Technische Universität Dresden

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Introduction to Formal Concept Analysis
Exercise Sheet 1, Winter Semester 2017/18

1 Set Theory

Exercise 1 (a piece of recapitulation)

Given the following hints and the universe $M := \{1, 2, 3, 4, 5, 6, 7, 8\}$, compute the sets A, B, C:

(a) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

(b) $B \cup C = \{1, 2, 4, 6, 8\}$

(c) $A \cup C = \{1, 2, 3, 4, 5, 7, 8\}$

(d) $A \cap B = \{2\}$

(e) $B \cap C = \{2, 4, 8\}$

(f) $A \cap C = \{2\}$

2 Logic

Exercise 2 (repetition first-order logic)

Formalize the following statements for natural numbers a, b, c, using only multiplication ("·"), equality ("=") and natural numbers ("0","1","2",...) besides the usual logical symbols ("¬", "∧", "∨", "→", "↔", "∀", "∃", variables and parentheses):

(i) a divides b.

(ii) a is odd.

(iii) a is common divisor of b and c

(iv) a is the gcd of b and c.

(v) a is a square number.

(vi) a is a prime number.

3 Derivation Operators and Formal Concepts

Exercise 3 (line diagram)

a) Recall: how is the derivation operator $(\cdot)'$ defined?

b) Let $\mathbb{K} = (G, M, I)$ be a formal context and let $A, B \subseteq G$. Prove the following statements:

1. $A \subseteq B$ implies $B' \subseteq A'$

2. $A \subseteq A''$

3. $A' = A'''$

4. For $C \in G$ and $D \in M$ holds: (C, D) is a formal concept if and only if there is some $E \subseteq G$ such that $C = E''$ and $D = E'$.

4 Formal Concept Analysis

Exercise 4 (Formal Context)

Regard the following formal context $\mathbb{K}$, given as a cross table:

	needs water to live	lives in water	lives on land	needs chlorophyll to produce food	two seed leaves	one seed leaf	can move around	has limbs	suckles its offspring
Leech	×	×					×		
Bream	×	×					×	×	
Frog	×	×	×				×	×	×
Spike-Weed	×	×		×		×			
Reed	×	×	×	×		×			
Bean	×		×	×	×				
Maize	×		×	×		×			

a) Specify the following sets:

    (*i*) {Bean}′

   (*ii*) {lives on land}′

  (*iii*) {two seed leaves}″

  (*iv*) {Frog, Maize}′

   (*v*) {needs chlorophyll to produce food, can move around}′

  (*vi*) {lives in water, lives on land}′

 (*vii*) {needs chlorophyll to produce food, can move around}″

b) Extend $\mathbb{K}$ with both an object and an attribute.