Exercise Sheet 1: Getting to Know Graphs

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Exercise 1.1. Show that the number of vertices of odd degree is even in every simple graph.

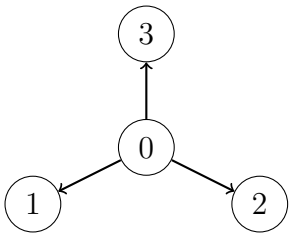
Exercise 1.2. Write a program that reads a directed graph from a file in the format of Exercise 0.3 (data available online[1]) and prints out the graph in METIS graph format:

$$\begin{array}{ccccc} n & m & & & \\ v_1 & n_1^1 & n_1^2 & \cdots & n_1^{d_1} \\ v_2 & n_2^1 & n_2^2 & \cdots & n_2^{d_1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ v_n & n_n^1 & n_n^2 & \cdots & n_n^{d_1} \end{array}$$

The first line consists of two integers n and m, separated by a space, where n is the number of vertices, and m is the total number of edges. Each of the following lines specifies the neighbours $n_i^1, n_i^2, \ldots, n_i^{d_i}$ of vertex v_i.

As an example, the directed star S_3 would be encoded as:

```
4 3
0 1 2 3
```



Exercise 1.3. A triangle in a directed graph is a simple directed path $v_1 \xrightarrow{e_1} v_2 \xrightarrow{e_2} v_3 \xrightarrow{e_3} v_1$.

Write a program that reads a directed graph G from a file in the format of Exercise 0.3 (data available online[1]) and prints out the number of triangles in G. How does the runtime of your program scale with the size of the input graph?

∗ **Exercise 1.4.** Let $G = \langle V, E \rangle$ be an undirected graph. Show that if G is triangle-free (i.e., there are no triangles in G), then

$$|E| \leq \left\lfloor \frac{|V|^2}{4} \right\rfloor .$$

[1] https://github.com/knowsys/Course-Knowledge-Graphs/tree/master/data/simple-graphs