TECHNISCHE UNIVERSITÄT DRESDEN

PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

Lecture 1

Sarah Gaggl

Dresden, 14th April 2020

What to expect

- The course has 12 lectures, 11 tutorials and a practical part
- Lecture is originally planned for Tuesday in DS 2, 9:20-10:50
- Tutorials are planned for Thursday in DS1, 7:30-9:00
- **Due to the COVID-19 situation the course will be held as an online course until the situation changes**
- Schedule and lecture material will be available at course web-page `https://iccl.inf.tu-dresden.de/web/Problem_Solving_and_Search_in_Artificial_Intelligence_(SS2020)`
- Additionally the **forum in OPAL** will be used for discussions related to lecture, tutorial and practical part
- The practical part consists of solving (implementing) a problem and its presentation. Should be performed in groups of two, assignment will be ready at April 28th.
- 3 fixed Dates for practical part (see web-page)
  1) Analysis of the problem; 28.04.2020
  2) Concept how to solve it, group building; 15.05.2020
  3) Presentation of solution - status report; 30.06.2020
- Deadline for practical part 20th July 2020

Agenda

1. Introduction
2. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
3. Local Search, Stochastic Hill Climbing, Simulated Annealing
4. Tabu Search
5. Answer-set Programming (ASP)
6. Constraint Satisfaction Problems (CSP)
7. Evolutionary Algorithms/ Genetic Algorithms
8. Structural Decomposition Techniques (Tree/Hypertree Decompositions)

What are the Ages of my Three Sons?

Two men meet on the street. One gives the other a puzzle

A: "All three of my sons celebrate their birthday this very day! So, can you tell me how old each of them is?"

B: "Sure, but you'll have to tell me something about them."

A: "The product of the ages of my sons is 36."

B: "That's fine but I need more than just this."

A: "The sum of their ages is equal to the number of windows in that building."

B: "Still, I need an additional hint to solve your puzzle."

A: "My oldest son has blue eyes."

B: "Oh, this is sufficient!"

What are the Ages of my Three Sons? ctd.

"The product of the ages of my sons is 36."

son 1	son 2	son 3
36	1	1
18	2	1
12	3	1
9	4	1
9	2	2
6	6	1
6	3	2
4	3	3

What are the Ages of my Three Sons? ctd.

"The sum of their ages is equal to the number of windows in that building."

son 1	son 2	son 3
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18	2	1
12	3	1
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$$
\begin{array}{ccccccc}
36 & + & 1 & + & 1 & = & 38 \\
18 & + & 2 & + & 1 & = & 21 \\
12 & + & 3 & + & 1 & = & 16 \\
9 & + & 4 & + & 1 & = & 14 \\
9 & + & 2 & + & 2 & = & 13 \\
6 & + & 6 & + & 1 & = & 13 \\
6 & + & 3 & + & 2 & = & 11 \\
4 & + & 3 & + & 3 & = & 10
\end{array}
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What was difficult on this problem?

Problem Solving

- Where to begin?
- You have to create the plan for generating a solution.
- Always consider all of the available data.
- Can you make connections between the goal and what is given?

Why are Some Problems Difficult to Solve?

- The number of possible solutions in the search space is too large for an exhaustive search.
- The problem is too complicated, and simplified models of the problem are useless.
- The evaluation function of the quality of a solution is noisy or varies with time, which requires an entire series of solutions.
- There are so many constraints that finding even one feasible answer is difficult, let alone searching for an optimal solution.
- The person solving the problem is inadequately prepared.

The Size of the Search Space

Boolean Satisfiability Problem (SAT)

Make a compound statement of Boolean variables evaluate to TRUE.

- For example, consider the following problem of 100 variables given in conjunctive normal form (CNF):

$$F(x) = (x_{17} \vee \neg x_{37} \vee x_{73}) \wedge (\neg x_{11} \vee \neg x_{56}) \wedge \cdots \wedge (x_2 \vee x_{43} \vee \neg x_{77} \vee \neg x_{89} \vee \neg x_{97}).$$

- Challenge: find the truth assignment for each variable x_i, for all $i = 1, \ldots 100$ s.t. $F(x) = \text{TRUE}$.

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Space of possible solutions.

- Any binary string of length 100 is a possible solution.
- Two choices for each variable, and taken over 100 variables, generates 2^{100} possibilities.

The Size of the Search Space ctd.

- Size of the search space $\mathcal{S}$ is $|\mathcal{S}| = 2^{100} \approx 10^{30} = 1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000$.
- The number of bacterial cells on Earth is estimated at around 5×10^{30}.
- If we had a computer that could test 1000 strings per second and could have started at the beginning of time itself, 15 billion years ago (Big Bang!) we would have examined fewer than 1% of all the possibilities by now!
- Trying out all alternatives is out of the question.
- Choice of which evaluation function to use.
- Solutions closer to the right answer should yield better evaluations than those who are far away.
- If we try a string x and $F(x)$ returns TRUE, we are done. But what if $F(x)$ returns FALSE?
- How to find a function which gives more than just "right" or "wrong"?

The Size of the Search Space ctd.

Traveling Salesperson Problem (TSP)

- Given n cities and the distances between each pair of cities;
- Traveling salesperson must visit every city exactly once and return home covering the shortest distance.

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Seach Space

- Set of permutations of n cities.
- $2n$ different ways (for symmetrical TSP) to represent one tour.
- There are $n!$ ways to permute n numbers.
- $|\mathcal{S}| = n!/(2n) = (n-1)!/2$

The Size of the Search Space ctd.

- $|\mathcal{S}| = n!/(2n) = (n-1)!/2$
- For any $n > 6$, number of possible solutions to the TSP with n cities is larger than the number of possible solutions to the SAT problem with n variables.
- For $n = 6$: $5!/2 = 60$ solutions to the TSP and $2^6 = 64$ solutions to a SAT.
- For $n = 7$: 360 solutions to the TSP and 128 to the SAT.
- Search space increases very quickly with increasing n.
- A 50-city TSP has more solutions than existing liters of water on the planet.
- However, the evaluation function for the TSP is more straightforward than for SAT.
- Table with distances between each pair of cities.
- After n addition operations we could calculate the distance of any candidate tour and use this to evaluate its merit.
- $cost = dist(15,3) + dist(3,11) + \cdots + dist(6,15)$

Modeling the problem

- We only find the solution to a model of the problem.
- All models are simplifications of the real world.
- Problem $\rightarrow$ Model $\rightarrow$ Solution
  1. Use an approximate model of a problem and find the precise solution: Problem $\rightarrow$ Model$_a$ $\rightarrow$ Solution$_p$(Model$_a$)
  2. Use a precise model of the problem and find an approximate solution: Problem $\rightarrow$ Model$_p$ $\rightarrow$ Solution$_a$(Model$_p$)
- Which one is better?

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- Which one is better?
- Solution$_a$(Model$_p$) is better than Solution$_p$(Model$_a$).

Change over time

Problems my change

- before you model them,
- while you derive a solution, and
- after you execute the solution.

TSP - Travel time between two cities depends on many factors:

- traffic lights
- slow-moving trucks
- flat tire
- weather
- many more...

Constraints

- Almost all practical problems pose constraints
- Two types of constraints:
  - Hard constraints, and
  - Soft constraints.
- Constraints make the search space smaller, but
  - It is hard to create operators that will act on feasible solution and generate in turn new feasible solutions that are an improvement of previous solution.
  - The geometry of search space gets tricky.

Constraints ctd.

Timetable of the classes at a college in one semester

We are given

- list of courses that are offered;
- list of students assigned to each class;
- professors assigned to each class;
- list of available classrooms, and information for size and other facilities that each offer.

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Construct timetables that fulfill hard constraints:

- Each class must be assigned to an available room that has enough seats and requisite facilities.
- Students who are enrolled in more than one class can not have their classes held at the same time on the same day.
- Professors can not be assigned to teach courses that overlap in time.

Constraints ctd.

Timetable - Soft Constraints:

- Courses that meets twice a week should preferably be assigned to Mondays and Wednesdays or Tuesdays and Thursdays.
- Courses that meets three times per week should preferably be assigned to Mondays, Wednesdays, and Fridays.
- Course time should be assigned so that students do not have to take final exams for multiple courses without any break in between.
- If more than one room satisfies the requirements for a course and is available at the designated time, the course should be assigned to the room with the capacity that is closest to the class size.

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- Any timetable that meets the hard constraints is feasible.
- The timetable has to be optimized in the light of soft constraints.
- Each soft constraint has to be quantified.
- We can evaluate two candidate assignments and decide that one is better than other.

Solve the Problem!

- Mr. Smith and his wife invited four other couples for a party.
- When everyone arrived, some of the people in the room shook hands with some of the others.
- Nobody shook hands with their spouse and nobody shook hands with the same person twice.
- After that, Mr. Smith asked everyone how many times they shook someone's hand.
- He received different answers from everybody.
- How many times did Mrs. Smith shake someone's hand?

Summary

Problem solving is difficult for several reasons:

- Complex problems often pose an enormous number of possible solutions.
- To get any sort of solution at all, we often have to introduce simplifications that make the problem tractable. As a result, the solutions that we generate may not be very valuable.
- The conditions of the problem change over time and might even involve other people who want to fail you.
- Real-world problems often have constraints that require special operations to generate feasible solutions.

References

Zbigniew Michalewicz and David B. Fogel.
How to Solve It: Modern Heuristics, volume 2. Springer, 2004.