Descriptional Complexity of Grammars Regulated by Context Conditions

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Abstract. This paper improves several well-known results concerning the descriptional complexity of grammars regulated by context conditions. Specifically, it proves that every recursively enumerable language is generated (A) by a context-conditional grammar of degree $(2, 1)$ with no more than seven conditional productions and eight nonterminals, (B) by a generalized forbidding grammar of degree two with no more than eight conditional productions and ten nonterminals, or (C) by a simple semi-conditional grammar of degree $(2, 1)$ with no more than nine conditional productions and ten nonterminals.

Key words: formal languages, descriptional complexity, context-conditional grammars, generalized forbidding grammars, simple semi-conditional grammars

1 Introduction

Grammars with derivations regulated by various context conditions have always represented an important investigation area of language theory (see [1] for an overview). The present paper continues with this vivid topic of language theory by investigating their descriptional complexity. Specifically, it studies the descriptional complexity of context-conditional, generalized forbidding, and simple semi-conditional grammars.

Recall that every recursively enumerable language is generated (1) by a context-conditional grammar, (2) by a generalized forbidding grammar of degree two with no more than thirteen conditional productions and fifteen nonterminals (see [2]), or (3) by a simple semi-conditional grammar of degree $(2, 1)$ with no more than ten conditional productions and twelve nonterminals (see [3]). This paper improves these results. Specifically, it proves that every recursively enumerable language is generated (A) by a context-conditional grammar of degree $(2, 1)$ with no more than seven conditional productions and eight nonterminals, (B) by a generalized forbidding grammar of degree two with no more than eight conditional productions and ten nonterminals, or (C) by a simple semi-conditional grammar of degree $(2, 1)$ with no more than nine conditional productions and ten nonterminals. In fact, we establish all these results for grammars with context conditions represented by strings consisting solely of nonterminals as opposed to the previous results that allow terminals to appear in them as well.

2 Preliminaries

In this paper, we assume that the reader is familiar with the theory of formal languages (see [4, 5]). For an alphabet V, V^* represents the free monoid generated by V. The unit of V^* is denoted by ε. Set $V^+ = V^* - \{\varepsilon\}$. For $w \in V^*$, $|w|$ denotes the length of w. Set $sub(w) = \{u : u \text{ is a subword of } w\}$. For a finite subset $W \subseteq V^*$, $\max(W)$ is the minimal nonnegative integer n such that $|x| \leq n$, for all $x \in W$.

Recall that every recursively enumerable language is generated by a grammar, G_1, in the *Geffert normal form* (see [6]) of the form

$$G_1 = (\{S, A, B, C\}, T, P \cup \{ABC \to \varepsilon\}, S),$$

where P contains context-free productions of the form

$$\begin{array}{ll} S \to uSa, & \text{where } u \in \{A, AB\}^*,\ a \in T, \\ S \to uSv, & \text{where } u \in \{A, AB\}^*,\ v \in \{BC, C\}^*, \\ S \to uv, & \text{where } u \in \{A, AB\}^*,\ v \in \{BC, C\}^*. \end{array}$$

In addition, any derivation in G_1 generating a terminal string is of the form $S \Rightarrow^* w_1w_2w$ by productions from P, where $w_1 \in \{A, AB\}^*$, $w_2 \in \{BC, C\}^*$, $w \in T^*$, and $w_1w_2w \Rightarrow^* w$ by the production $ABC \to \varepsilon$.

3 Definitions

A *context-conditional grammar*, G, is a quadruple $G = (N, T, P, S)$, where N is a nonterminal alphabet, T is a terminal alphabet, $N \cap T = \emptyset$, $S \in N$ is the start symbol, and P is a finite set of productions of the form $(X \to \alpha, Per, For)$, where $X \in N$, $\alpha \in (N \cup T)^*$, and $Per, For \subseteq (N \cup T)^+$ are finite sets. If $Per \cup For \neq \emptyset$, then the production is said to be *conditional*. G has *degree* (i, j) if for all productions $(X \to \alpha, Per, For) \in P$, $\max(Per) \leq i$ and $\max(For) \leq j$. For $x \in (N \cup T)^+$ and $y \in (N \cup T)^*$, x *directly derives* y according to the production $(X \to \alpha, Per, For) \in P$, denoted by $x \Rightarrow y$, if $x = x_1Xx_2$, $y = x_1\alpha x_2$, for some $x_1, x_2 \in (N \cup T)^*$, $Per \subseteq sub(x)$ and $For \cap sub(x) = \emptyset$. As usual, $\Rightarrow$ is extended to $\Rightarrow^i$, for $i \geq 0$, $\Rightarrow^+$, and $\Rightarrow^*$. The language generated by a context-conditional grammar, G, is defined as $\mathcal{L}(G) = \{w \in T^* : S \Rightarrow^* w\}$.

Let $G = (N, T, P, S)$ be a context-conditional grammar. If $Per = \emptyset$, for all productions $(X \to \alpha, Per, For) \in P$, then G is said to be a *generalized forbidding grammar*. In this case, $(X \to \alpha, For)$ is written instead of $(X \to \alpha, \emptyset, For)$, and G is said to have degree i if G has degree $(0, i)$ as a context-conditional grammar.

A *simple semi-conditional grammar*, G, is a quadruple $G = (N, T, P, S)$, where N is a nonterminal alphabet, T is a terminal alphabet, $N \cap T = \emptyset$, $S \in N$ is the start symbol, and P is a finite set of productions of the form $(X \to \alpha, u, v)$, where $X \in N$, $\alpha \in (N \cup T)^*$, $u, v \in (N \cup T)^+ \cup \{0\}$, $0 \notin N \cup T$ is a special symbol, and $0 \in \{u, v\}$. If $u \neq 0$ or $v \neq 0$, then the production is said to be *conditional.* G has *degree* (i, j) if for all productions $(X \to \alpha, u, v) \in P$, $u \neq 0$ implies $|u| \leq i$ and $v \neq 0$ implies $|v| \leq j$. For $x \in (N \cup T)^+$ and $y \in (N \cup T)^*$, x *directly derives* y according to the production $(X \to \alpha, u, v) \in P$, denoted by $x \Rightarrow y$, if $x = x_1 X x_2$, $y = x_1 \alpha x_2$, for some $x_1, x_2 \in (N \cup T)^*$, and $u \neq 0$ implies that $u \in sub(x)$ and $v \neq 0$ implies that $v \notin sub(x)$. As in the previous definition, $\Rightarrow$ is extended to $\Rightarrow^i$, for $i \geq 0$, $\Rightarrow^+$, and $\Rightarrow^*$. The language generated by a simple semi-conditional grammar, G, is defined as $\mathcal{L}(G) = \{w \in T^* : S \Rightarrow^* w\}$.

4 Main Results

This section presents the main results of this paper.

4.1 Context-Conditional Grammars

Theorem 1. *Every recursively enumerable language is generated by a context-conditional grammar of degree* $(2, 1)$ *with no more than 7 conditional productions and 8 nonterminals.*

Proof. Let L be a recursively enumerable language. Then, there is a grammar $G_1 = (\{S, A, B, C\}, T, P \cup \{ABC \to \varepsilon\}, S)$ in the Geffert normal form such that $L = \mathcal{L}(G_1)$. Construct the grammar

$$G = (\{S, A, B, C, A', B', C', B''\}, T, P' \cup P'', S),$$

where

$$P' = \{(X \to \alpha, \emptyset, \emptyset) : X \to \alpha \in P\},$$

and P'' contains the following seven conditional productions:

1. $(A \to A', \emptyset, \{A', B''\})$,
2. $(B \to B', \emptyset, \{B', B''\})$,
3. $(C \to C', \emptyset, \{C', B''\})$,
4. $(B' \to B'', \{A'B', B'C'\}, \emptyset)$,
5. $(A' \to \varepsilon, \{B''\}, \emptyset)$,
6. $(C' \to \varepsilon, \{B''\}, \emptyset)$,
7. $(B'' \to \varepsilon, \emptyset, \{A', C'\})$.

To prove that $\mathcal{L}(G_1) \subseteq \mathcal{L}(G)$, consider a derivation $S \Rightarrow^* wABCw'v \Rightarrow ww'v$ in G_1 by productions from P and the only one production $ABC \to \varepsilon$, where $w, w' \in \{A, B, C\}^*$ and $v \in T^*$. Then, $S \Rightarrow^* wABCw'v$ in G by productions from P'. By productions 1, 2, 3, 4, 5, 6, and 7,

$$\begin{aligned} wABCw'v &\Rightarrow wA'BCw'v \\ &\Rightarrow wA'B'Cw'v \\ &\Rightarrow wA'B'C'w'v \\ &\Rightarrow wA'B''C'w'v \\ &\Rightarrow wB''C'w'v \\ &\Rightarrow wB''w'v \\ &\Rightarrow ww'v. \end{aligned}$$

The inclusion follows by induction.

To prove that $\mathcal{L}(G_1) \supseteq \mathcal{L}(G)$, consider a terminal derivation. Notice that to eliminate a nonterminal, there is B'' in the derivation. From production 4 and the observation that there is no more than one A', B', C' in the derivation (see productions 1, 2, 3), there cannot be a terminal between any two nonterminals. Therefore, the derivation is of the form $S \Rightarrow^* w_1w_2w_3$ in G by productions from P', where $w_1 \in \{A, AB\}^*$, $w_2 \in \{BC, C\}^*$, and $w_3 \in T^*$, and $w_1w_2w_3 \Rightarrow^* w_3$. Note that before S is eliminated, there is no occurrence of the substring ABC in the derivation. Then, $S \Rightarrow^* w_1w_2w_3$ in G_1 by productions from P. We prove that $w_1w_2w_3 \Rightarrow^* w_3$ in G_1.

For $w_1w_2 = \varepsilon$, the proof is done. For $w_1w_2 \neq \varepsilon$, there is B in w_1w_2; otherwise, B'' cannot be obtained and no nonterminal can be eliminated. To obtain B'', production 4 is applied. Therefore, $w_1w_2 = wABCw'$, where $w \in \{A, AB\}^*$ and $w' \in \{BC, C\}^*$; otherwise, the conditions of production 4 are not met. Thus, at the beginning, only productions 1, 2, and 3 are applicable. Then, only production 4 is applicable, and, after that, only productions 5 and 6 are applicable. Finally, only production 7 is applicable;

$$wABCw'w_3 \Rightarrow^3 wA'B'C'w'w_3 \Rightarrow wA'B''C'w'w_3 \Rightarrow^2 wB''w'w_3 \Rightarrow ww'w_3.$$

Thus, if $S \Rightarrow^* w_1w_2w_3 \Rightarrow^* w_3$ in G, where $w_1 \in \{A, AB\}^*$, $w_2 \in \{BC, C\}^*$, and $w_3 \in T^*$, then $S \Rightarrow^* w_1w_2w_3 \Rightarrow^* w_3$ in G_1. □

4.2 Generalized Forbidding Grammars

Theorem 2. *Every recursively enumerable language is generated by a generalized forbidding grammar of degree 2 with no more than 8 conditional productions and 10 nonterminals.*

Proof. Let L be a recursively enumerable language. Then, there is a grammar $G_1 = (\{S, A, B, C\}, T, P \cup \{ABC \to \varepsilon\}, S)$ in the Geffert normal form such that $L = \mathcal{L}(G_1)$. Construct the grammar

$$G = (\{S, S', Z, A, B, C, A', B', C', \#\}, T, P' \cup P'', S),$$

where P' contains productions of the form

$(S \to ZS'Z, \emptyset)$,
$(S' \to uS'ZaZ, \emptyset)$ if $S \to uSa \in P$, $u \in \{A, AB\}^*$, $a \in T$,
$(S' \to uS'v, \emptyset)$ if $S \to uSv \in P$, $u \in \{A, AB\}^*$, $v \in \{B, BC\}^*$,
$(S' \to uv, \emptyset)$ if $S \to uv \in P$, $u \in \{A, AB\}^*$, $v \in \{B, BC\}^*$,

and P'' contains the following eight conditional productions:

1. $(A \to \#A', \{\#, S'\})$,
2. $(B \to B', \{B', \#, S'\})$,
3. $(C \to C', \{C', \#, S'\})$,
4. $(A' \to \varepsilon, \{A'\}\{A, B, C, C', Z\})$,
5. $(B' \to \varepsilon, \{B'\}\{A, B, C, Z\} \cup \{A, B, C, C', Z\}\{B'\})$,
6. $(C' \to \varepsilon, \{A', B'\} \cup \{A, B, C, Z\}\{C'\})$,
7. $(\# \to \varepsilon, \{A', B', C'\})$,
8. $(Z \to \varepsilon, \{S', A, A', B, B', C, C'\})$.

To prove that $\mathcal{L}(G_1) \subseteq \mathcal{L}(G)$, consider a derivation $S \Rightarrow^* wABCw'v \Rightarrow ww'v$ in G_1 by productions from P and the only one application of the production $ABC \to \varepsilon$, where $w, w' \in \{A, B, C\}^*$ and $v \in T^*$. Then, $S \Rightarrow^* ZwABCw'Zv'$ in G by productions from P', where $v' \in (T \cup \{Z\})^*$ is such that $h(v') = v$, for a homomorphism $h : (T \cup \{Z\})^* \to T^*$ defined as $h(a) = a$, for $a \in T$, and $h(Z) = \varepsilon$. By productions 3, 2, 1, 4, 5, 6, and 7,

$$\begin{aligned}
ZwABCw'Zv' &\Rightarrow ZwABC'w'Zv' \\
&\Rightarrow ZwAB'C'w'Zv' \\
&\Rightarrow Zw\#A'B'C'w'Zv' \\
&\Rightarrow Zw\#B'C'w'Zv' \\
&\Rightarrow Zw\#C'w'Zv' \\
&\Rightarrow Zw\#w'Zv' \\
&\Rightarrow Zww'Zv'.
\end{aligned}$$

The inclusion follows by induction and, eventually, by production 8.

To prove that $\mathcal{L}(G_1) \supseteq \mathcal{L}(G)$, observe that if there is a string of the form $Z\{B', C'\}$ as a substring of a sentential form, then neither of productions 5 and 6 is applicable to the rightmost nonterminal of this string—there is Z before the nonterminal. Thus, we can assume that $S \Rightarrow^* Zw_1w_2Zw_3$ in G, by productions from P', and that $Zw_1w_2Zw_3 \Rightarrow^* h(w_3)$, where $w_1 \in \{A, AB\}^*$, $w_2 \in \{BC, C\}^*$, and $w_3 \in (T \cup \{Z\})^*$. Notice that before S and S' are eliminated, there is no occurrence of ABC in the sentential form (see [6]), and, moreover, no production from P'' can be applied. Then, $S \Rightarrow^* w_1w_2h(w_3)$ in G_1 by productions from P. We prove that $w_1w_2h(w_3) \Rightarrow^* h(w_3)$.

By induction on the length of w_1w_2, we prove that $w_1w_2 = w_1'ABCw_2'$, for some $w_1' \in \{A, AB\}^*$ and $w_2' \in \{BC, C\}^*$, or $w_1w_2 = \varepsilon$. In any derivation step, there is no more than one A', B', C', and no X', for $X \in \{A, B, C\}$, is generated while there is $\#$ in the sentential form (see productions 1, 2, 3). Moreover, $\#$ is eliminated after all primed nonterminals are eliminated (see production 7). We prove that A, B, and C are in $sub(w_1w_2)$, for $w_1w_2 \neq \varepsilon$.

1. $A \in sub(w_1w_2)$: to eliminate A, A has to be rewritten to A'. Then, B' has to follow A' (by production 4) and C' has to follow B' (by production 5).
2. $B \in sub(w_1w_2)$: to eliminate B, B has to be rewritten to B'. Then, A' or $\#$ has to be before B' and C' has to follow B' (by production 5).

3. $C \in sub(w_1w_2)$: to eliminate C, C has to be rewritten to C'. Then, # has to be before C' (by production 6)—that is, $A \in sub(w_1w_2)$; otherwise, this case is analogical to 1.

In all above cases, $ABC \in sub(w_1w_2)$. Thus, $w_1w_2 = w_1'ABCw_2'$, for some $w_1' \in \{A, AB\}^*$ and $w_2' \in \{BC, C\}^*$.

We prove that while ABC is eliminated, no other nonterminal is eliminated, and then # is removed.

First, only productions 1, 2, and 3 are applicable.

(i) If production 1 is applied, then productions 2 and 3 are not applicable because there is # in the sentential form. Also, production 4 is not applicable because A' is followed by A, B, C, or Z. Thus, the derivation is blocked.

(ii) Assume that production 2 is applied first. Then, there is B' in the sentential form. Notice that production 5 is not applicable because B' is followed by A, B, C, or Z. Thus, only productions 1 and 3 are applicable. To apply production 5, # or A' has to be before B' and C' has to follow B'. If production 1 is applied, then production 3 is not applicable—C' cannot be generated. Moreover, if there is $\#A'B'\{A, B, C, Z\}$ as a substring of the sentential form, then A' can be eliminated (by production 4). However, no other production is applicable. Thus, the sequence of productions in the derivation is 2, 3, and 1.

(iii) Assume that production 3 is applied first. Then, there is C' in the sentential form. Notice that production 6 is not applicable because A, B, C, or Z is before C'. To apply production 6, # has to be before C'. Thus, only productions 1 and 2 are applicable. If production 1 is applied, then production 2 is not applicable. To eliminate A', A' has to be followed by B' (see production 4)—a contradiction; there is no B' in the sentential form. Therefore, production 2 had to be applied before production 1. Thus, the sequence of productions in the derivation is 3, 2, and 1.

After the sequence of productions 2, 3, 1, or 3, 2, 1, productions 4 and 5 are applicable if and only if $\#A'B'C'$ is a substring of the sentential form (see productions 4 and 5). Notice that no other productions are applicable. Thus,

$$w_1'ABCw_2'h(w_3) \Rightarrow^2 w_1'AB'C'w_2'h(w_3) \Rightarrow w_1'\#A'B'C'w_2'h(w_3).$$

After the application of productions 4 and 5 (in this order, otherwise A' cannot be eliminated),

$$w_1'\#A'B'C'w_2'h(w_3) \Rightarrow w_1'\#B'C'w_2'h(w_3) \Rightarrow w_1'\#C'w_2'h(w_3),$$

only production 6 is applicable,

$$w_1'\#C'w_2'h(w_3) \Rightarrow w_1'\#w_2'h(w_3).$$

If $w_1'w_2' \neq \varepsilon$, then only production 7 is applicable because there is no A', B', C' in the sentential form. If $w_1'w_2' = \varepsilon$, then also production 8 is applicable. However, it is easy to see that it does not matter whether some Zs are eliminated before # is removed. Then,

$$w_1'\#w_2'h(w_3) \Rightarrow^+ w_1'w_2'h(w_3).$$

As a result, by the induction hypothesis,

$$w'_1 ABC w'_2 h(w_3) \Rightarrow^* w'_1 w'_2 h(w_3) \Rightarrow^* h(w_3).$$

Thus, if $S \Rightarrow^* Zw_1w_2Zw_3 \Rightarrow^* h(w_3)$ in G, where $w_1 \in \{A, AB\}^*$, $w_2 \in \{BC, C\}^*$, and $w_3 \in (T \cup \{Z\})^*$, then $S \Rightarrow^* w_1w_2h(w_3) \Rightarrow^* h(w_3)$ in G_1. Hence, the other inclusion holds. □

4.3 Simple Semi-Conditional Grammars

Theorem 3. *Every recursively enumerable language is generated by a simple semi-conditional grammar of degree* $(2, 1)$ *with no more than 9 conditional productions and 10 nonterminals.*

Proof. Let L be a recursively enumerable language. Then, there is a grammar $G_1 = (\{S, A, B, C\}, T, P \cup \{ABC \to \varepsilon\}, S)$ in the Geffert normal form such that $L = \mathcal{L}(G_1)$. Construct the grammar

$$G = (\{S, A, B, C, \#, B', C', \$, B'', C''\}, T, P' \cup P'', S),$$

where

$$P' = \{(S \to \alpha, 0, 0) : S \to \alpha \in P\}$$

and P'' contains the following nine conditional productions:

1. $(A \to \#, 0, \#)$,
2. $(B \to B', 0, B')$,
3. $(C \to C', 0, C')$,
4. $(B' \to B'', \#B', 0)$,
5. $(C' \to C'', B''C', 0)$,
6. $(B'' \to \varepsilon, B''C'', 0)$,
7. $(\# \to \$, \#C'', 0)$,
8. $(C'' \to \varepsilon, \$, 0)$,
9. $(\$ \to \varepsilon, 0, C'')$.

To prove that $\mathcal{L}(G_1) \subseteq \mathcal{L}(G)$, consider a derivation $S \Rightarrow^* wABCw'v \Rightarrow ww'v$ in G_1 by productions from P with only one application of the production $ABC \to \varepsilon$, where $w, w' \in \{A, B, C\}^*$ and $v \in T^*$. Then, $S \Rightarrow^* wABCw'v$ in G by productions from P. By productions 3, 2, 1, 4, 5, 6, 7, 8, and 9,

$$\begin{aligned}
wABCw'v &\Rightarrow wABC'w'v \\
&\Rightarrow wAB'C'w'v \\
&\Rightarrow w\#B'C'w'v \\
&\Rightarrow w\#B''C'w'v \\
&\Rightarrow w\#B''C''w'v \\
&\Rightarrow w\#C''w'v \\
&\Rightarrow w\$C''w'v \\
&\Rightarrow w\$w'v \\
&\Rightarrow ww'v.
\end{aligned}$$

The inclusion follows by induction.

To prove that $\mathcal{L}(G_1) \supseteq \mathcal{L}(G)$, consider a terminal derivation. Let $X \in \{A, B, C\}$ be in a sentential form. To eliminate X, there are following three possibilities:

1. if $X = A$, then there has to be C (by production 7) and B (by production 5) in the sentential form;
2. if $X = B$, then there has to be A (by production 4) and C (by production 6) in the sentential form;
3. if $X = C$, then there has to be B (by production 5) and A (by production 8) in the sentential form.

In all above cases, there are A, B, and C in the sentential form. By productions 1, 2, and 3, there can be no more than one $\#$, B', and C' in the sentential form. By productions 4 and 5, $\#$ is before B' and C' follows this B'. We prove that in any terminal derivation, there is no terminal symbol between any two nonterminals. More precisely, there is no substring of the form $T\{BC, C\}$. Assume that aB, for some $a \in T$, is a substring of the sentential form. Then, B is rewritten to B' and B' cannot be rewritten to B'' because $\#$ is before aB'. Similarly, if there is aC in the sentential form, for some $a \in T$, then C is rewritten to C' and aC' cannot be rewritten to aC'' because there is never B'' followed by C'. Thus, any terminal derivation in G is of the form

(*) $\quad S \Rightarrow^* w_1 \# w_2 B' w_3 C' w_4 w$

by productions from P and productions 1, 2, 3, and

$$\Rightarrow^* w$$

where $w_1 \in \{A, B\}^*$, $w_2, w_3 \in \{A, B, C, S\}^*$, $w_4 \in \{B, C\}^*$, and $w \in T^*$. We prove that $S \notin sub(w_2 w_3)$. To rewrite B' (by production 4), $w_2 = \varepsilon$. Thus,

(**) $\quad w_1 \# B' w_3 C' w_4 w \Rightarrow w_1 \# B'' w_3 C' w_4 w$

and, also, production 2 is applicable. However, to rewrite C' (by production 5), $w_3 = \varepsilon$. Thus,

$$\Rightarrow^+ w_1 \# B'' C'' w_4 w$$

where $w_1 \in \{A, B, B'\}^*$, $w_4 \in \{B, B', C\}^*$. Thus, $\#B'C'$ is a substring of $w_1 \# w_2 B' w_3 C' w_4 w$, and $\#B'C'$ was obtained from ABC.

Next, we prove that no other nonterminal is eliminated while ABC is eliminated. Besides a possible application of productions 2 and 3, only production 6 is applicable. Thus,

$$\Rightarrow^+ w_1 \# C'' w_4 w$$

where $w_1 \in \{A, B, B'\}^*$, $w_4 \in \{B, B', C, C'\}^*$. Besides a possible application of productions 2 and 3, only production 7 is applicable. Thus,

$$\Rightarrow^+ w_1 \$ C'' w_4 w$$

where $w_1 \in \{A, B, B'\}^*$, $w_4 \in \{B, B', C, C'\}^*$. Besides a possible application of productions 1, 2, 3, and 4, only production 8 is applicable. Thus,

$$\Rightarrow^+ w_1 \$ w_4 w$$

where $w_1 \in \{A, \#, \#B'', B, B'\}^*$, $w_4 \in \{B, B', C, C'\}^*$. Besides a possible application of productions 1, 2, 3, and 4, only production 9 is applicable. Thus,

$$\Rightarrow^+ w_1 w_4 w$$

where $w_1 \in \{A, \#, \#B'', B, B'\}^*$, $w_4 \in \{B, B', C, C'\}^*$. Thus,

$$\Rightarrow^* uvw$$

by productions 1, 2, and 3, if they are applicable. Then,

$$uvw \in \{u_1 \# B' C' u_4 w : u_1 \in \{A, B\}^*, u_4 \in \{B, C\}^*\}$$

$$\cup \{v_1 \# B'' C' v_4 w : v_1 \in \{A, B, B'\}^*, v_4 \in \{B, B', C\}^*\}$$

or $uv = \varepsilon$. Thus, the string ABC, and only the string, was eliminated. By induction (see (*) and (**)), the inclusion holds. □

Can the results achieved in this paper be established for fewer nonterminals or conditionals productions with the same (or even less) degree?

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