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# Relative Observability in Coordination Control

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***Abstract*— Relative observability was introduced and studied in the framework of partially observed discrete-event systems as a condition stronger than observability and weaker than normality. Unlike observability, relative observability is closed under language unions, which makes it interesting for practical applications. In this paper, we investigate this notion in the framework of coordination control. We prove that conditional normality is stronger than conditional relative observability, hence it can be used in coordination control instead of conditional normality. We present a distributive procedure to compute a conditionally controllable and conditionally observable sublanguage of the specification that contains the supremal conditionally relative observable sublanguage.**

## I. Introduction

Supervisory control theory of discrete-event systems was developed in [10] as a formal approach to solve the safety issue. Coordination control was proposed for modular discrete-event systems in [9] as a trade-off between a purely modular control synthesis, which is in many cases unrealistic, and a global control synthesis, which is naturally prohibitive for complexity reasons. The idea is to compute a coordinator that takes care of the communication between subsystems. This approach was further developed in [6], [8]. In [6], a procedure for the distributive computation of the supremal conditionally-controllable sublanguages (the necessary and sufficient condition for the existence of a solution) of prefix-closed specifications and controllers with complete observations was proposed. The approach was later extended to non-prefix-closed specifications in [8], and for partial observations in [4].

Relative observability was introduced and studied in [1] in the framework of partially observed discrete-event systems as a condition stronger than observability and weaker than normality. It was shown to be closed under language unions, which makes it an interesting notion that can replace normality in practical applications.

In this paper, we study relative observability in the coordination control framework. We introduce and discuss the concept of *conditional relative observability* in the coordination control framework and show that it is closed under language unions. We prove that the previously defined notion of conditional normality [4] implies conditional relative observability, which means that conditional relative observability can be used in coordination control with partial observations instead of conditional normality. We present a distributive/parallel procedure to compute a conditionally controllable and conditionally observable sublanguage of the specification that contains the supremal conditionally relative observable sublanguage.

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## II. Preliminaries

We briefly recall the basic elements of supervisory control theory. The reader is referred to [2] for more details. Let $\Sigma$ be a finite nonempty set of *events*, and let $\Sigma^*$ denote the set of all finite words over $\Sigma$. The *empty word* is denoted by $\varepsilon$.

A *generator* is a quadruple $G = (Q, \Sigma, f, q_0)$, where $Q$ is a finite nonempty set of *states*, $\Sigma$ is an *event set*, $f : Q \times \Sigma \to Q$ is a *partial transition function*, and $q_0 \in Q$ is the *initial state*. As usual, the transition function is extended to the domain $Q \times \Sigma^*$ by induction. The language *generated* by $G$ is the set $L(G) = \{s \in \Sigma^* \mid f(q_0, s) \in Q\}$.

A *language* $L$ over an event set $\Sigma$ is a set $L \subseteq \Sigma^*$ such that there exists a generator $G$ with $L(G) = L$.

A *(natural) projection* $P : \Sigma^* \to \Sigma_o^*$, for some $\Sigma_o \subseteq \Sigma$, is a homomorphism defined so that $P(a) = \varepsilon$, for $a \in \Sigma \setminus \Sigma_o$, and $P(a) = a$, for $a \in \Sigma_o$. The *inverse image* of $P$, denoted by $P^{-1} : \Sigma_o^* \to 2^{\Sigma^*}$, is defined as $P^{-1}(s) = \{w \in \Sigma^* \mid P(w) = s\}$. The definition is naturally extended to languages. The *projection of a generator* $G$ is a generator $P(G)$ whose behavior satisfies $L(P(G)) = P(L(G))$.

A *controlled generator* is a structure $(G, \Sigma_c, P, \Gamma)$, where $G$ is a generator over $\Sigma$, $\Sigma_c \subseteq \Sigma$ is the set of *controllable events*, $\Sigma_u = \Sigma \setminus \Sigma_c$ is the set of *uncontrollable events*, $P : \Sigma^* \to \Sigma_o^*$ is the projection to the set of *observable events*, and $\Gamma = \{\gamma \subseteq \Sigma \mid \Sigma_u \subseteq \gamma\}$ is the *set of control patterns*. A *supervisor* for the controlled generator $(G, \Sigma_c, P, \Gamma)$ is a map $S : P(L(G)) \to \Gamma$. A *closed-loop system* associated with the controlled generator $(G, \Sigma_c, P, \Gamma)$ and the supervisor $S$ is defined as the smallest language $L(S/G) \subseteq \Sigma^*$ such that

1) $\varepsilon \in L(S/G)$ and
2) if $s \in L(S/G)$, $sa \in L(G)$, and $a \in S(P(s))$, then also $sa \in L(S/G)$.

Let $G$ be a generator over an event set $\Sigma$, and let $K \subseteq L(G)$ be a specification (a language). The aim of supervisory control theory is to find a supervisor $S$ such that $L(S/G) = K$. Such a supervisor exists if and only if $K$ is

1) *controllable* with respect to $L(G)$ and $\Sigma_u$, that is, $K\Sigma_u \cap L(G) \subseteq K$ and
2) *observable* with respect to $L(G)$, $\Sigma_o$, and $\Sigma_c$, that is, for all words $s, s' \in \Sigma^*$ such that $Q(s) = Q(s')$, for a

projection $Q : \Sigma^* \to \Sigma_o^*$, it holds that, for all $\sigma \in \Sigma$, if $s\sigma \in K$, $s' \in K$, and $s'\sigma \in L(G)$, then $s'\sigma \in K$.

Note that it is sufficient to consider $\sigma \in \Sigma_c$ in the definition of observability, since for $\sigma \in \Sigma_u$ the condition follows from controllability, cf. [2].

The parallel composition of two languages $L_1 \subseteq \Sigma_1^*$ and $L_2 \subseteq \Sigma_2^*$ is defined by

$$L_1 \parallel L_2 = P_1^{-1}(L_1) \cap P_2^{-1}(L_2) \subseteq \Sigma^*$$

where $P_i : \Sigma^* \to \Sigma_i^*$, for $i = 1, 2$, are projections to local event sets. In terms of generators, $L(G_1 \parallel G_2) = L(G_1) \parallel L(G_2)$, see [2].

## III. Coordination Control Framework

A language $K \subseteq (\Sigma_1 \cup \Sigma_2)^*$ is *conditionally decomposable* with respect to event sets $\Sigma_1$, $\Sigma_2$, and $\Sigma_k$, where $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_k$, if

$$K = P_{1+k}(K) \parallel P_{2+k}(K)$$

where $P_{i+k} : (\Sigma_1 \cup \Sigma_2)^* \to (\Sigma_i \cup \Sigma_k)^*$ is a projection, for $i = 1, 2$. Note that $\Sigma_k$ can always be extended in polynomial time [5] so that $K$ becomes conditionally decomposable, while to find the minimal extension with respect to set inclusion is NP-hard [8].

Now we recall the coordination control problem that is discussed in this paper.

*Problem 1:* Consider generators $G_1$ and $G_2$ over the event sets $\Sigma_1$ and $\Sigma_2$, respectively, and a generator $G_k$ (called a *coordinator*) over the event set $\Sigma_k$ satisfying the inclusions $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_k \subseteq \Sigma_1 \cup \Sigma_2$. Let $K \subseteq L(G_1 \parallel G_2 \parallel G_k)$ be a specification language. Assume that $K$ is conditionally decomposable with respect to $\Sigma_1$, $\Sigma_2$, and $\Sigma_k$. The aim of coordination control is to determine supervisors $S_1$, $S_2$, and $S_k$ such that $L(S_k/G_k) \subseteq P_k(K)$ and $L(S_i/[G_i \parallel (S_k/G_k)]) \subseteq P_{i+k}(K)$ for $i = 1, 2$, and

$$L(S_1/[G_1 \parallel (S_k/G_k)]) \parallel L(S_2/[G_2 \parallel (S_k/G_k)]) = K\,.$$

◇

One possible way to construct a coordinator is to set

$$G_k = P_k(G_1) \parallel P_k(G_2)$$

cf. [6], [8] for more details. An advantage of this construction is that the coordinator does not affect the system, that is,

$$L(G_1 \parallel G_2 \parallel G_k) = L(G_1 \parallel G_2)\,.$$

The notion of conditional controllability introduced in [9] and further studied in [6], [8] plays the central role in coordination control.

Let $G_1$ and $G_2$ be generators over the event sets $\Sigma_1$ and $\Sigma_2$, respectively, and let $G_k$ be a coordinator over the event set $\Sigma_k$. Let $P_k : \Sigma^* \to \Sigma_k^*$ and $P_{i+k} : \Sigma^* \to (\Sigma_i \cup \Sigma_k)^*$ be projections. Let $\Sigma_{i,u} = \Sigma_i \cap \Sigma_u$ denote the set of uncontrollable events of the event set $\Sigma_i$. A language $K \subseteq L(G_1 \parallel G_2 \parallel G_k)$ is *conditionally controllable* with respect to generators $G_1$, $G_2$, $G_k$ and uncontrollable event sets $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$ if

1) $P_k(K)$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$ and
2) $P_{i+k}(K)$ is controllable with respect to $L(G_i) \parallel P_k(K)$ and $\Sigma_{i+k,u}$, for $i = 1, 2$, where $\Sigma_{i+k,u} = (\Sigma_i \cup \Sigma_k) \cap \Sigma_u$.

The supremal conditionally controllable sublanguage always exists and equals to the union of all conditionally controllable sublanguages [8].

For coordination control, the notion of conditional observability is of the same importance as observability for supervisory control theory.

Let $G_1$ and $G_2$ be generators over the event sets $\Sigma_1$ and $\Sigma_2$, respectively, and let $G_k$ be a coordinator over $\Sigma_k$. A language $K \subseteq L(G_1 \parallel G_2 \parallel G_k)$ is *conditionally observable* with respect to generators $G_1, G_2, G_k$, controllable sets $\Sigma_{1,c}$, $\Sigma_{2,c}$, $\Sigma_{k,c}$, and projections $Q_{1+k}$, $Q_{2+k}$, $Q_k$, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, if

1) $P_k(K)$ is observable with respect to $L(G_k)$, $\Sigma_{k,c}$, and $Q_k$, and
2) $P_{i+k}(K)$ is observable with respect to $L(G_i) \parallel P_k(K)$, $\Sigma_{i+k,c}$, and $Q_{i+k}$, for $i = 1, 2$, where $\Sigma_{i+k,c} = \Sigma_c \cap (\Sigma_i \cup \Sigma_k)$.

*Theorem 2 ([4]):* Consider the setting of Problem 1. Then there exist the required supervisors $S_1$, $S_2$, $S_k$ if and only if the specification $K$ is

1) conditionally controllable with respect to $G_1$, $G_2$, $G_k$ and $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$ and
2) conditionally observable with respect to $G_1$, $G_2$, $G_k$, event sets $\Sigma_{1,c}$, $\Sigma_{2,c}$, $\Sigma_{k,c}$, and projections $Q_{1+k}$, $Q_{2+k}$, $Q_k$ from $\Sigma_i^*$ to $\Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$. ■

## IV. Conditional Relative Observability

As mentioned above, relative observability ($C$-observability) was introduced and studied in [1] as a weaker condition than normality, but stronger than observability. It was shown there that supremal relatively observable sublanguages exist. In this section, we introduce the notion of conditional $C$-observability as a counterpart of relative observability for coordination control. First, we recall the definition of relative observability.

Let $K \subseteq C \subseteq L(G)$. The language $K$ is *$C$-observable* with respect to a plant $G$ and a projection $Q : \Sigma^* \to \Sigma_o^*$ if for all words $s, s' \in \Sigma^*$ such that $Q(s) = Q(s')$, it holds that, for all $\sigma \in \Sigma$, if $s\sigma \in K$, $s' \in C$ and $s'\sigma \in L(G)$, then $s'\sigma \in K$. For $C = K$ the definition thus coincides with the definition of observability.

*Definition 3:* Let $G_1$ and $G_2$ be generators over the event sets $\Sigma_1$ and $\Sigma_2$, respectively, and let $G_k$ be a coordinator over the event set $\Sigma_k$. Let $K \subseteq C \subseteq L(G_1 \parallel G_2 \parallel G_k)$. The language $K$ is *conditionally $C$-observable* with respect to generators $G_1, G_2, G_k$ and projections $Q_{1+k}, Q_{2+k}, Q_k$, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, if

1) $P_k(K)$ is $P_k(C)$-observable with respect to $L(G_k)$ and $Q_k$, and
2) $P_{i+k}(K)$ is $P_{i+k}(C)$-observable with respect to the plant $L(G_i) \parallel L(G_k)$ and $Q_{i+k}$, for $i = 1, 2$.

By definition, if $K' \subseteq K$ is conditionally $C$-observable, then it is also conditionally $K$-observable.

We now show that the supremal conditionally relative observable sublanguage always exists.

*Theorem 4:* For a given $C$, the supremal conditionally $C$-observable sublanguage always exists and equals to the union of all conditionally $C$-observable sublanguages.

*Proof:* Let $I$ be an index set. For $i \in I$, let $K_i \subseteq C$ be a conditionally $C$-observable sublanguage of $K \subseteq L(G_1 \parallel G_2 \parallel G_k)$ with respect to $G_1$, $G_2$, $G_k$ and projections $Q_{1+k}$, $Q_{2+k}$, $Q_k$. To prove that $\cup_{i\in I} K_i$ is conditionally $C$-observable, note that $P_k(\cup_{i\in I} K_i)$ is $P_k(C)$-observable with respect to $L(G_k)$ and $Q_k$, since if $sa \in P_k(\cup_{i\in I} K_i) = \cup_{i\in I} P_k(K_i)$, $s' \in P_k(C)$, $s'a \in L(G_k)$, and $Q_k(s) = Q_k(s')$, then $sa \in P_k(K_i)$, for some $i \in I$. Then $P_k(C)$-observability of $P_k(K_i)$ with respect to $L(G_k)$ and $Q_k$ implies that $s'a \in P_k(K_i) \subseteq P_k(\cup_{i\in I} K_i) = P_k(\cup_{i\in I} K_i)$. The case for $P_{j+k}(\cup_{i\in I} K_i)$, for $j = 1, 2$, is analogous. ∎

We now recall the definitions of normality and conditional normality.

Let $G$ be a generator over the event set $\Sigma$, and let $Q : \Sigma^* \to \Sigma_o^*$ be a projection. A language $K \subseteq L(G)$ is *normal* with respect to $L(G)$ and $Q$ if

$$K = Q^{-1}Q(K) \cap L(G)\,.$$

It is known that normality implies observability [2].

Let $G_1$ and $G_2$ be generators over the event sets $\Sigma_1$ and $\Sigma_2$, respectively, and let $G_k$ be a coordinator over $\Sigma_k$. A language $K \subseteq L(G_1 \parallel G_2 \parallel G_k)$ is *conditionally normal* with respect to generators $G_1$, $G_2$, $G_k$ and projections $Q_{1+k}$, $Q_{2+k}$, $Q_k$, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, if

1) $P_k(K)$ is normal with respect to $L(G_k)$ and $Q_k$, and
2) $P_{i+k}(K)$ is normal with respect to $L(G_i) \parallel P_k(K)$ and $Q_{i+k}$, for $i = 1, 2$.

The following theorem compares the notions. The main point is to show that we do not need conditional normality in coordination control, because the weaker condition of conditional relative observability can be used instead.

*Theorem 5:* The following holds:

1) Conditional normality implies conditional relative observability.
2) Conditional relative observability implies conditional observability.

*Proof:* Implication (2) follows from [1], where it was shown that relative observability implies observability. We now prove (1). Let $K \subseteq C \subseteq L(G_1 \parallel G_2 \parallel G_k)$ be such that $K$ is conditionally normal with respect to generators $G_1$, $G_2$, $G_k$ and projections $Q_{1+k}$, $Q_{2+k}$, $Q_k$. Then, the assumption that $P_k(K)$ is normal with respect to $L(G_k)$ implies that $P_k(K)$ is $P_k(C)$-observable with respect to $L(G_k)$ by [1]. Moreover, for $i = 1, 2$, we have that $P_{i+k}(K)$ is normal with respect to $L(G_i) \parallel P_k(K)$. By Lemma 12, $L(G_i) \parallel P_k(K)$ is normal with respect to $L(G_i) \parallel L(G_k)$. Hence, by the transitivity of normality (Lemma 13), $P_{i+k}(K)$ is normal with respect to $L(G_i) \parallel L(G_k)$. Then, by [1], we obtain that $P_{i+k}(K)$ is $P_{i+k}(C)$-observable with respect to $L(G_i) \parallel L(G_k)$, which was to be shown. ∎

We have shown that the supremal conditionally controllable and conditionally relative observable sublanguage exists. We now present conditions under which a conditionally controllable and conditionally observable sublanguage containing the supremal conditionally controllable and conditionally relative observable sublanguage can be computed in a distributed/parallel way.

Consider the setting of Problem 1 and define the languages

$$\sup \mathrm{CRO}_k = \sup \mathrm{CRO}(P_k(K), L(G_k)) \quad (1)$$
$$\sup \mathrm{CRO}_{i+k} = \sup \mathrm{CRO}(P_{i+k}(K), L(G_i) \parallel \sup \mathrm{CRO}_k)$$

for $i = 1, 2$, where $\sup \mathrm{CRO}(K, L)$ denotes the supremal sublanguage of $K$ that is controllable (with respect to $L$ and the corresponding event set of uncontrollable events) and $(K \cap L)$-observable (with respect to $L$ and the corresponding projection to observable events).

The way how to compute the supremal relatively observable sublanguage is discussed in [1]. For $K \subseteq L$, let

$$\sup \mathrm{cCRO} = \sup \mathrm{cCRO}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$$

denote the supremal conditionally controllable and conditionally $K$-observable sublanguage of $K$ with respect to $L = L(G_1 \parallel G_2 \parallel G_k)$, the sets of uncontrollable events $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, and projections $Q_{1+k}$, $Q_{2+k}$, $Q_k$, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$.

We now show the following inclusion.

*Lemma 6:* Consider the notation above. Then

$$\sup \mathrm{cCRO} \subseteq \sup \mathrm{CRO}_{1+k} \parallel \sup \mathrm{CRO}_{2+k}\,. \quad (2)$$

*Proof:* To prove this, we show that $P_{i+k}(\sup \mathrm{cCRO}) \subseteq \sup \mathrm{CRO}_{i+k}$, for $i = 1, 2$. By the definition of conditional controllability, $P_{i+k}(\sup \mathrm{cCRO}) \subseteq P_{i+k}(K)$ is controllable with respect to $L(G_i) \| P_k(\sup \mathrm{cCRO})$. Since the language $P_k(\sup \mathrm{cCRO}) \subseteq P_k(K)$ is controllable and $P_k(K)$-observable with respect to $L(G_k)$, $P_k(\sup \mathrm{cCRO}) \subseteq \sup \mathrm{CRO}_k$. Thus, $P_k(\sup \mathrm{cCRO})$ is controllable with respect to $\sup \mathrm{CRO}_k \subseteq L(G_k)$. Then, by Lemma 9, the language $L(G_i) \parallel P_k(\sup \mathrm{cCRO})$ is controllable with respect to the plant $L(G_i) \parallel \sup \mathrm{CRO}_k$, and the transitivity of controllability (Lemma 10) implies that $P_{i+k}(\sup \mathrm{cCRO})$ is controllable with respect to $L(G_i) \parallel \sup \mathrm{CRO}_k$.

Furthermore, by the definition of conditional relative observability, $P_{i+k}(\sup \mathrm{cCRO})$ is $P_{i+k}(K)$-observable with respect to $L(G_i) \parallel L(G_k)$, hence it is also $C$-observable with respect to $L(G_i) \parallel L(G_k)$, for every $P_{i+k}(\sup \mathrm{cCRO}) \subseteq C \subseteq P_{i+k}(K)$. As $P_{i+k}(\sup \mathrm{cCRO}) \subseteq L(G_i) \parallel \sup \mathrm{CRO}_k$, we also obtain that $P_{i+k}(\sup \mathrm{cCRO})$ is $C'$-observable with respect to $L(G_i) \parallel \sup \mathrm{CRO}_k$, for every $P_{i+k}(\sup \mathrm{cCRO}) \subseteq C' \subseteq P_{i+k}(K) \cap (L(G_i) \parallel \sup \mathrm{CRO}_k)$, which means that $P_{i+k}(\sup \mathrm{cCRO}) \subseteq \sup \mathrm{CRO}_{i+k}$. ∎

Note that the language

$$\sup \mathrm{CRO}_{1+k} \parallel \sup \mathrm{CRO}_{2+k}$$

is controllable and observable, by Lemmas 9 and 11, hence it is a solution of our problem that always contains the supremal conditionally controllable and conditionally relatively observable sublanguage $\sup \mathrm{cCRO}$. Thus, we have computed a solution that is in general larger than the supremal conditionally controllable and conditionally relatively observable sublanguage. We now compare it with the supremal language $\sup \mathrm{cCRO}$.

*Reaching supremal languages*

If the coordinator part of $\sup \mathrm{CRO}_{1+k} \parallel \sup \mathrm{CRO}_{2+k}$ is conditionally controllable and conditionally observable, then the computed language coincides with the supremal conditionally controllable and conditionally relatively observable sublanguage.

*Theorem 7:* Consider the setting of Problem 1 and the languages defined in (1). Let

$$M = \sup \mathrm{CRO}_{1+k} \parallel \sup \mathrm{CRO}_{2+k}$$

and $L = L(G_1 \parallel G_2 \parallel G_k)$. If $P_k(M)$ is controllable and $P_k(M)$-observable with respect to $L(G_k)$, $\Sigma_{k,u}$, and $Q_k$, then $M$ is conditionally controllable with respect to $G_1$, $G_2$, $G_k$ and $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, and conditionally observable with respect to $G_1$, $G_2$, $G_k$ and $Q_{1+k}$, $Q_{2+k}$, $Q_k$. Moreover, it contains the language $\sup \mathrm{cCRO}$.

*Proof:* We have $M \subseteq P_{1+k}(K) \| P_{2+k}(K) = K$ by conditional decomposability. Moreover, $P_k(M)$ is controllable and observable with respect to $L(G_k)$, $\Sigma_{k,u}$, and $Q_k$ by the assumptions.

Furthermore, $P_{1+k}(M) = \sup \mathrm{CRO}_{1+k} \parallel P_k(M)$ is controllable with respect to $[L(G_1) \parallel \sup \mathrm{CRO}_k] \parallel P_k(M) = L(G_1) \parallel P_k(M)$ by Lemma 9. To show that the language $P_{1+k}(M) \subseteq P_{1+k}(K) \cap (L(G_1) \parallel \sup \mathrm{CRO}_k)$ is observable, let $a \in \Sigma_{1+k}$, $sa$, $s' \in P_{1+k}(M)$, $s'a \in L(G_1) \parallel P_k(M) \subseteq L(G_1) \parallel \sup \mathrm{CRO}_k$, and $Q_{1+k}(s) = Q_{1+k}(s')$. By the $(P_{1+k}(K) \cap (L(G_1) \parallel \sup \mathrm{CRO}_k))$-observability of $\sup \mathrm{CRO}_{1+k}$, we have that $s'a \in \sup \mathrm{CRO}_{1+k}$. We now have two cases:

(i) If $a \in \Sigma_1 \setminus \Sigma_k$, then we immediately have that $P_k(s'a) = P_k(s') \in P_k(M) \subseteq P_k(\sup \mathrm{CRO}_{2+k})$;

(ii) If $a \in \Sigma_k$, then $P_k(s)a \in P_k(M)$, $P_k(s') \in P_k(M)$, and $P_k(s')a \in L(G_k)$ imply (by the $P_k(M)$-observability of $P_k(M)$) that $P_k(s'a) \in P_k(M) \subseteq P_k(\sup \mathrm{CRO}_{2+k})$.

Thus, in both cases, we have that $s'a \in \sup \mathrm{CRO}_{1+k} \parallel P_k(\sup \mathrm{CRO}_{2+k}) = P_{1+k}(M)$.

The case of $P_{2+k}(M)$ is analogous, hence $M$ is conditionally controllable with respect to $G_1$, $G_2$, $G_k$ and $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, and conditionally observable with respect to $G_1$, $G_2$, $G_k$ and $Q_{1+k}$, $Q_{2+k}$, $Q_k$.

Finally, $\sup \mathrm{cCRO} \subseteq \sup \mathrm{CRO}_{1+k} \parallel \sup \mathrm{CRO}_{2+k}$ as shown in Lemma 6. ■

There is a serious drawback in Theorem 7. Namely, the controllability and $P_k(M)$-observability conditions might be quite restrictive (although controllability was shown weaker than previously used conditions). A natural approach is then to impose these conditions by an additional, a posteriori supervisor. It is well known from basic supervisory control theory that for any controllable and observable sublanguage there always exists a supervisor under partial observations that can impose this language for the controlled system. It appears that if $P_k(M)$ is not controllable or $P_k(M)$-observable with respect to $L(G_k)$, $\Sigma_{k,u}$, $Q_k$, then we can synthesize a supervisor under partial observations on the alphabet $\Sigma_k$, where $L(G_k)$ is the plant and $P_k(M)$ is the specification. In particular, the supremal controllable and $P_k(M)$-observable sublanguage of $P_k(M)$ with respect to $L(G_k)$ always exists. Implementation issues for supervisors achieving relative observability are discussed in [1]. However, to allow for parallel computations, we define the a posteriori supervisor

$$\begin{aligned}\mathrm{CRO}'_k = &\sup \mathrm{CRO}(P_k(\sup \mathrm{CRO}_{1+k}), L(G_k)) \\ &\cap \sup \mathrm{CRO}(P_k(\sup \mathrm{CRO}_{2+k}), L(G_k))\end{aligned}$$

for imposing controllability and observability with respect to $L(G_k)$. Then we have the following result.

*Proposition 8:* Consider the notation introduced in and below (1) and in Theorem 7. Then the language $\mathrm{CRO}'_k \parallel M = \sup \mathrm{cCRO}$ is the supremal sublanguage of $K$ that is conditionally controllable and conditionally observable with respect to $G_1$, $G_2$, $G_k$ and $Q_{1+k}$, $Q_{2+k}$, $Q_k$.

*Proof:* We show that $M' = \mathrm{CRO}'_k \parallel M$ is conditionally controllable and conditionally observable. To do this, note that $P_k(M') = \mathrm{CRO}'_k \parallel P_k(M) = \mathrm{CRO}'_k$ and, by definition of $\mathrm{CRO}'_k$ and Lemmas 9 and 11, $P_k(M')$ is controllable and observable with respect to $L(G_k)$. Furthermore, for $i = 1, 2$, $P_{i+k}(M') = \mathrm{CRO}'_k \parallel P_{i+k}(M) = \mathrm{CRO}'_k \parallel P_k(M) \parallel \sup \mathrm{CRO}_{i+k} = \mathrm{CRO}'_k \parallel \sup \mathrm{CRO}_{i+k}$. By Lemmas 9 and 11, $P_{i+k}(M')$ is controllable and observable with respect to $L(G_k) \parallel [L(G_i) \parallel \sup \mathrm{CRO}_k] = L(G_i) \parallel \sup \mathrm{CRO}_k$. Since $P_k(M') = \mathrm{CRO}'_k \subseteq \sup \mathrm{CRO}_k$, we have that the language $P_{i+k}(M')$ is controllable and observable with respect to $L(G_i) \parallel P_k(M')$.

To prove the opposite implication, note that it holds, for $i = 1, 2$, that $P_{i+k}(\sup \mathrm{cCRO}) \subseteq \sup \mathrm{CRO}_{i+k}$. Thus, it remains to show that $P_k(\sup \mathrm{cCRO}) \subseteq \mathrm{CRO}'_k$ also holds. However, $P_k(\sup \mathrm{cCRO}) \subseteq P_k(\sup \mathrm{CRO}_{i+k}) \subseteq P_k(K)$ follows from above and, since the language $P_k(\sup \mathrm{cCRO})$ is, by definition, controllable and $P_k(K)$-observable with respect to $L(G_k)$, we obtain that $P_k(\sup \mathrm{cCRO})$ is a subset of $\mathrm{CRO}'_k$. ■

The advantage of Proposition 8 is that there are no restrictive conditions on the computation of a conditionally controllable and conditionally relatively observable sublanguage. Thus, one could directly apply Proposition 8 instead of verifying the conditions of Theorem 7.

It is worth noticing that the previous result shows that the supremal conditionally controllable and conditionally relative observable sublanguages is always conditionally decomposable, therefore it can potentially be computed in a distributed way.

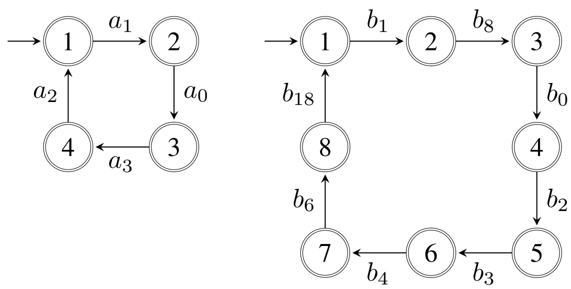


Fig. 1. Generators for AGV 1 ($L_1$) and AGV 2 ($L_2$)

Finally, let us point out that for systems with too many components it is not realistic to have only a single (centralized) coordinator, because too many events have to be included into the coordinator alphabet to make the specification conditionally decomposable. Therefore, we have recently proposed a multi-level coordination control architecture with a hierarchical structure of groups of subsystems, their respective coordinators and supervisors. For more details, the reader is referred to [7].

### A. *Example*

We have chosen a part of the AGV example of [1] to illustrate the concept of conditional relative observability. Namely, we consider the first two of the five AGVs on Fig. 1 and the corresponding conflict zone 1 specification on the left of Fig. 2, which aims to avoid collisions between AGV 1 and AGV 2. Moreover, we consider prefix-closed (generated) languages of all automata. We have renamed the events in such a way that events $1i$ of AGV 1 are called $a_i$, $i = 1,2,3,0$, and events $2j$ of AGV 2 are called $b_j$ with the exception of 18 and 28 that are called $b8$ and $b18$, respectively.

We apply our coordination control framework to impose the specification (denoted $K$). Since the specification $K$ is not conditionally decomposable, we have to include events $a_1, a_3, b_0, b_3$ into $\Sigma_k$. The corresponding coordinator is then $L_k = P_k(L_1) \parallel P_k(L_2)$ as depicted on the right of Fig. 2. It turns out that $P_k(K)$ is even larger than $L_k$, i.e., no supervisor for the coordinator is needed, meaning that $\sup \mathrm{C}_k = L_k$.

Then we decompose the supervisory control problem for the global plant into two subproblems: imposing $P_{1+k}(K)$ for the plant $L_1 \parallel L_k$ and imposing $P_{2+k}(K)$ for the plant $L_2 \parallel L_k$. It appears that $P_{1+k}(K)$ is not included in $L_1 \parallel L_k$.

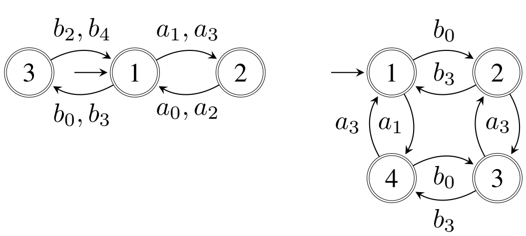


Fig. 2. Generator for the specification $K$ and the coordinator $L_k$

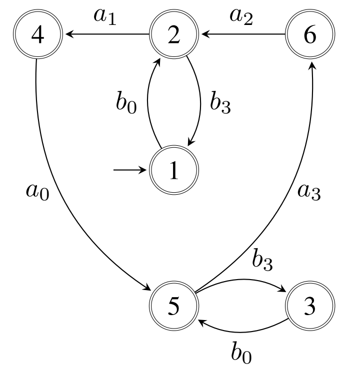


Fig. 3. Generator for $\sup \mathrm{CRO}_{1+k}$

Therefore, we now consider $P_{1+k}(K) \cap (L_1 \parallel L_k)$ and $P_{2+k}(K) \cap (L_2 \parallel L_k)$ as new specifications. It turns that $P_{1+k}(K) \cap (L_1 \parallel L_k)$ is not controllable with respect to $L_1 \parallel L_k$. We then compute $\sup \mathrm{C}_{1+k}$, which is not normal with respect to $L_1 \parallel L_k$ and $Q_{1+k}$. However, supremal normal sublanguage need not be computed, because $\sup \mathrm{C}_{1+k}$ is observable, thus relatively observable with respect to $L_1 \parallel L_k$ and $Q_{1+k}$. Otherwise stated, the supervisor $\sup \mathrm{CRO}_{1+k} = \sup \mathrm{C}_{1+k}$, see Fig. 3.

Similarly, $\sup \mathrm{C}_{2+k}$ of Fig. 4 is not normal with respect to $L_2 \parallel L_k$ and $Q_{2+k}$, but it is relatively observable. This shows the advantage of using $\sup \mathrm{CRO}_{i+k}$ over the supremal controllable and normal sublanguage: the former ones are strictly more permissive. The first supervisor has 6 states and 8 transitions and the second supervisor has 16 states and 28 transitions.

Finally, the condition of Theorem 7 is not satisfied, since, although the language $P_k(\sup \mathrm{CRO}_{1+k})$ is controllable and $P_k(\sup \mathrm{CRO}_{1+k})$-observable with respect to $L_k$, the language $P_k(\sup \mathrm{CRO}_{2+k})$ is not controllable with respect to $L_k$. Therefore, a new supervisor is needed and Proposition 8 can be applied to compute it.

The local supervisors for $L_i \parallel L_k$, for $i = 1,2$, are then $\sup \mathrm{CRO}_{1+k}$ and

$$\sup \mathrm{CRO}_{2+k} \parallel \sup \mathrm{CRO}(P_k(\sup \mathrm{CRO}_{2+k}), L(G_k)) .$$

## V. Auxiliary Results

This section provides auxiliary results needed in the paper.

*Lemma 9 (Proposition 4.6 in [3]):* For $i = 1,2$, let $K_i \subseteq L_i$ over an event set $\Sigma_i$ be languages such that $K_i$ is controllable with respect to $L_i$ and $\Sigma_{i,u}$. Let $\Sigma = \Sigma_1 \cup \Sigma_2$. Then the parallel composition $K_1 \parallel K_2$ is controllable with respect to $L_1 \parallel L_2$ and $\Sigma_u$. ■

*Lemma 10 ([6]):* Let $K \subseteq L \subseteq M$ be languages over $\Sigma$ such that $K$ is controllable with respect to $L$ and $\Sigma_u$, and $L$ is controllable with respect to $M$ and $\Sigma_u$. Then $K$ is controllable with respect to $M$ and $\Sigma_u$. ■

*Lemma 11:* For $i = 1,2$, let $K_i \subseteq L_i$ over an event set $\Sigma_i$ be languages such that $K_i$ is observable with respect to $L_i$ and $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$. Then the parallel composition $K_1 \parallel K_2$

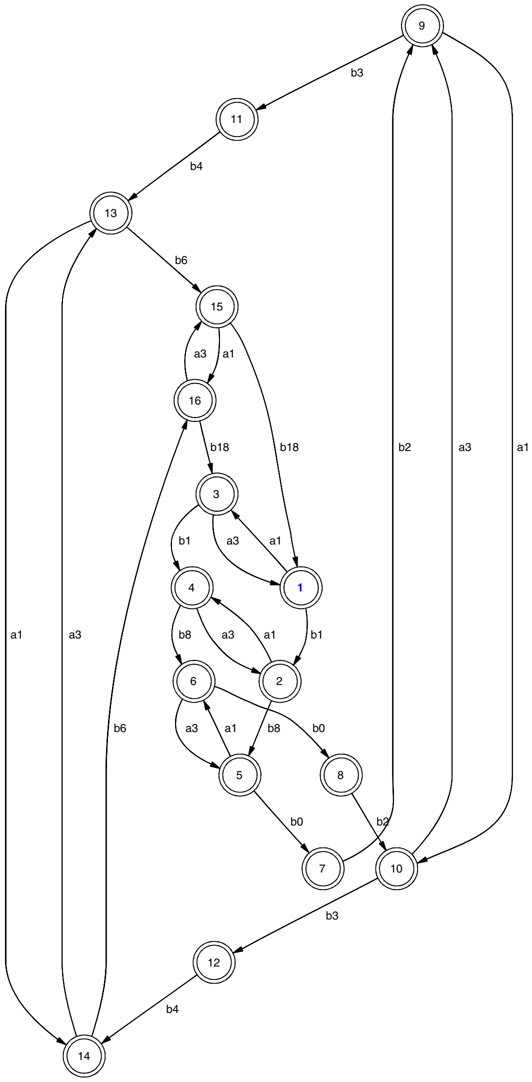


Fig. 4. Generator for $\sup \mathrm{CRO}_{2+k}$

is observable with respect to $L_1 \parallel L_2$ and $Q : (\Sigma_1 \cup \Sigma_2)^* \to (\Sigma_{1,o} \cup \Sigma_{2,o})^*$.

*Proof:* Let $s, s' \in \Sigma^*$ be such that $Q(s) = Q(s')$. Let $\sigma \in \Sigma$ and assume that $s\sigma, s' \in K_1 \parallel K_2$ and $s'\sigma \in L_1 \parallel L_2$. Let $P_i : (\Sigma_1 \cup \Sigma_2)^* \to \Sigma_i^*$, for $i = 1, 2$, be a projection. Then $P_i(s\sigma), P_i(s') \in K_i$ and $P_i(s'\sigma) \in L_i$ imply that $P_i(s'\sigma) \in K_i$, by observability of $K_i$ with respect to $L_i$. Thus, we have $s'\sigma \in K_1 \parallel K_2$. ∎

*Lemma 12:* For $i = 1, 2$, let $K_i \subseteq L_i$ over an event set $\Sigma_i$ be languages such that $K_i$ is normal with respect to $L_i$ and $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$. Then the parallel composition $K_1 \parallel K_2$ is normal with respect to $L_1 \parallel L_2$ and $Q : (\Sigma_1 \cup \Sigma_2)^* \to (\Sigma_{1,o} \cup \Sigma_{2,o})^*$.

*Proof:* By definition, we have that $Q^{-1}Q(K_1 \parallel K_2) \cap L_1 \parallel L_2 \subseteq Q_1^{-1}Q_1(K_1) \parallel Q_2^{-1}Q_2(K_2) \parallel L_1 \parallel L_2 = K_1 \parallel K_2$, where the equality is by normality of $K_1$ and $K_2$. As the other inclusion always holds, the proof is complete. ∎

*Lemma 13:* Let $K \subseteq L \subseteq M$ be languages such that $K$ is normal with respect to $L$ and $Q$, and $L$ is normal with respect to $M$ and $Q$. Then $K$ is normal with respect to $M$ and $Q$.

*Proof:* By the assumption, $Q^{-1}Q(K) \cap L = K$ and $Q^{-1}Q(L) \cap M = L$, hence $Q^{-1}Q(K) \cap M \subseteq Q^{-1}Q(L) \cap M = L$. Thus, $Q^{-1}Q(K) \cap M = Q^{-1}Q(K) \cap M \cap L = K \cap M = K$. ∎

## VI. Conclusion

We introduced the notion of conditional relative observability and studied the coordinated computation of the supremal conditionally controllable and conditionally relative observable sublanguage of the specification. Note that there exist conditions, namely the observer and OCC (LCC) properties, fulfilled by a modification of the coordinator event set, that imply the assumptions of Theorem 7 for controllability. However, to the best of our knowledge, no similar conditions are known for observability.

Finally, note that the approach presented here can be generalized to non-prefix-closed languages, provided the languages are nonconflicting. The verification of this property is known to be PSPACE-complete [11] if the number of components is unlimited, whereas it can be verified in nondeterministic logarithmic space, that is, in polynomial time, if the number of components is fixed. The result should be read so that the polynomial space is still sufficient. Note that when handling large systems, the space is the critical complexity issue. In some cases, nonconflictingness can be even imposed by coordinators on subalphabets, which leads to savings on complexity, cf. [8].

## Acknowledgments

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