy a model-based semantics, we can defined the corresponding entailment operator $\models_c$ by

$$\mathcal{L} \models_c (A \to B) \iff (\forall \mathbb{K} \colon \mathcal{L} \subseteq R(\mathbb{K}, c) \implies (A \to B) \in R(\mathbb{K}, c)),$$

where the formal contexts all have the same set M of attributes.

It is quite easy to see that we do not need all such rules to still be able to do all the reasoning. If we have rules $A \to B$ and $A \to B \cup C$ which are both valid in $\mathbb{K}$ using c, then the latter suffices.

We denote maximal such sets with a special name. Let $P \subseteq M$. We call Q an *c-extension* of P in $\mathbb{K}$ if and only if Q is $\subseteq$-maximal with respect to

$$R(\mathbb{K}, c) \models_c (P \to Q).$$

It thus suffices to know all the rules

$$\{\, P \to Q \mid Q \text{ is a } c\text{-extension of } P \text{ in } \mathbb{K} \,\}.$$

It is also easy to see that for rules $P \to Q$ whose confidence is not 1 in $\mathbb{K}$, it is enough for the sets P and Q to be intents (note that extensions are always intents). In other words, the set

$$\{\, P \to P'' \mid P \subseteq M \,\} \cup \\ \{\, P \to Q \mid Q \text{ } c\text{-extension of } P, \operatorname{conf}_{\mathbb{K}}(P \to Q) \neq 1 \text{ and } P, Q \in \operatorname{Int}(\mathbb{K}) \,\}.$$

is complete for $R(\mathbb{K}, c)$. Of course, instead of $\{\, P \to P'' \mid P \subseteq M \,\}$, we could choose any base of $\operatorname{Th}(\mathbb{K})$.

Note that every set P has a c-extension, since $\operatorname{conf}_{\mathbb{K}}(P \to P) = 1$ and $\operatorname{conf}_{\mathbb{K}}(\cdot \to \cdot)$ is antitone in its second argument. More precisely, we have the following characterization of c-extensions.

Proposition 1. *Let $\mathbb{K} = (G, M, I)$ be a finite and non-empty formal context, and let $c \in [0, 1]$. Let $P \subseteq M$. Then a set Q is an c-extension of P in $\mathbb{K}$ if and only if Q is maximal with respect to $\operatorname{conf}_{\mathbb{K}}(P \to Q) \geqslant c$.*

But note that in contrast to the case $c = 1$, a set P can have multiple c-extensions if $c \neq 1$.

Example 4. Let us consider the formal context

	m	n
1	×	
2	×	×
3	×	×
4	×	×
5		×

and choose $c = 4/5$. Then the set $P = \varnothing$ has two c-extensions, namely $\{\, m \,\}$ and $\{\, n \,\}$.

It is quite easy to see that this example can be generalized to work for every value $c \in [0, 1)$ and for every number of c-extensions.

Also note that in contrast to the classical case, c-extensions are *not* closed under normal entailment.

Example 5. Consider the formal context from Example 4 again, and let $c = 3/4$. Then the c-extensions of $\varnothing$ are $\{ m \}, \{ n \}$, but both sets on their part have the set $\{ m, n \}$ as c-extension.

5 Conclusions and Future Research

We have presented a formalization of evidence based non-monotonic reasoning based on formal concept analysis. To this end, we have used formal contexts to model the set of experiences a person has. Using the notion of confidence, we were able to give a precise formulation of what it means that in "almost all" experiences obtained so far a certain conclusion was correct. Based on this, we have shown that this form of inference indeed yields a non-monotonic formalism.

Albeit the author is quite sure that this approach of combining formal concept analysis and non-monotonic reasoning is original, he is aware of the fact that the general idea underlying this approach is not new. It is thus one of the major next steps in investigating this approach to find connections to existing frameworks for non-monotonic reasoning. Moreover, our formalization yields a connection between formal concept analysis and non-monotonic reasoning. Thus, if we can find that our idea is similar to existing ones, it might be the case that methods from formal concept analysis could be helpful in solving tasks in these existing approaches. Conversely, it is possible that ideas and results from non-monotonic reasoning could be applied to issues of formal concept analysis.

Moreover, as we have already indicated in our considerations above, our formalization could yield a method which allows for the extraction of non-monotonic rules which could be used by other formalisms like default logic. In particular, we have given a first "base" of non-monotonic rules of a formal context, which however might be too large to be practically relevant. A smaller base, maybe comparable to the *canonical base* [2] known in formal concept analysis, maybe of practical interest.

Acknowledgments The author wants to thank both Bernhard Ganter and Gerd Brewka for their encouraging discussions on this topic.

References

[1] R. Agrawal, T. Imielinski, and A. Swami. "Mining Association Rules between Sets of Items in Large Databases". In: *Proceedings of the ACM SIGMOD International Conference on Management of Data.* 1993, pp. 207–216.

[2] Bernhard Ganter and Rudolf Wille. *Formal Concept Analysis: Mathematical Foundations.* Berlin-Heidelberg: Springer, 1999.

[3] John McCarthy. "Circumscription–A form of non-monotonic reasoning". In: *Artificial Intelligence* 13.1–2 (1980). Special Issue on Non-Monotonic Logic, pp. 27–39.

[4] R. Reiter. "A logic for default reasoning". In: *Artificial Intelligence* 13.1–2 (1980). Special Issue on Non-Monotonic Logic, pp. 81–132.