Technische Universität Dresden

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Introduction to Formal Concept Analysis
Exercise Sheet 9, Winter Semester 2017/18

Exercise 1 (attribute exploration)

This is a theme from elementary geometry: discussing *pairs of squares*. Here are two possible such pairs:

 and

The two squares on the left *overlap*, because they have an inner point in common. They are also *parallel* in the sense that each side of one square is parallel to some side of the other. The two squares in the second diagram do not have these properties. Instead, they have a *common vertex* (but no *common edge*, not even a *common segment* of an edge). They are not *disjoint*, since they share a point.

We have collected a small list consisting of six attributes,

overlap, *parallel*, *disjoint*, *common vertex*, *common edge*, *common segment*,

and we have seen an object (a pair of squares) with the attribute combination

overlap, *parallel*,

and another one with the attribute "combination"

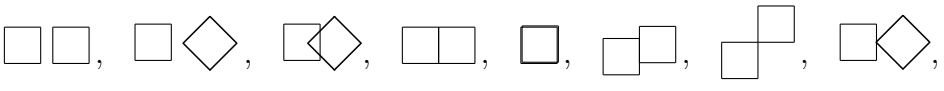
common vertex.

You may wonder which other attribute combinations are possible. Some, like

overlap, *parallel*, *disjoint*, *common vertex*,

are obviously not.

The idea of going through all possible cases is not very inviting. You would have to check 2^n cases for n attributes, which makes already 64 for our toy example. What one usually does is to come up with more examples, like



hoping to have intuitively included all possibilities.

Use attribute exploration, starting with a formal context containing all the above examples as objects and perform attribute exploration to find out which implications hold among the six attributes and find an example set which refutes all other implications.

Solution:

considered set A	closed? $A = A^{JJ}$?	general? $A^{JJ} = A^{II}$?	action	new object or implication
$\emptyset$	yes		next q	
{ce}	no	yes	new imp	ce $\rightarrow$ pa, cv, cs
{cs}	no	yes	new imp	cs $\rightarrow$ pa
{cv}	yes		next q	
{pa}	yes		next q	
{pa, cs}	yes		next q	
{pa, cv}	yes		next q	
{pa, cv, cs}	no	yes	new imp	pa, cv, cs $\rightarrow$ ce
{pa, cv, cs, ce}	yes		next q	
{ov}	yes		next q	
{ov, cv}	no	no	new obj	
{ov, cv}	yes		next q	
{ov, pa}	yes		next q	
{ov, pa, cs}	no	no	new obj	
{ov, pa, cs}	yes		next q	
{ov, pa, cv}	no	yes	new imp	ov, pa, cv $\rightarrow$ cs, ce
{ov, pa, cv, cs, ce}	yes		next q	
{di}	yes		next q	
{di, cv}	no	yes	new imp	di, cv $\rightarrow$ *all*
{di, pa}	yes		next q	
{di, pa, cs}	no	yes	new imp	di, pa, cs $\rightarrow$ *all*
{di, ov}	no	yes	new imp	di, ov $\rightarrow$ *all*
M	yes		end.	