Logic on MARS: Ontologies for Generalised Property Graphs

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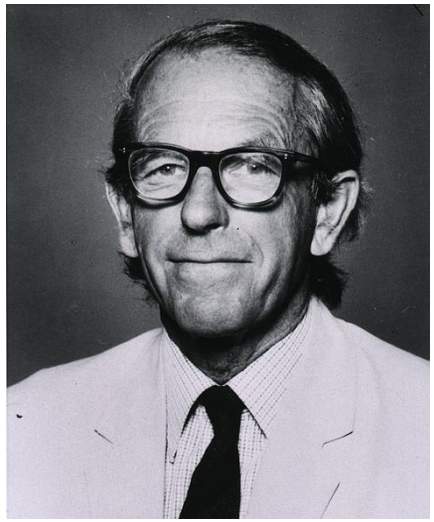
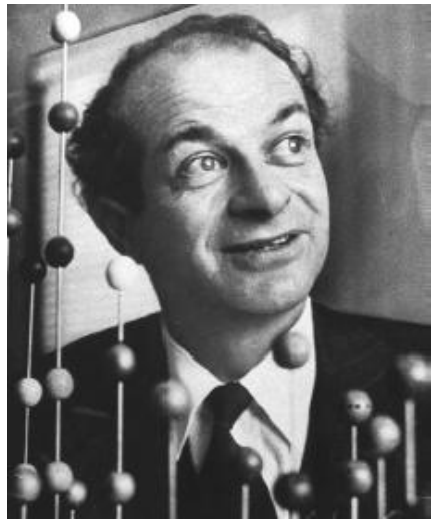
Full paper: `https://iccl.inf.tu-dresden.de/web/MARS/en`

What do these people have in common?



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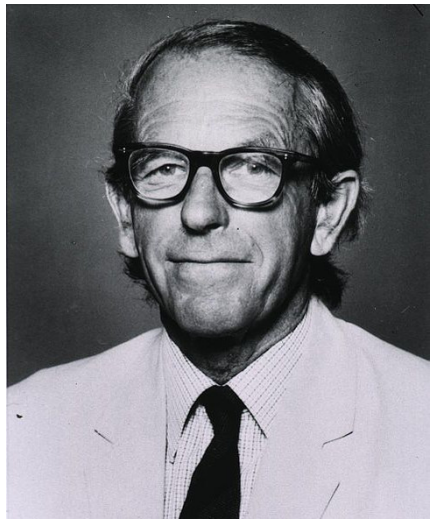
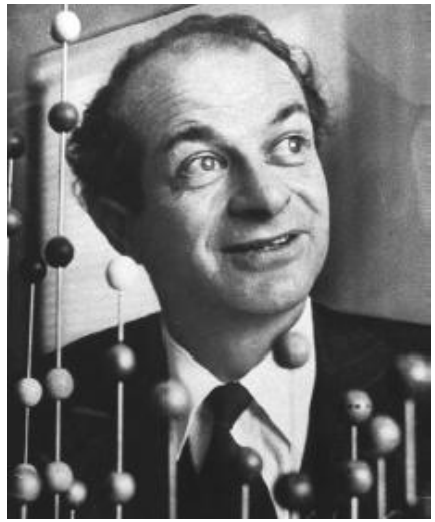
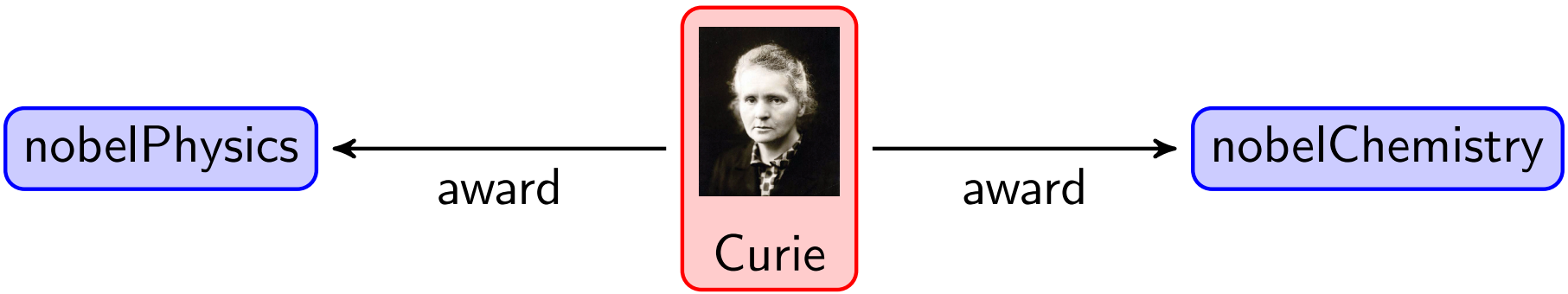



People that have won two Nobel prizes

Laureates in Knowledge Graphs

Wikidata: a free and open Knowledge Graph



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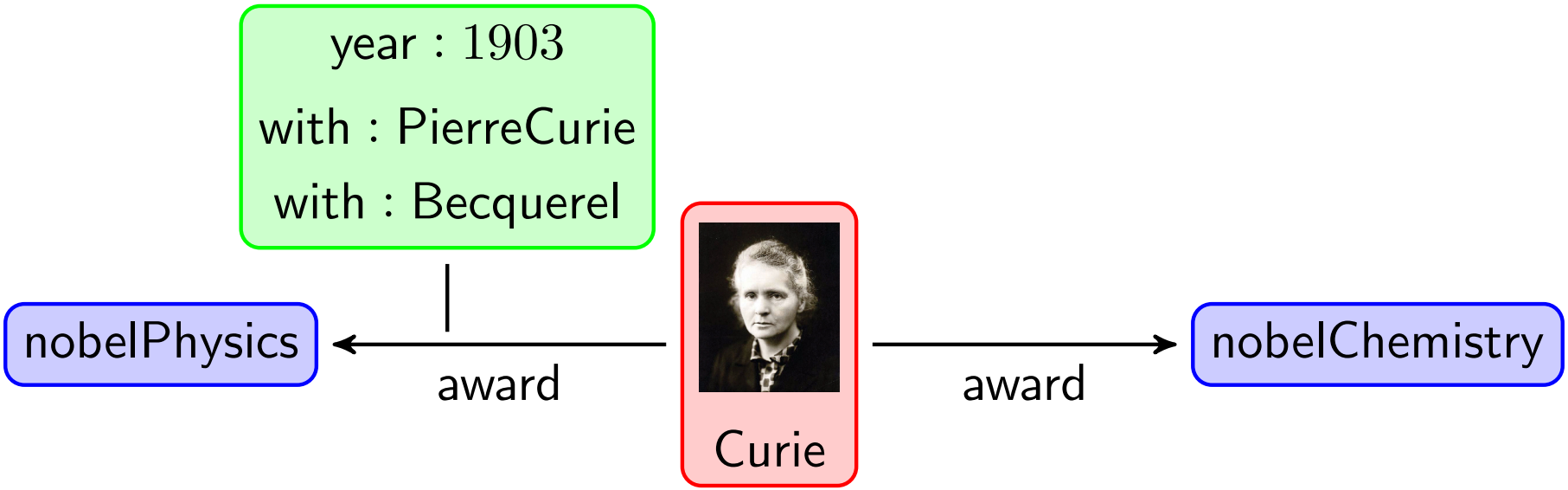

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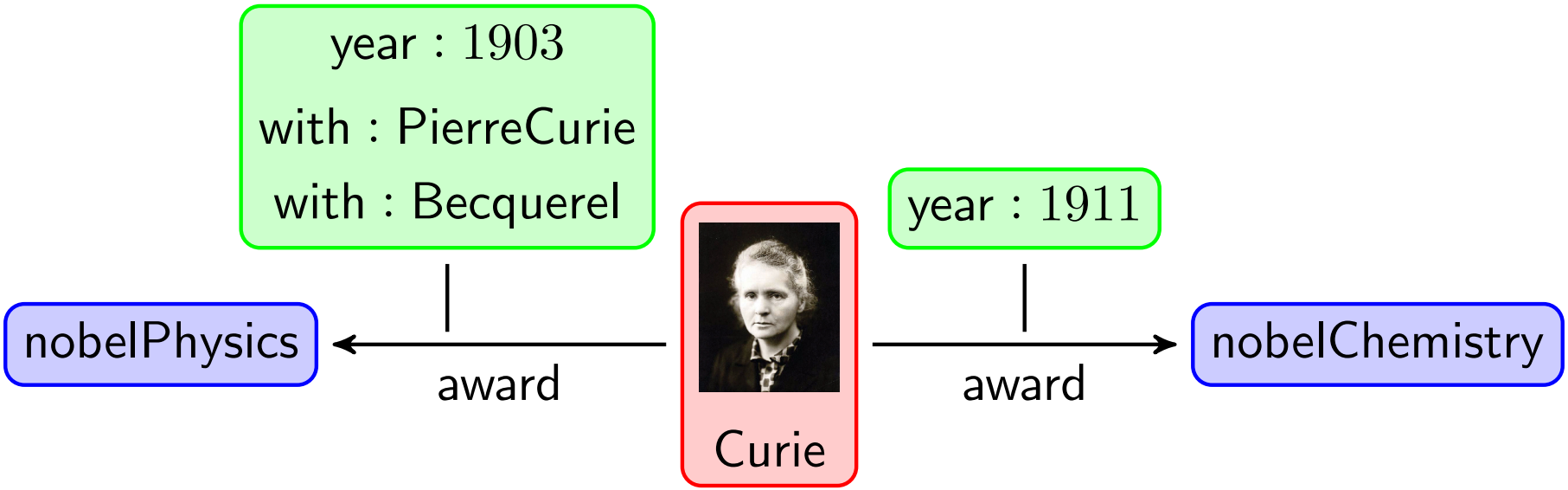

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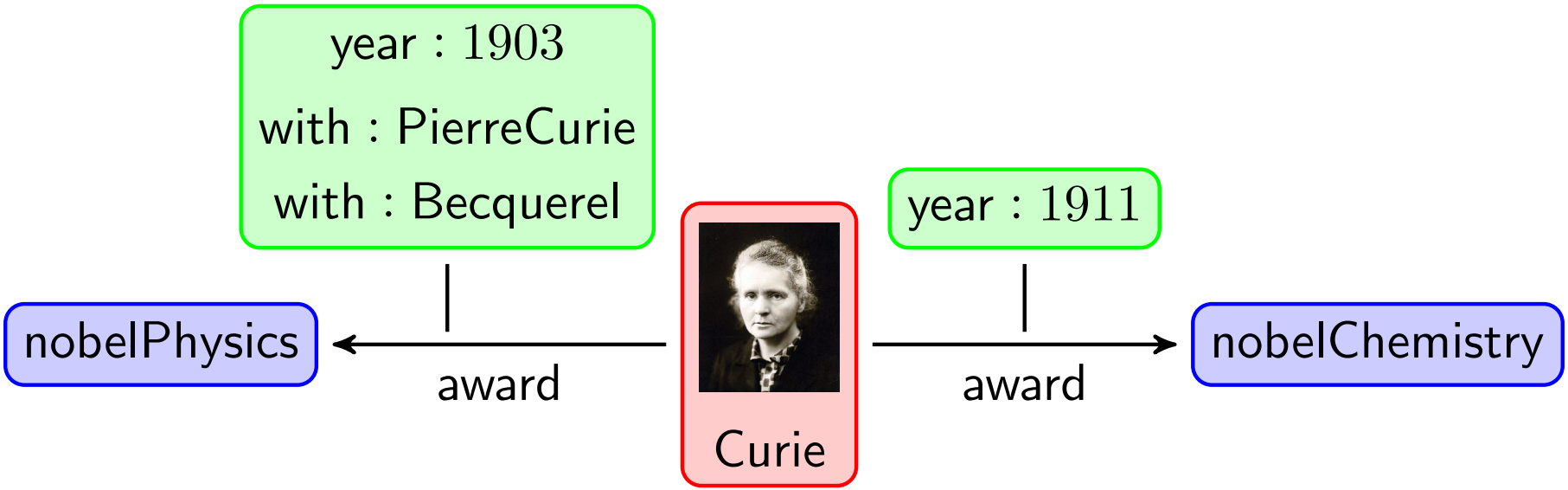

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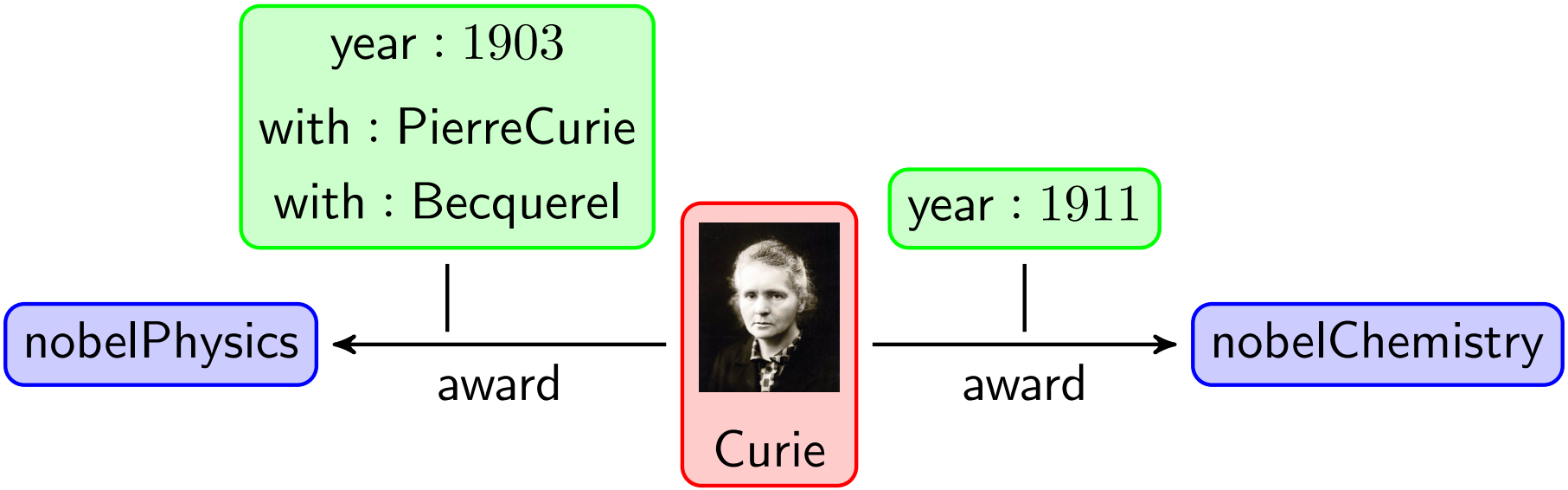

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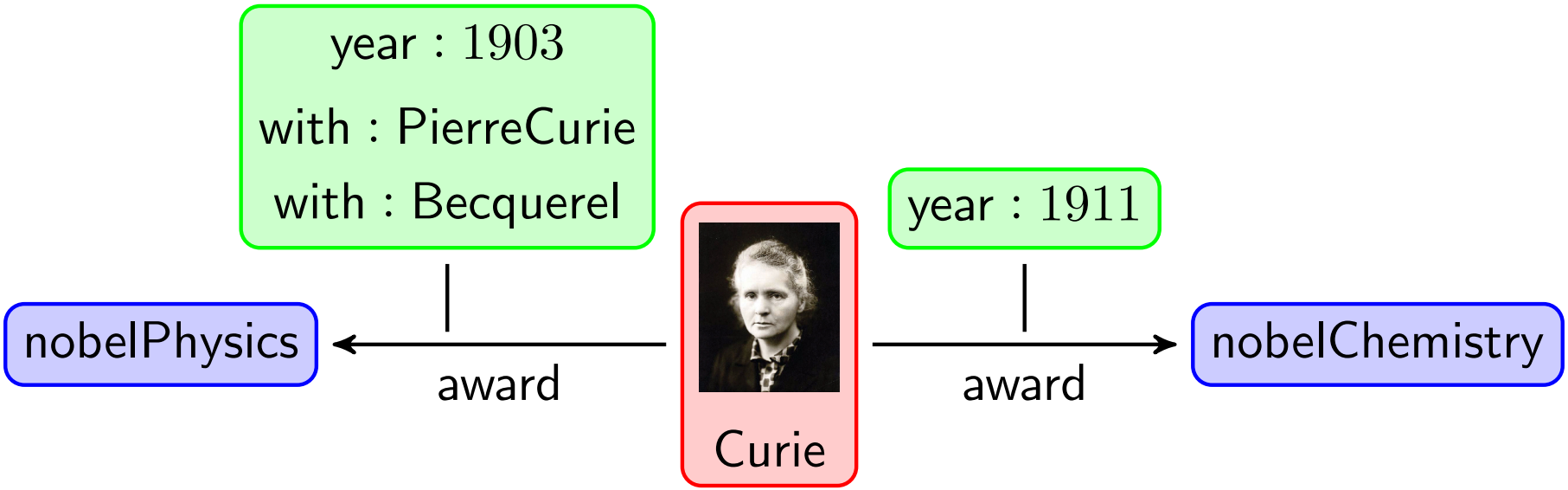

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MARS: multi-attributed relational structure (annotated hypergraph)

Annotation-aware reasoning

Example: for sole "award" winners, infer "laureate," i.e.,

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$\rightsquigarrow$ **Rule**:

$$\forall x, y. \forall S.\ \mathsf{award}(x, y)@S \wedge (\lfloor\rfloor \setminus \lfloor \mathsf{with} : + \rfloor)(S) \rightarrow \mathsf{laureate}(y, x)@S$$

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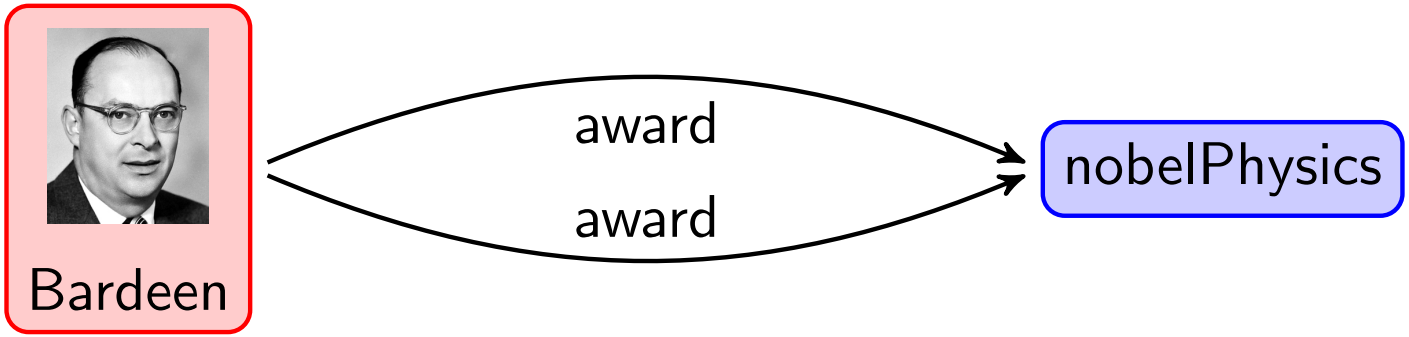
Specifier $(\lfloor\rfloor \setminus \lfloor \mathsf{with} : + \rfloor)$: any annotation set without attribute "with"

✓ $\{\mathsf{year} : 1911\}$

✗ $\{\mathsf{year} : 1903, \mathsf{with} : \mathsf{PierreCurie}, \mathsf{with} : \mathsf{Becquerel}\}$

What else do we need?



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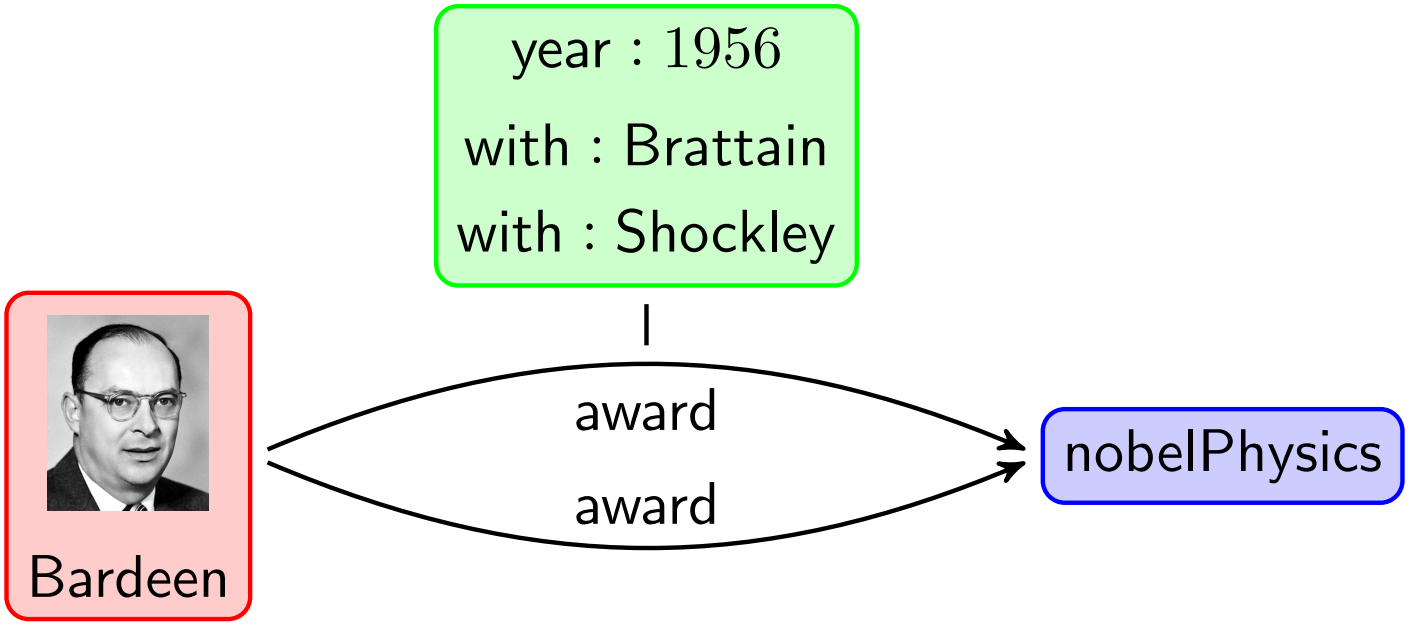

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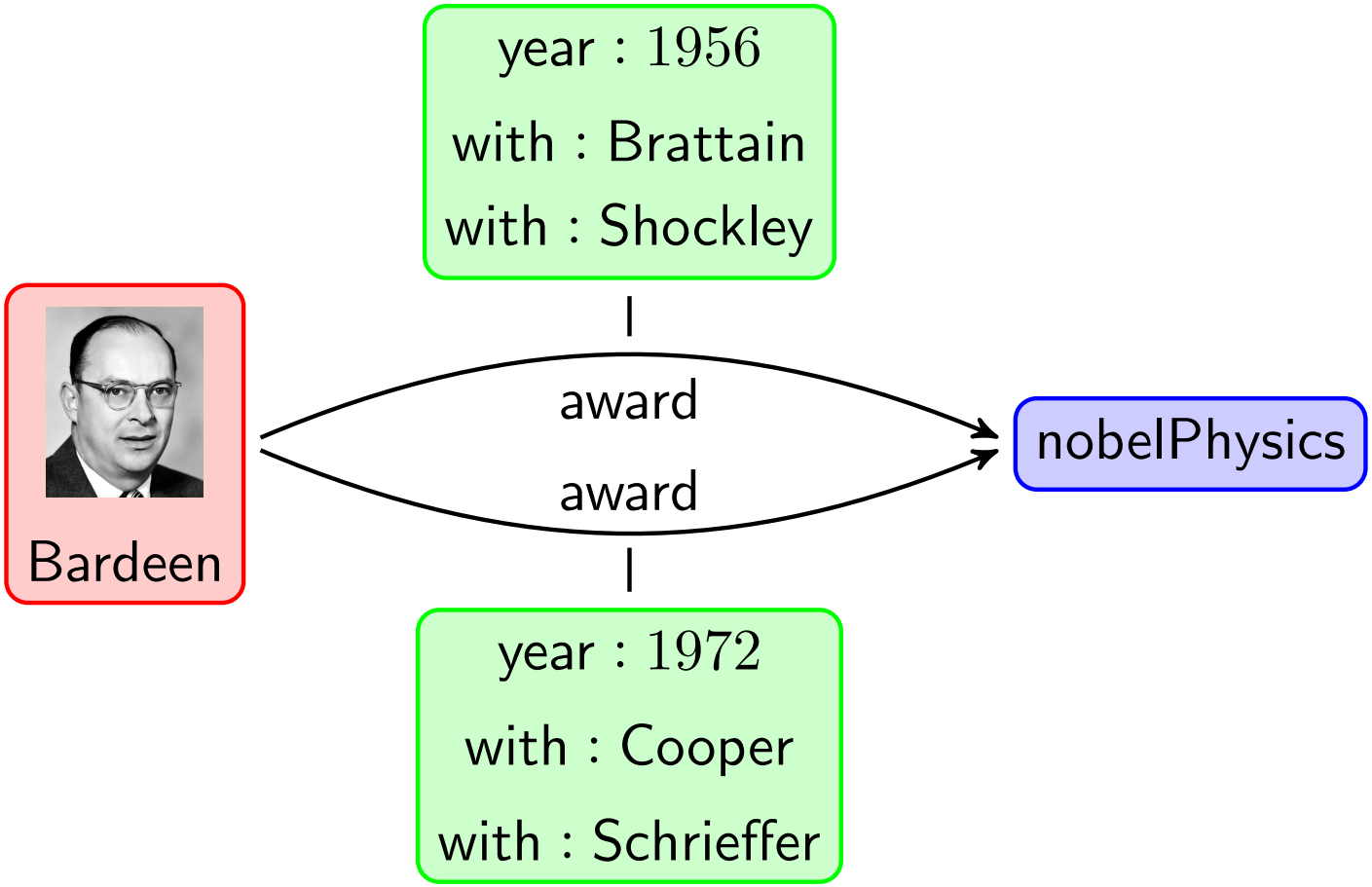

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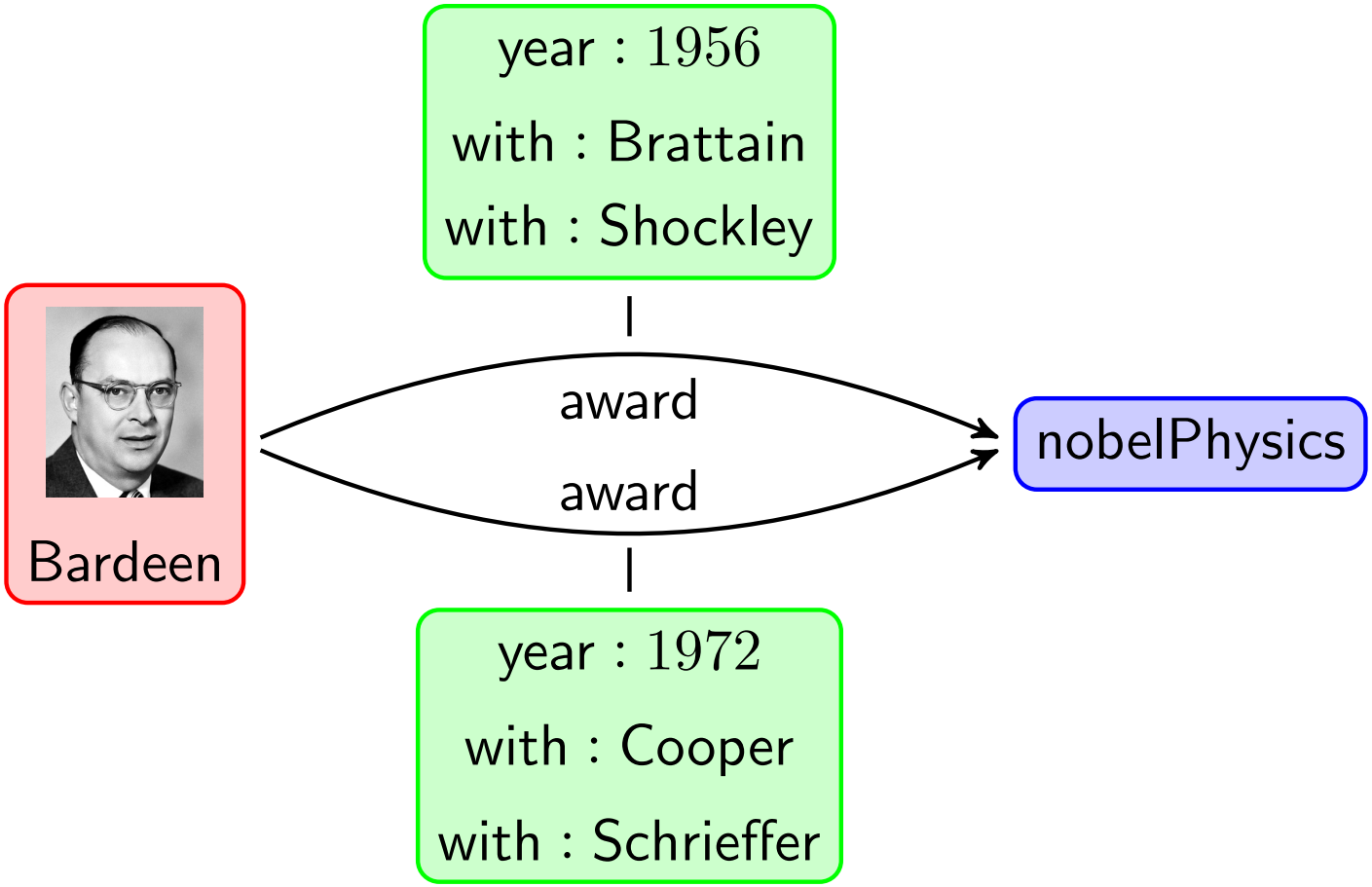

Goal: derive "award" for co-laureates $\rightsquigarrow$ copying annotation sets is not enough, we need to compute a new annotation set

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MAPL has the same expressivity as weak second-order logic.
$\rightsquigarrow$ *Entailment for MAPL theories is not semi-decidable.*

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Idea: Encode arbitrary arity predicates in annotation sets

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A decidable fragment

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with function definition $\mathbf{CoLaureate}(U, v, w)$:

$$\Rightarrow \mathrm{insert}(\mathsf{with} : v)$$

$$\lfloor \mathsf{with} : o \rfloor(U), o \not\approx w \Rightarrow \mathrm{insert}(\mathsf{with} : o)$$

$$\lfloor \mathsf{year} : d \rfloor(U) \Rightarrow \mathrm{insert}(\mathsf{year} : d)$$

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MARPL entailment is PTime*-complete for data complexity if the size of annotation sets is bounded.*

Conclusion & Outlook

Summary:

MAPL general, second-order based framework for attributed logics; not semi-decidable

MARPL decidable, rule-shaped fragment; ExpTime-complete for data & combined complexity

Future Work:

- Create attributed ontologies, e.g., for Wikidata
- Implement a MARPL reasoner
- Identify more expressive decidable fragments of MAPL
- Study attributed versions of other KR formalisms
- Classify data complexities, identify tractable fragments