Pushing the Boundaries of Tractable Multiperspective Reasoning

A Deduction Calculus for Standpoint $\mathscr{EL}$+

Lucía Gómez Álvarez, Sebastian Rudolph, Hannes Strass

International Center for Computational Logic
TECHNISCHE UNIVERSITÄT DRESDEN

Motivation

Multiperspective Reasoning

Motivation: Knowledge Integration

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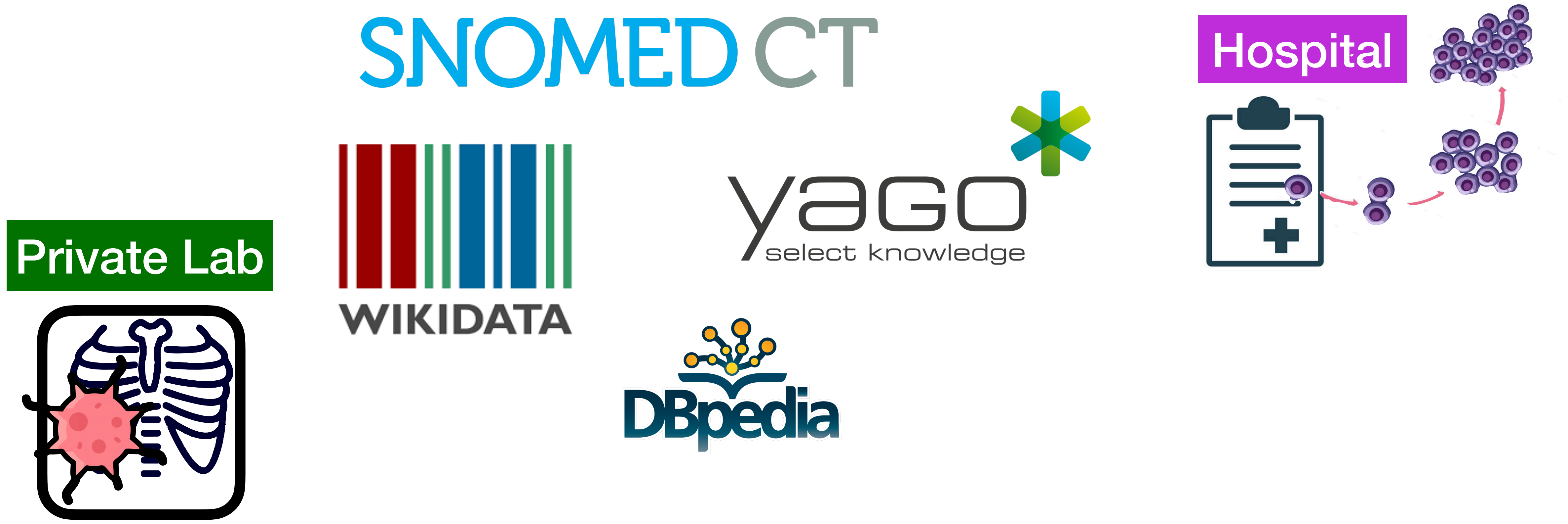
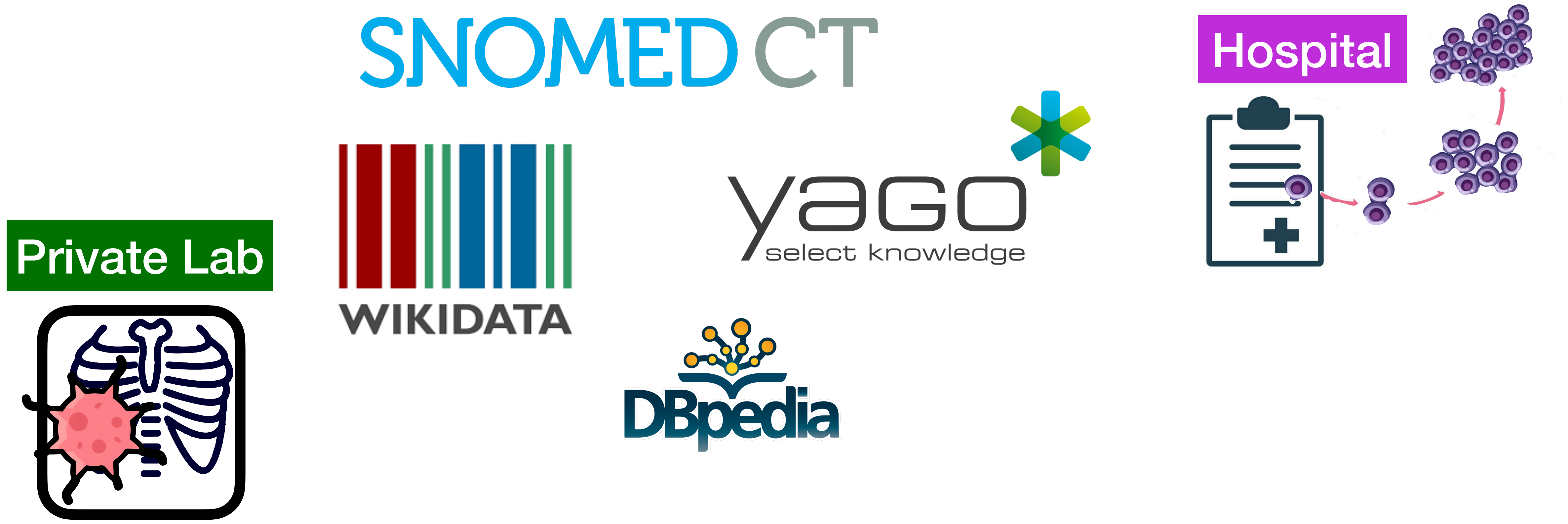


Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available



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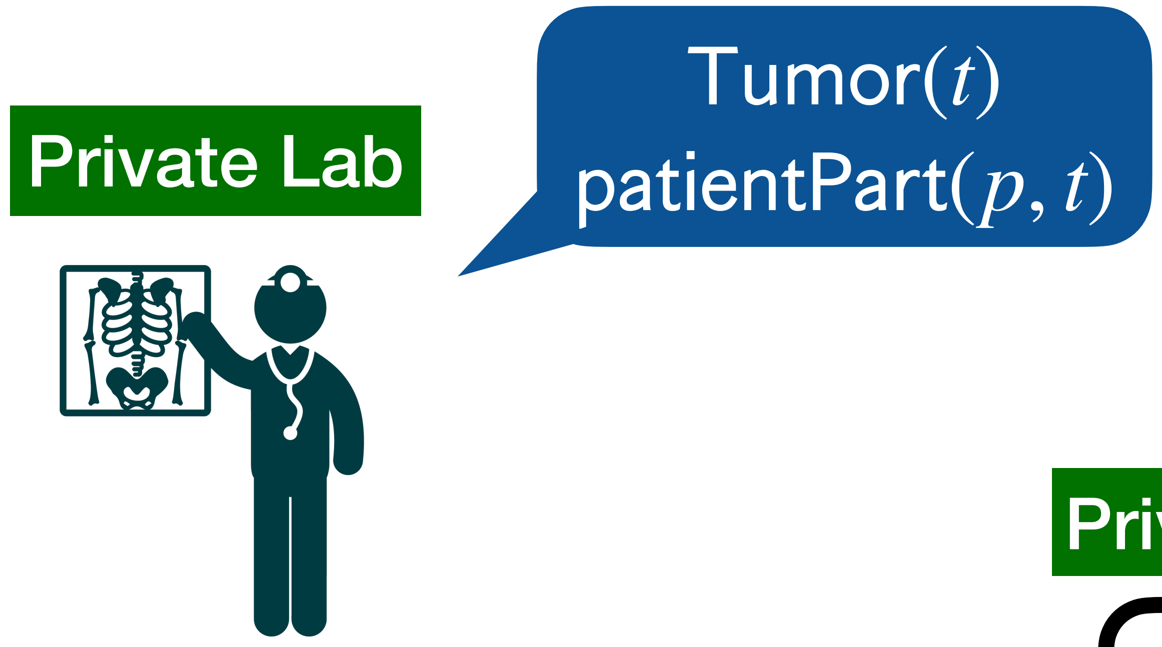
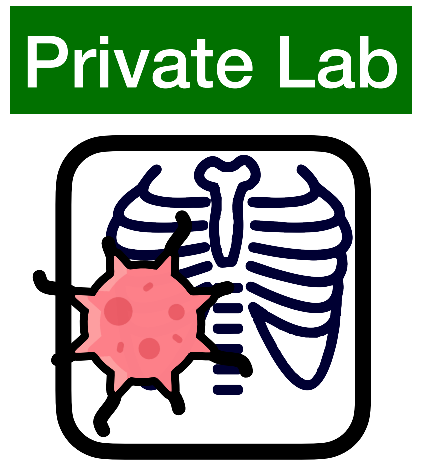


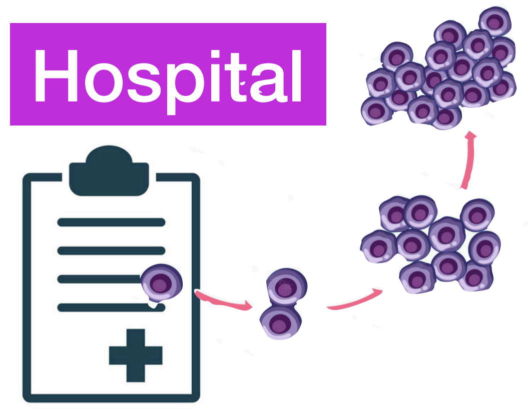


SNOMED CT

WIKIDATA

yago select knowledge

DBpedia



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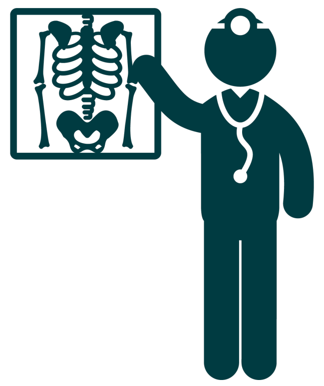
Tumor(t)
patientPart(p, t)



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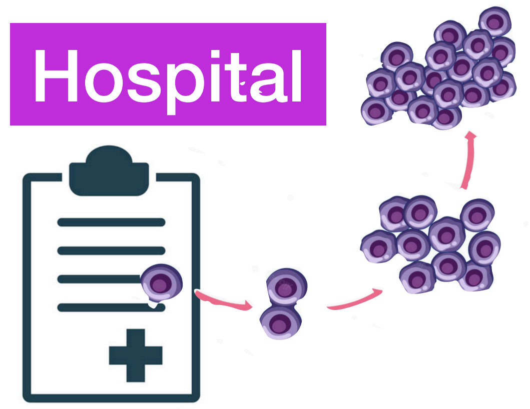

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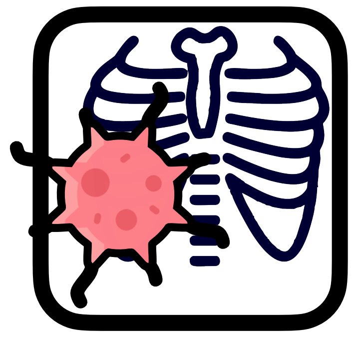

HighRisk(p)

Hospital

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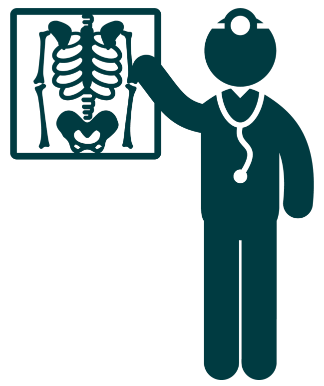

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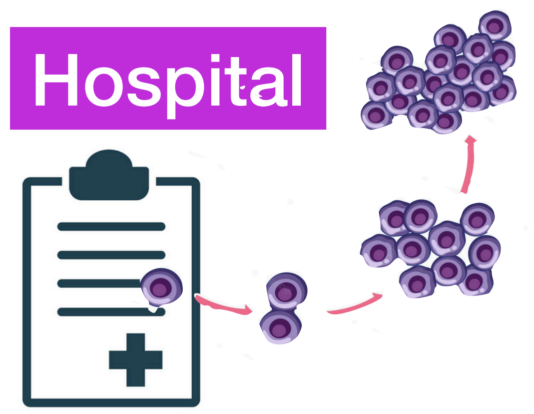

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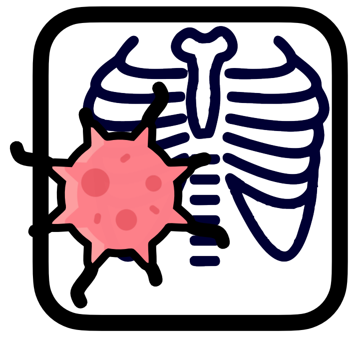

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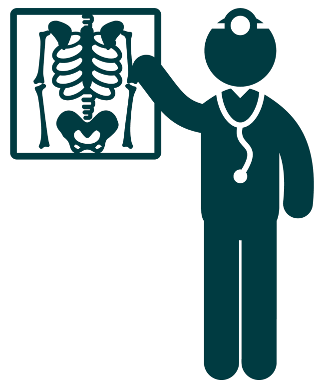
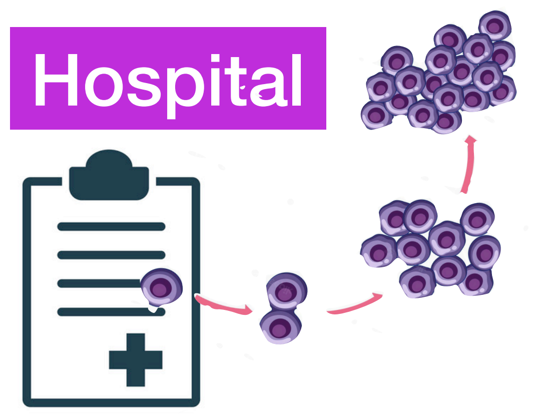


SNOMED CT

Process ⊓ Tissue ⊑ ⊥

Hospital



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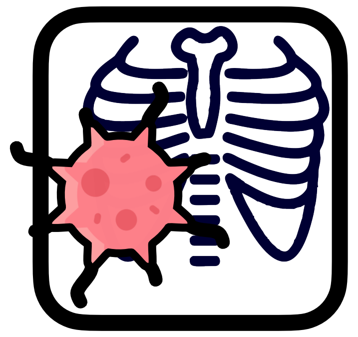

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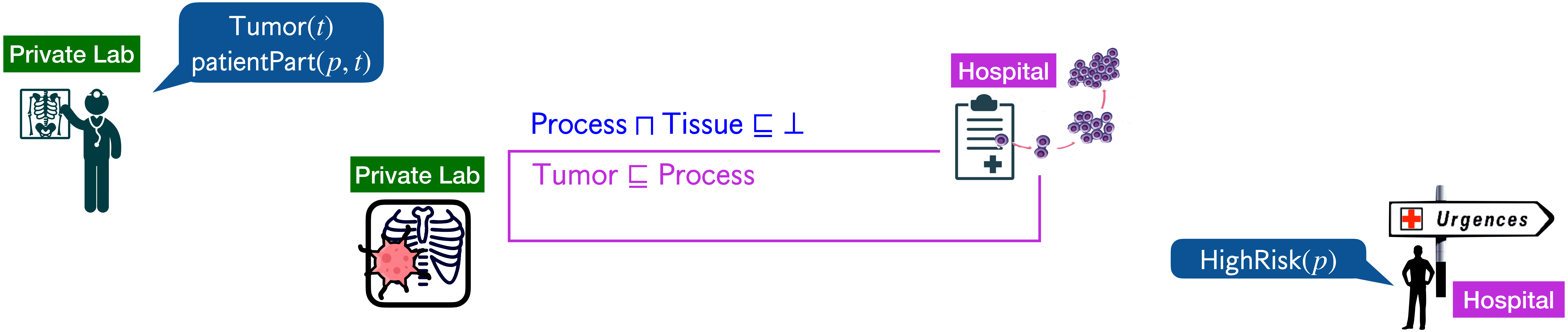

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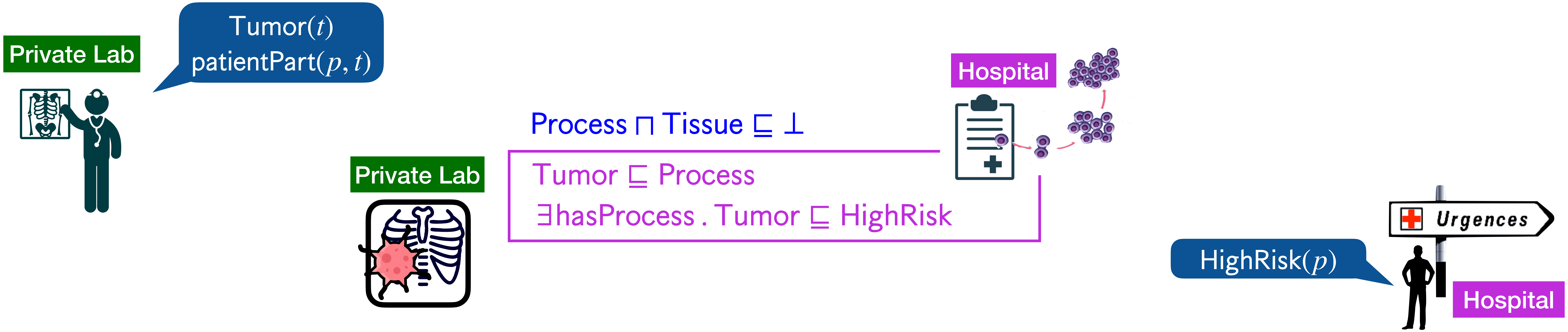

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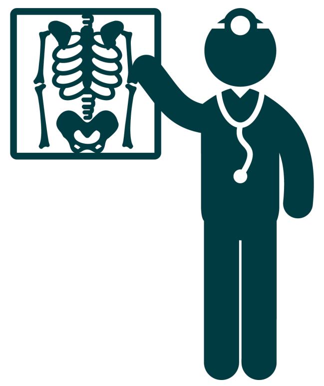
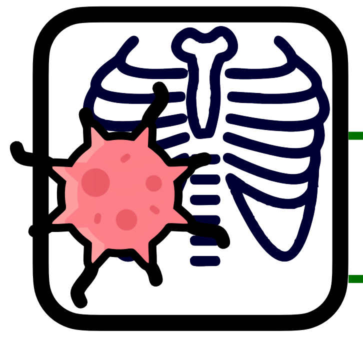


$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

Private Lab



$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$

$\exists \mathsf{hasProcess}.\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$

$\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}$

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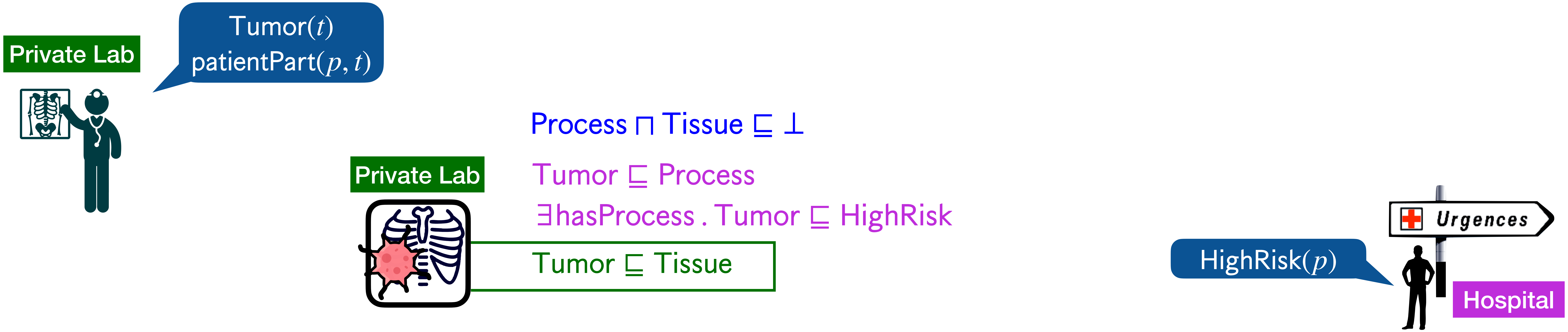

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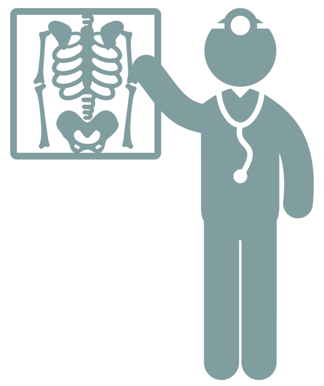

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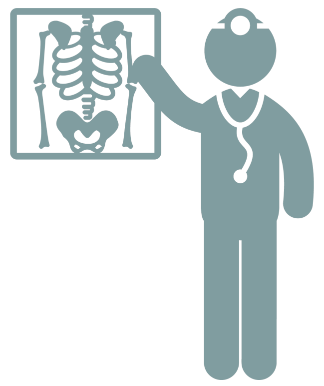

$\mathsf{Tumor}(t)$
$\mathsf{patientPart}(p, t)$

Knowledge Fusion

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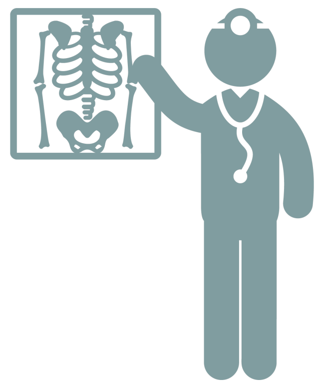

Knowledge Fusion

Process $\sqcap$ Tissue $\sqsubseteq \bot$

Tumor $\sqsubseteq$ Process
$\exists$hasProcess . Tumor $\sqsubseteq$ HighRisk

Tumor $\sqsubseteq$ Tissue

HighRisk(p)

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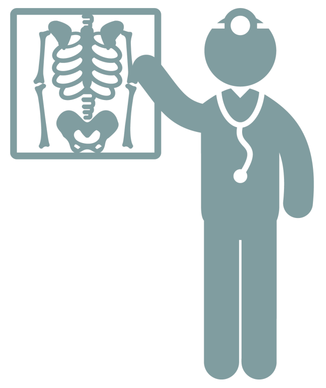

Knowledge Fusion

Process ⊓ Tissue ⊑ ⊥

Tumor ⊑ Process

∃hasProcess . Tumor ⊑ HighRisk

Tumor ⊑ Tissue

HighRisk(p)

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$\text{Tumor}(t)$ → $\text{Process}(t)$

$\text{Tumor}(t)$ → $\text{Tissue}(t)$

Private Lab

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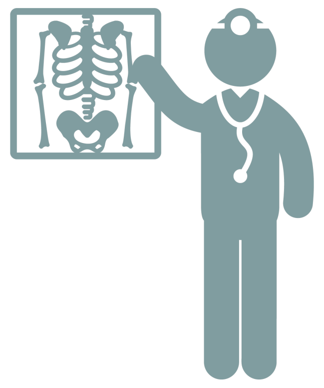

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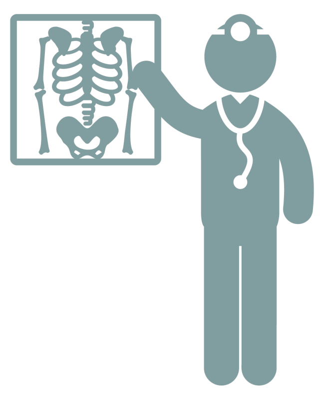

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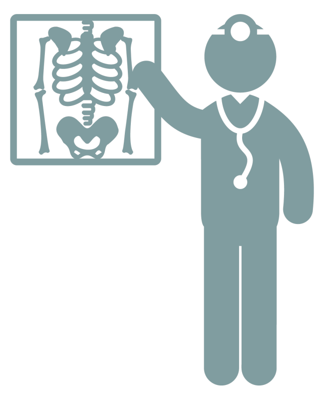

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Challenge: combining diverse (potentially conflicting) sources without weakening them

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➡ **Multimodal logic** characterised by simplified Kripke semantics

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$\Box_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\Diamond_L [\text{Tumor}] \sqsubseteq \Box_L [\text{Tissue}]$

$\Diamond_H [\text{Tumor}] \sqsubseteq \Box_H [\text{Process}]$

$\Box_H [\exists \text{hasProcess} . \text{Tumor} \sqsubseteq \text{HighRisk}]$

$\Box_e$ Unequivocal to e
$\Diamond_e$ Conceivable to e

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$\square_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\lozenge_L [\text{Tumor}] \sqsubseteq \square_L [\text{Tissue}]$

$\lozenge_H [\text{Tumor}] \sqsubseteq \square_H [\text{Process}]$

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$(L \cup H) \preceq S$

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(L and H inherit the axioms of S)

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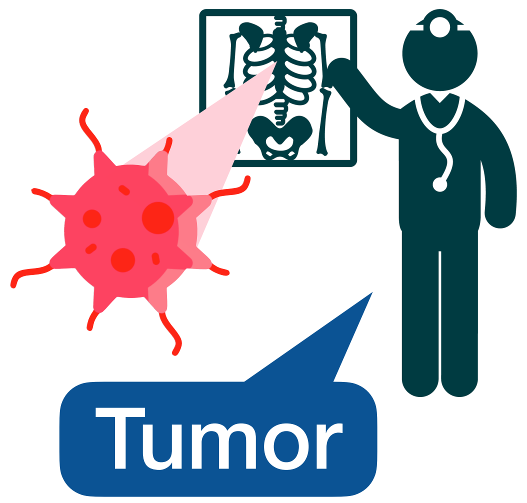
$\Diamond_L \text{Tumor}(t)$ $\Box_L \text{patientPart}(p, t)$

Private Lab



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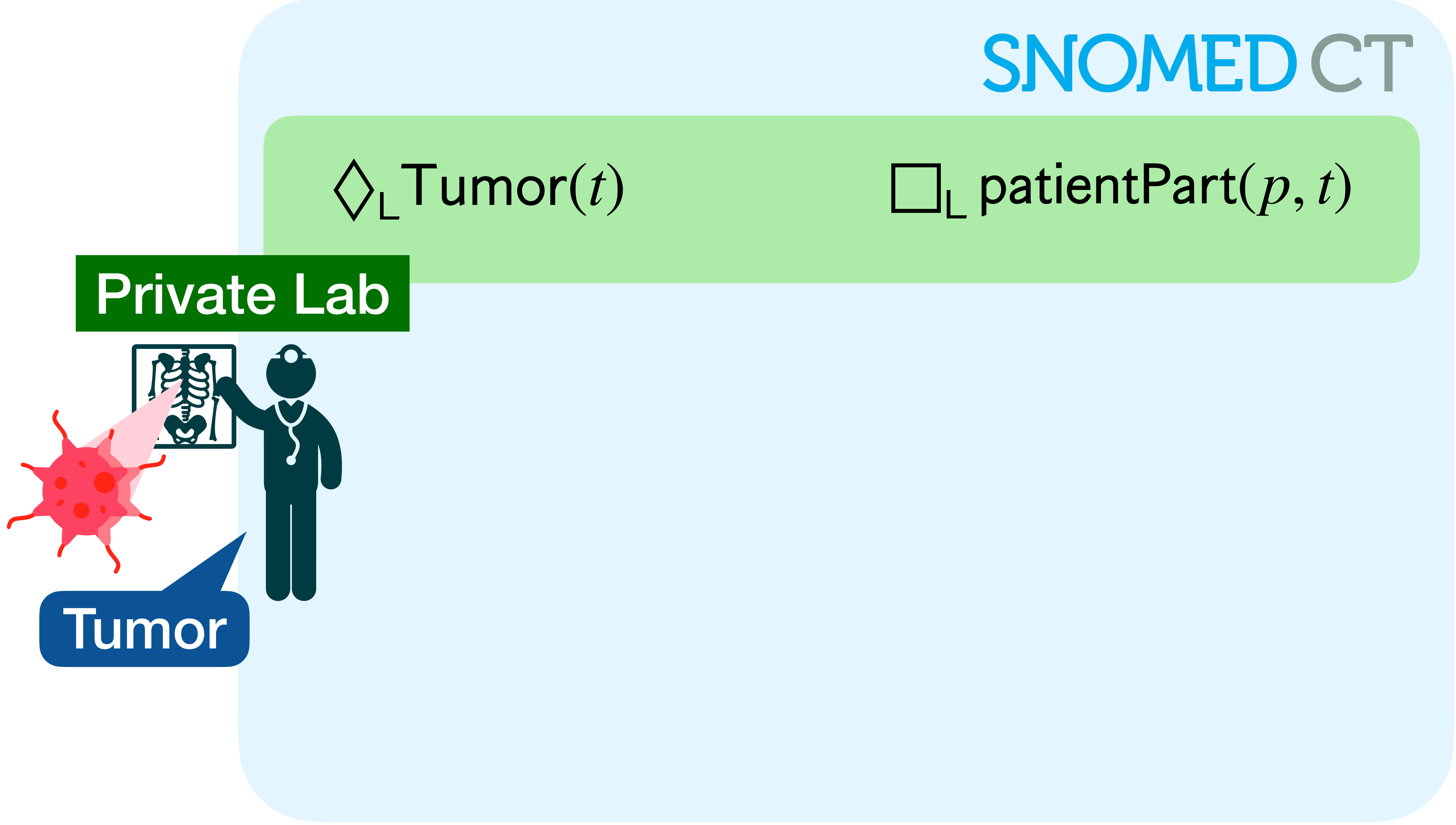

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SNOMED CT

$\lozenge_L \text{Tumor}(t)$ $\square_L \text{patientPart}(p, t)$

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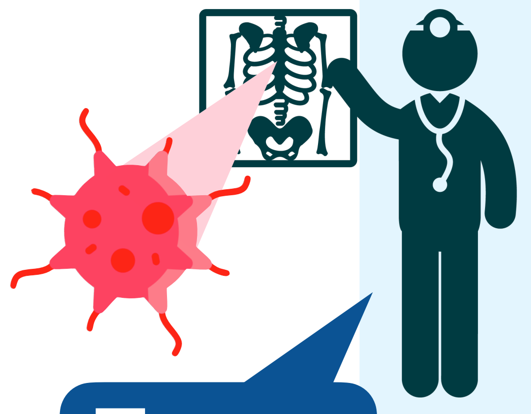

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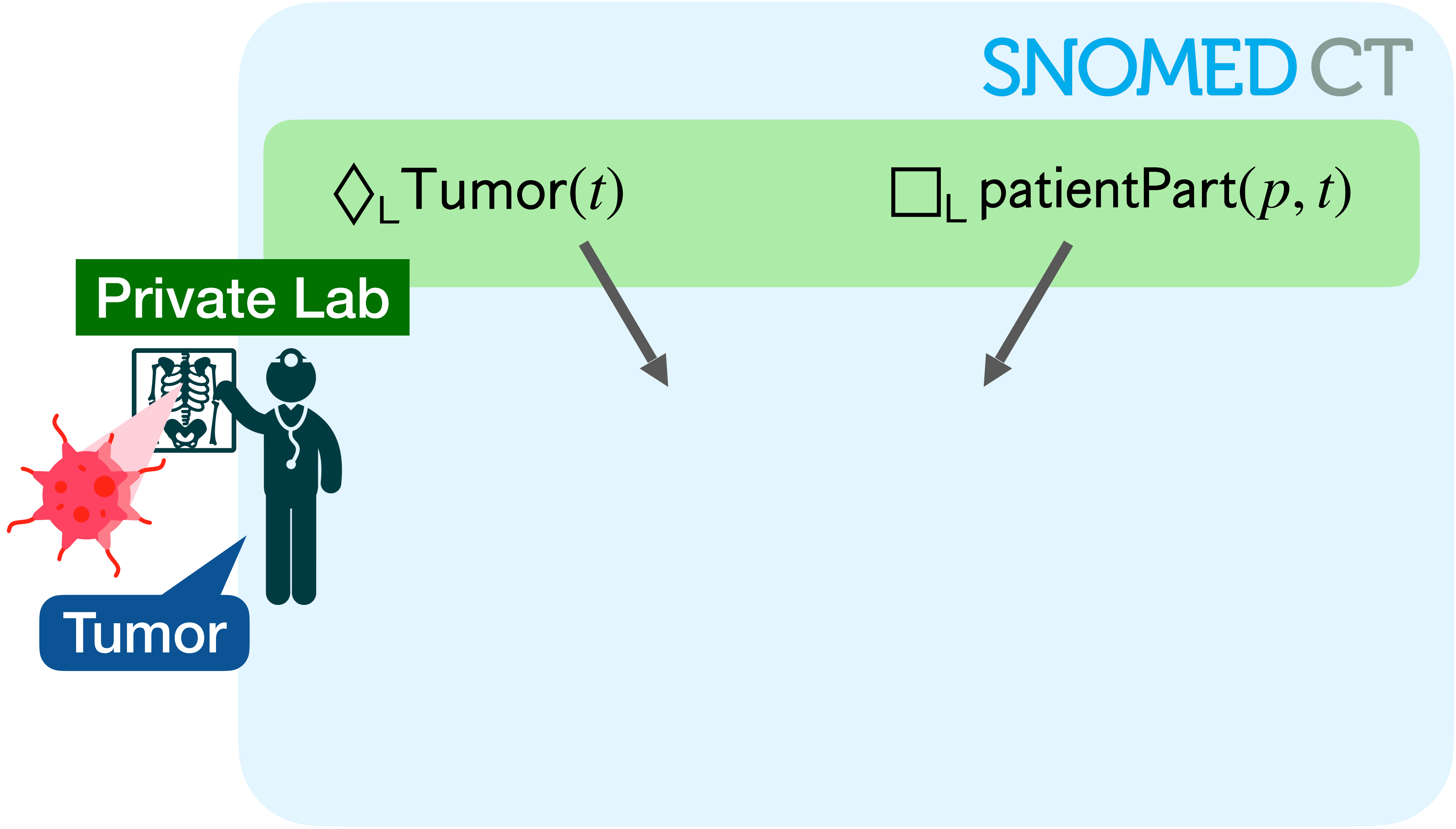

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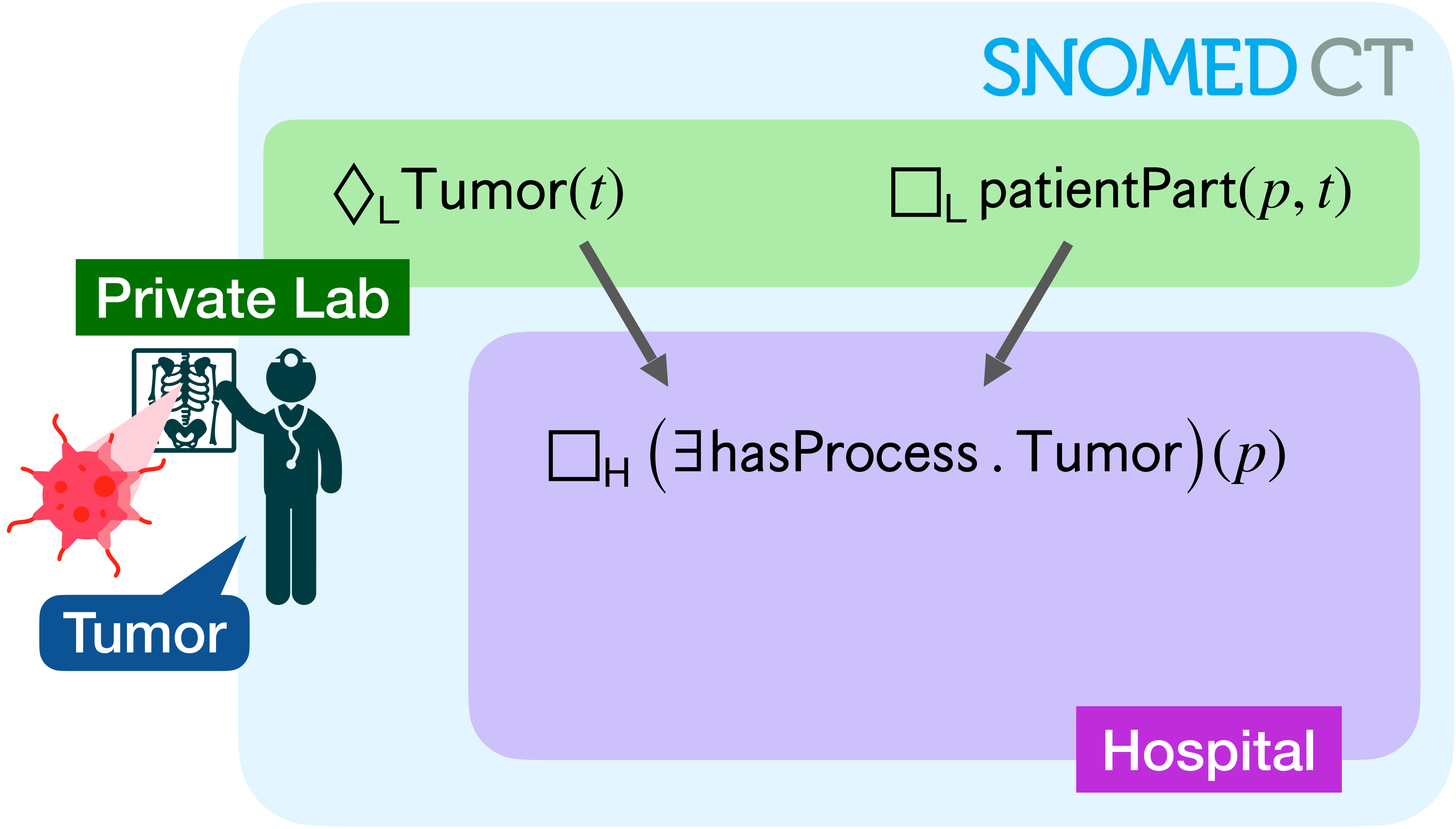

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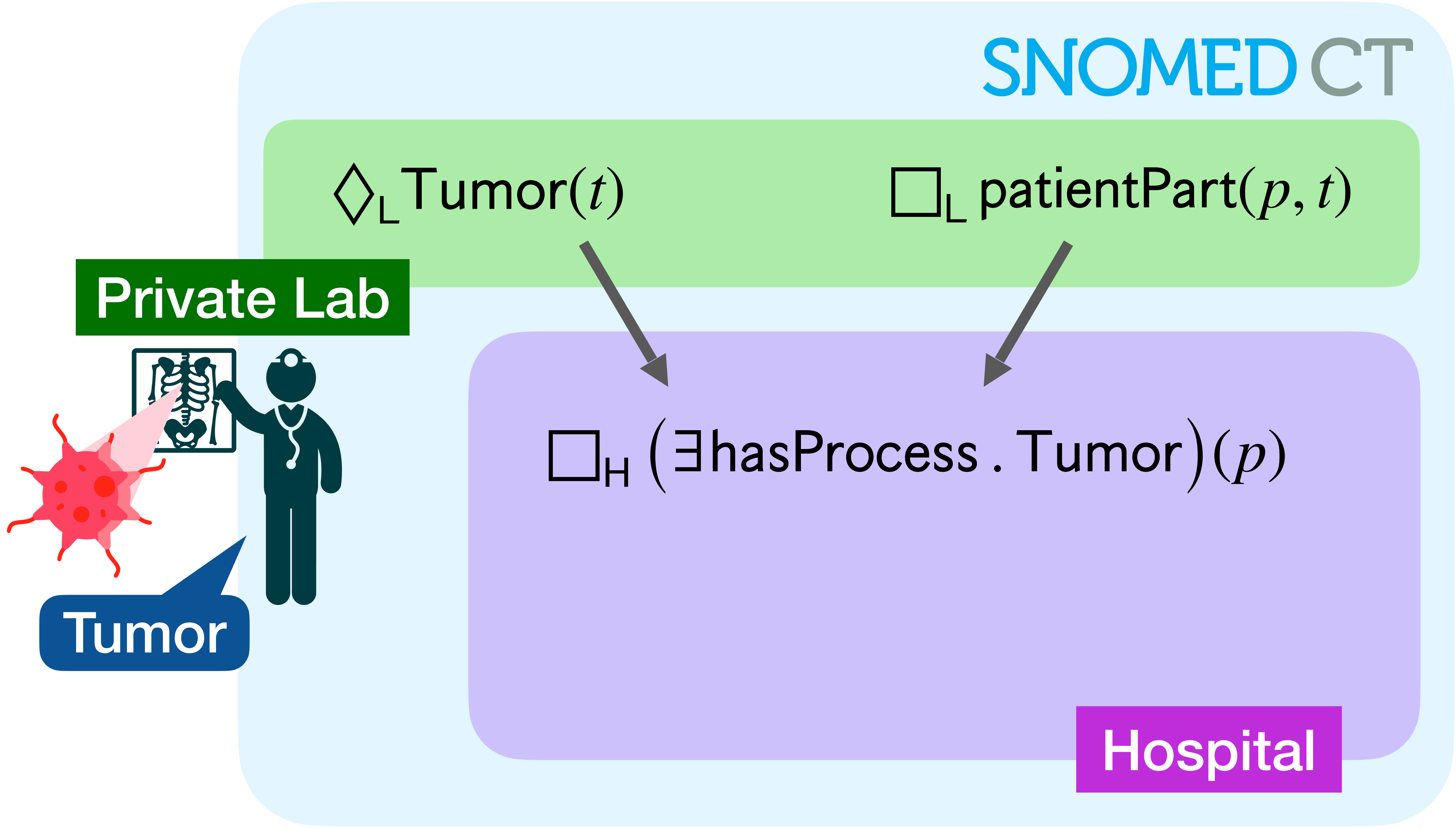

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➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to "points of view" (standpoints)

$\Box_S\,[\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\Diamond_L\,[\text{Tumor}] \sqsubseteq \Box_L\,[\text{Tissue}]$

$\Diamond_H\,[\text{Tumor}] \sqsubseteq \Box_H\,[\text{Process}]$

$\Box_H\,[\exists\,\text{hasProcess}\,.\,\text{Tumor} \sqsubseteq \text{HighRisk}]$

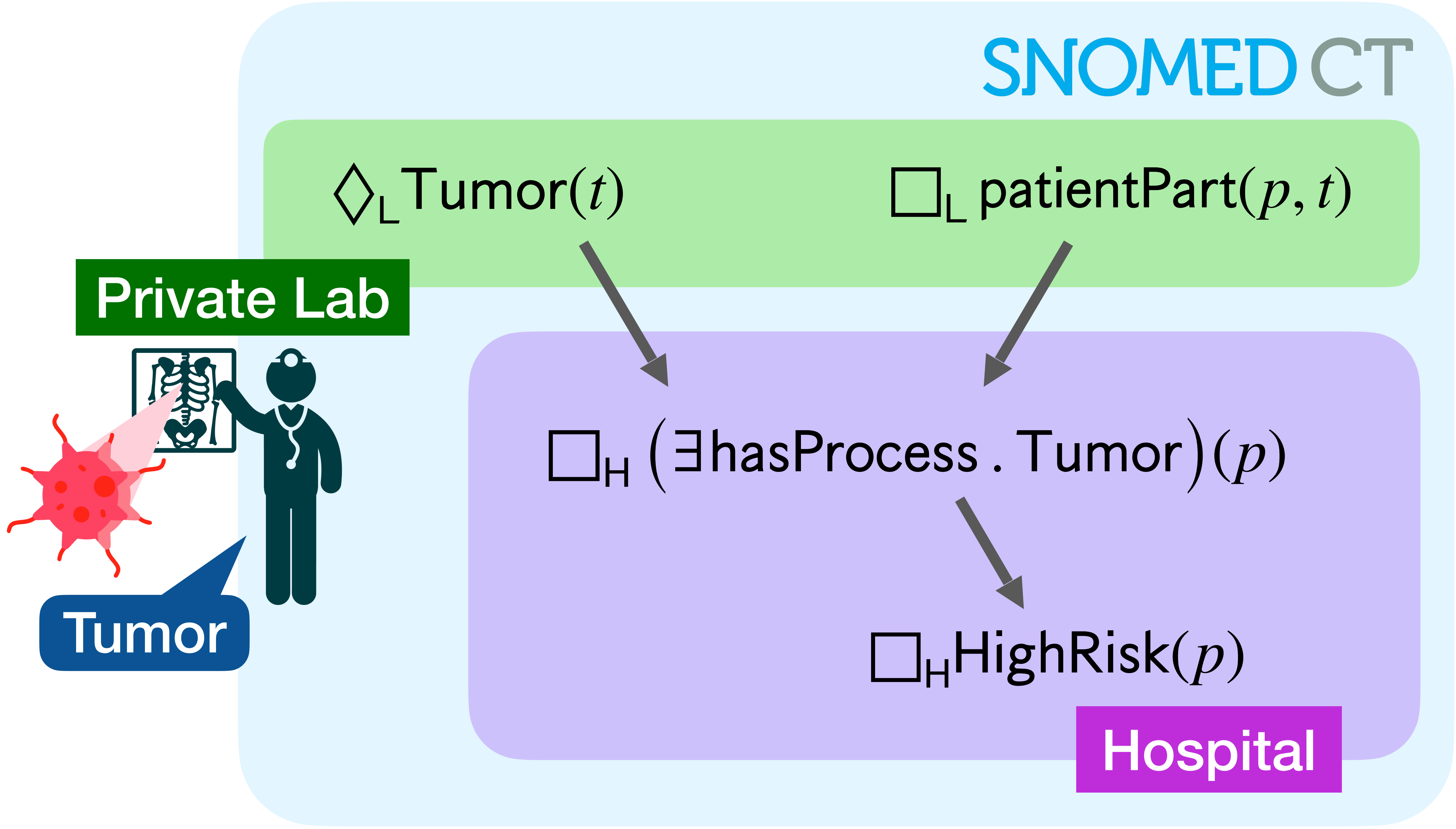
$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$\Diamond_L\,[\exists\,\text{patientPart}\,.\,\text{Tumor}] \sqsubseteq \Box_H\,[\exists\,\text{hasProcess}\,.\,\text{Tumor}]$

CONSISTENT

$\Box_e$ Unequivocal to e

$\Diamond_e$ Conceivable to e



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to “points of view” (standpoints)

CONSISTENT

$\square_e$ Unequivocal to e
$\lozenge_e$ Conceivable to e

$\square_S\,[\mathrm{Process} \sqcap \mathrm{Tissue} \sqsubseteq \bot]$

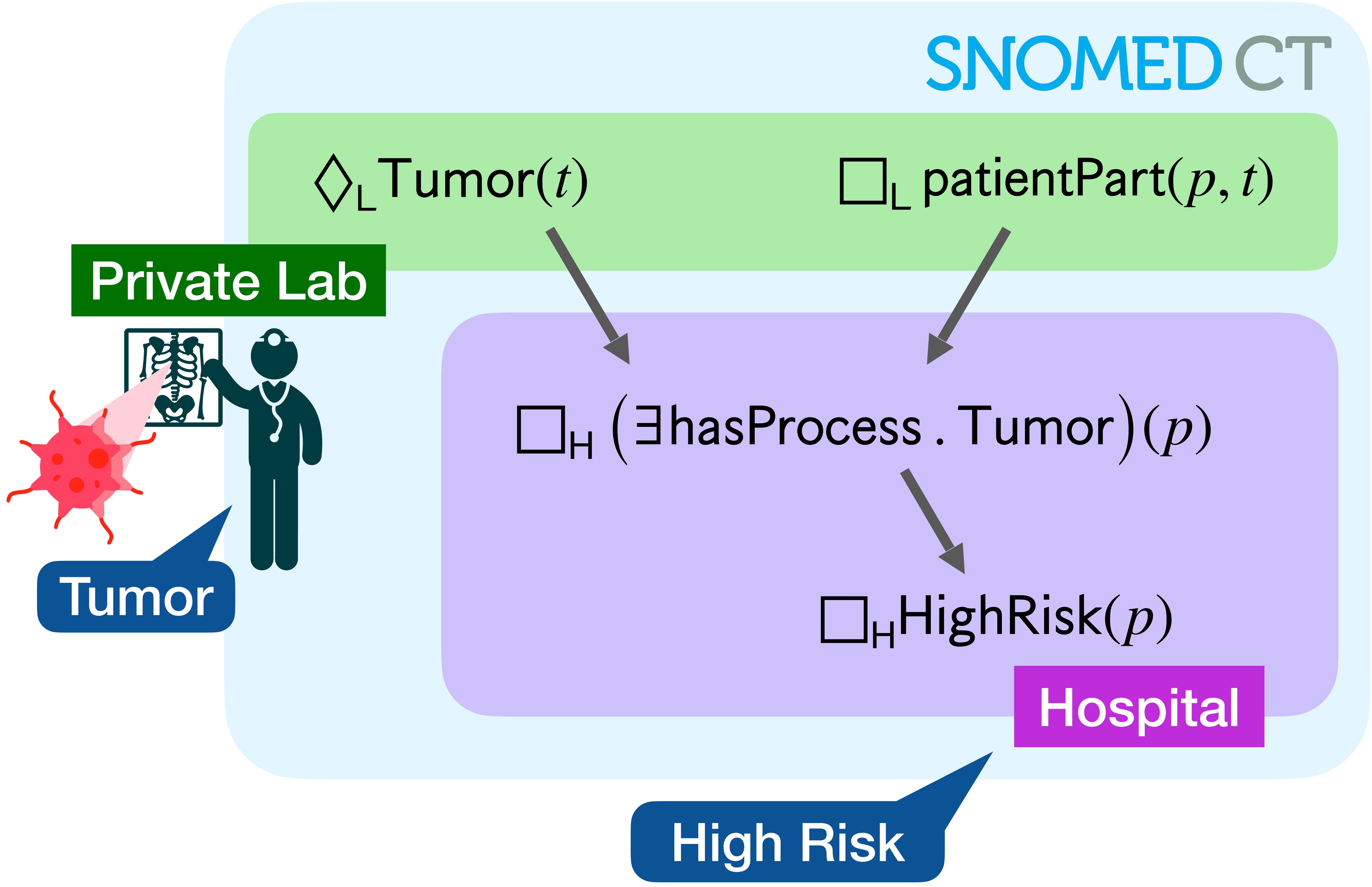
$\lozenge_L\,[\mathrm{Tumor}] \sqsubseteq \square_L\,[\mathrm{Tissue}]$

$\lozenge_H\,[\mathrm{Tumor}] \sqsubseteq \square_H\,[\mathrm{Process}]$

$\square_H\,[\exists \mathrm{hasProcess}.\mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}]$

$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$\lozenge_L\,[\exists \mathrm{patientPart}.\mathrm{Tumor}] \sqsubseteq \square_H\,[\exists \mathrm{hasProcess}.\mathrm{Tumor}]$



Standpoint $\mathscr{EL}$+

The description logic $\mathscr{EL}$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The description logic $\mathscr{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathrm{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_C$, $r \in N_R$

The description logic $\mathcal{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_C, r \in N_R$

Tissue

The description logic $\mathscr{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

Process ⊓ Tissue

The description logic $\mathcal{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_C, r \in N_R$

Tissue

Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

The description logic $\mathcal{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$$

With $A \in N_C$, $r \in N_R$

Tissue

Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$

(Tumor $\sqsubseteq$ Tissue)

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \, . \, C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} \, . \, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart} \, . \, \mathsf{Tumor})(p)$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$$

With $A \in N_C$, $r \in N_R$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue)

($\exists$patientPart . Tumor)(p)

Semantics:

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$$

With $A \in N_C$, $r \in N_R$

Tissue

Process $\sqcap$ Tissue $\quad\quad$ $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$
- Assertions: $C(a)$, $\quad r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue})$ $\quad\quad$ $(\exists \text{patientPart} . \text{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$

With $A \in N_C, r \in N_R$

Tissue

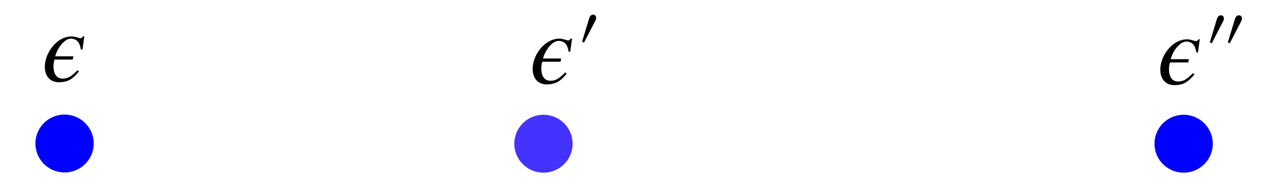
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathcal{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_C, r \in N_R$

Tissue

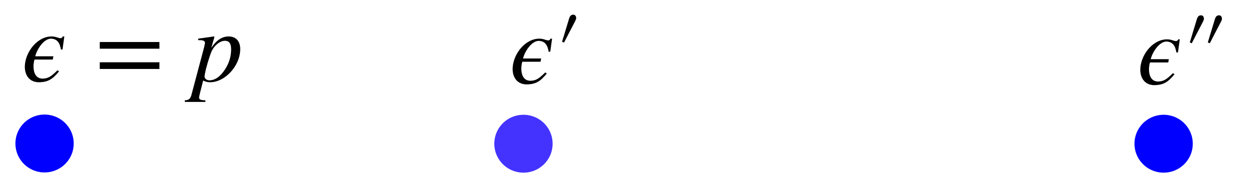
Process $\sqcap$ Tissue $\qquad \exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$
- Assertions: $C(a), \quad r(a,b)$

$(\text{Tumor} \sqsubseteq \text{Tissue}) \qquad (\exists\text{patientPart}\,.\,\text{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$$

With $A \in N_C$, $r \in N_R$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

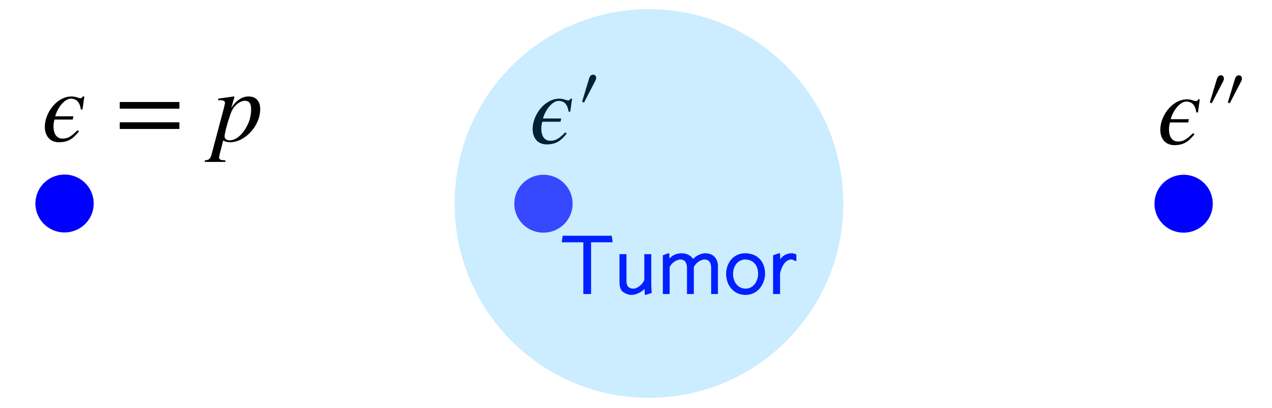
The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue)

($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue

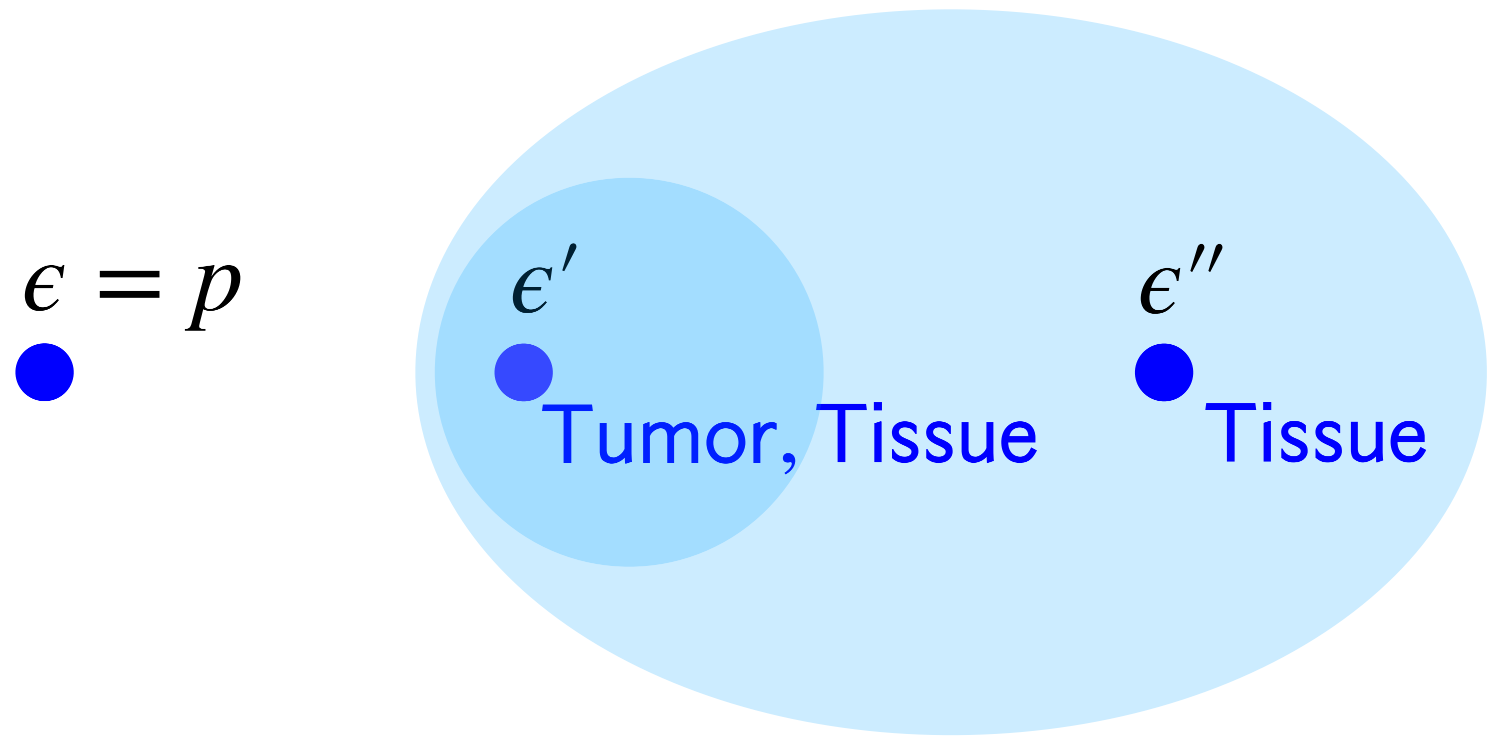
$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart} \,.\, \mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_C$, $r \in N_R$

Tissue

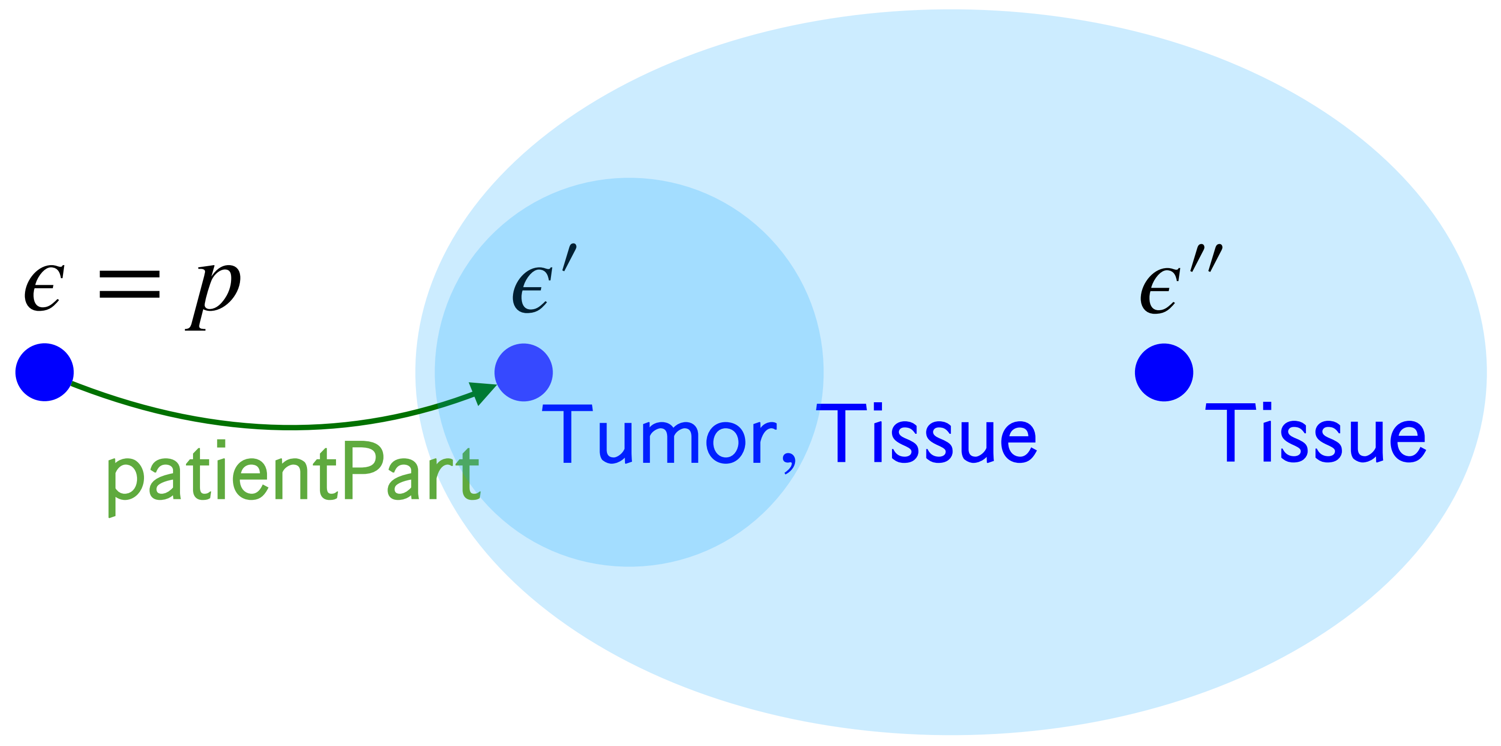
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue)   ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_C, r \in N_R$

Tissue

Process $\sqcap$ Tissue

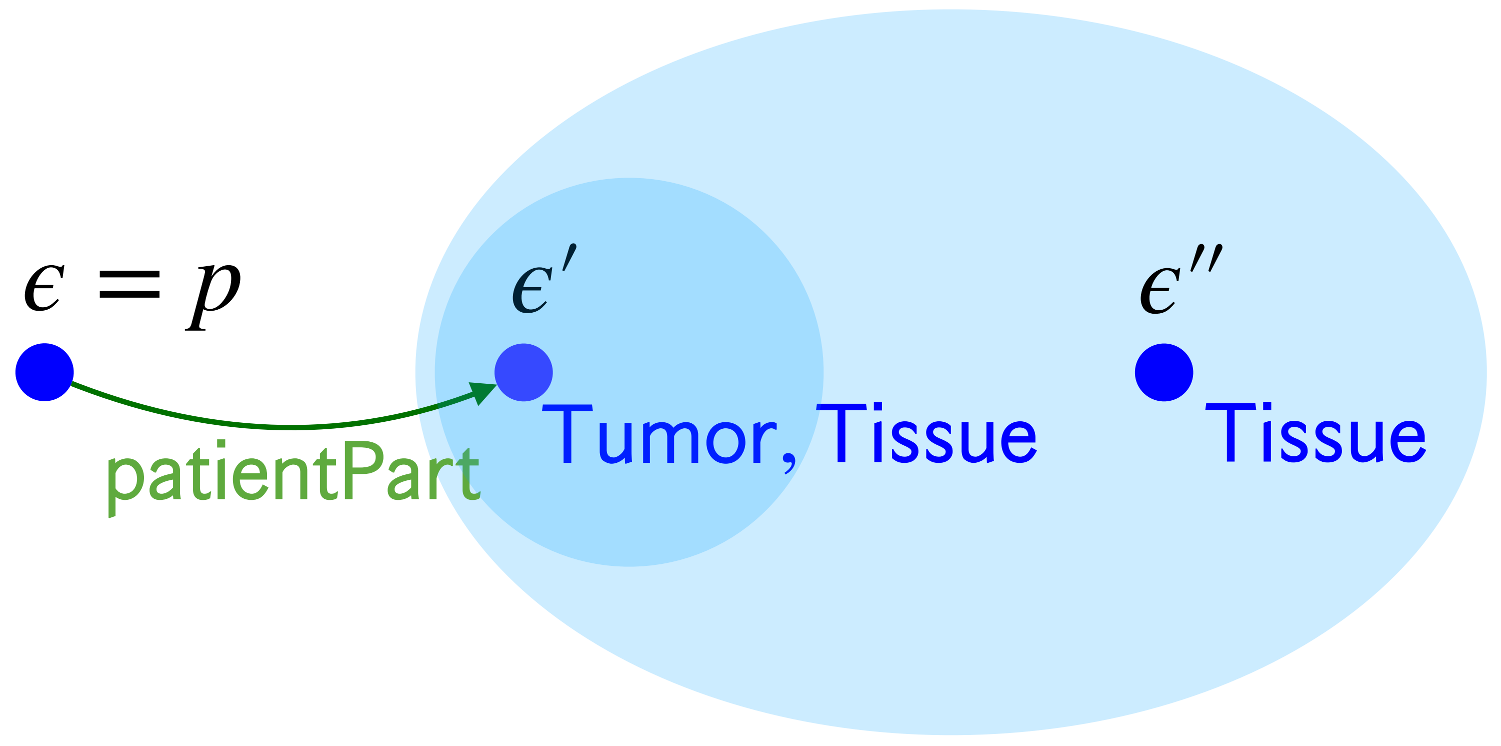
$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$

With $A \in N_C, r \in N_R$

Tissue

Process $\sqcap$ Tissue

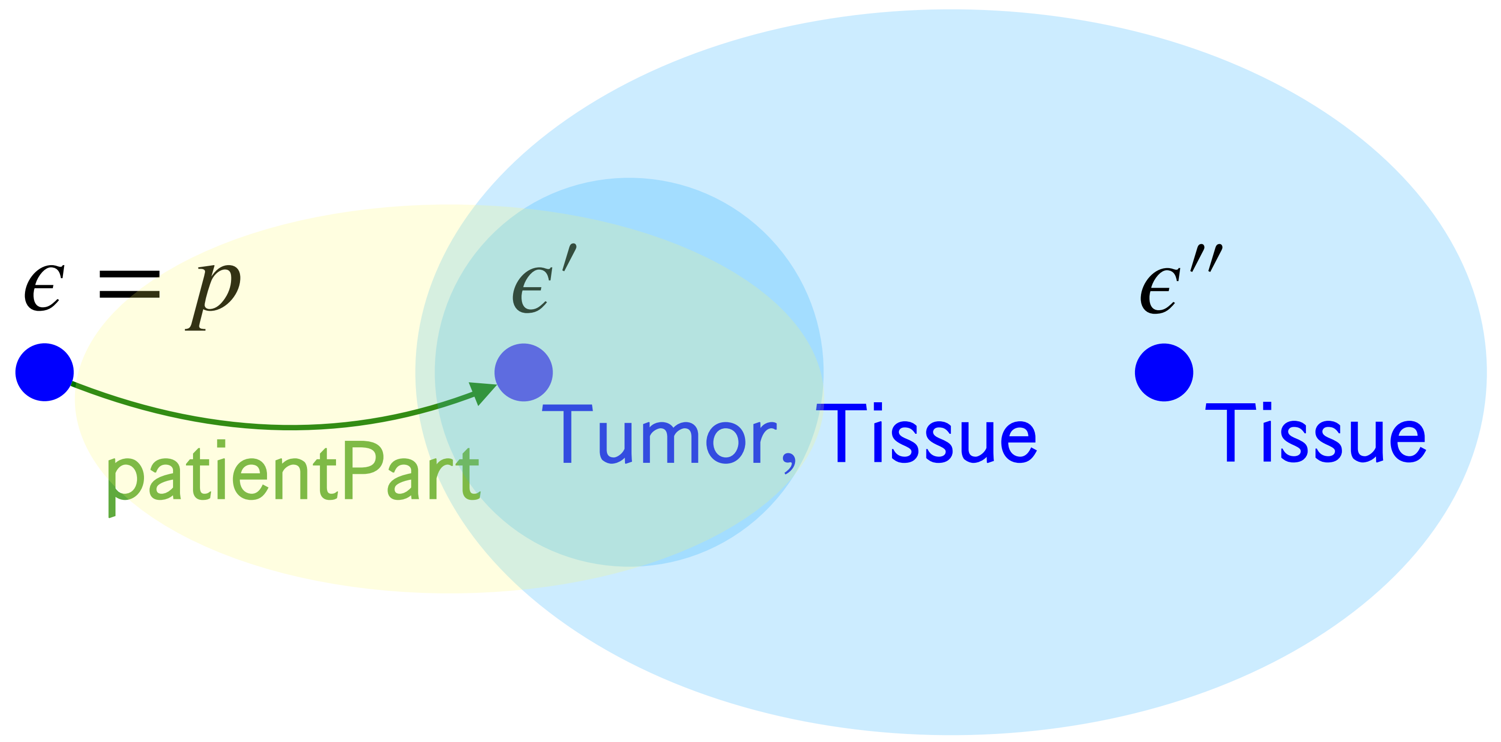
$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

Process $\sqcap$ Tissue $\quad\quad \exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$
- Assertions: $C(a), \quad r(a, b)$

(Tumor $\sqsubseteq$ Tissue) $\quad\quad$ ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$$

With $A \in N_C, r \in N_R$

Tissue

Process $\sqcap$ Tissue　　$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue})$　　$(\exists\text{patientPart} . \text{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists r \,.\, Self$$

With $A \in N_C, r \in N_R$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{EL}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists r \,.\, \mathit{Self}$$

With $A \in N_C, r \in N_R$

Tissue $\exists \mathsf{diagnoses} \,.\, \mathsf{Self}$

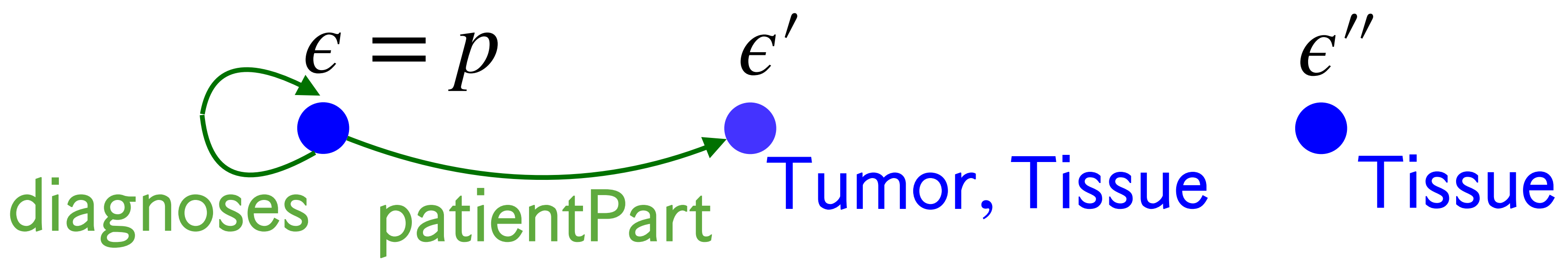
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart} \,.\, \mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,Self$$

With $A \in N_C, r \in N_R$

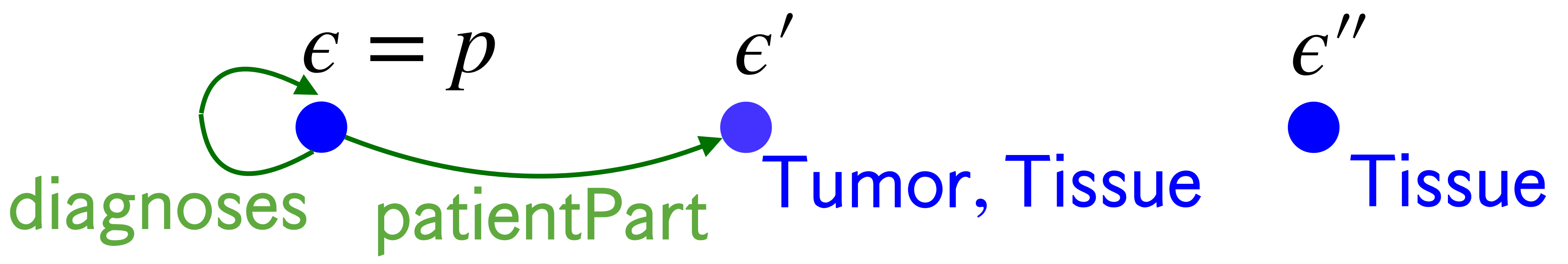
Tissue $\exists$diagnoses . Self
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$+

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self}$

With $A \in N_C, r \in N_R$

Tissue $\quad \exists \text{diagnoses}\,.\,\text{Self}$

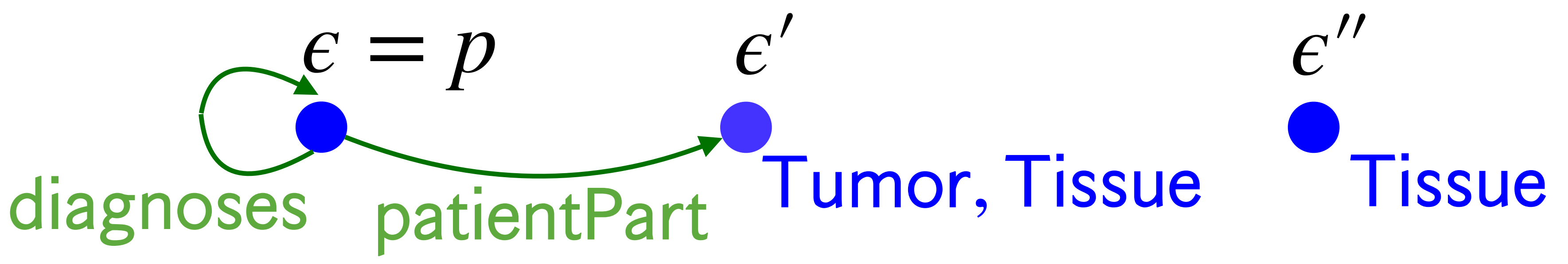
$\text{Process} \sqcap \text{Tissue} \quad \exists \text{patientPart}\,.\,\text{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue}) \quad (\text{patientPart} \circ \text{hasPart} \sqsubseteq \text{patientPart})$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathcal{EL}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . Self$

With $A \in N_C, r \in N_R$

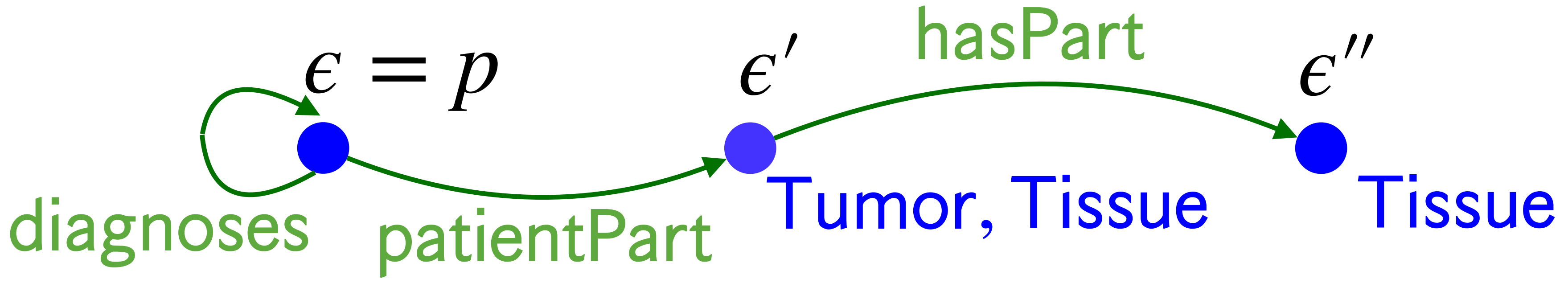
Tissue $\exists$diagnoses . Self
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) (patientPart $\circ$ hasPart $\sqsubseteq$ patientPart)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r.C \mid \exists r.\mathit{Self}$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

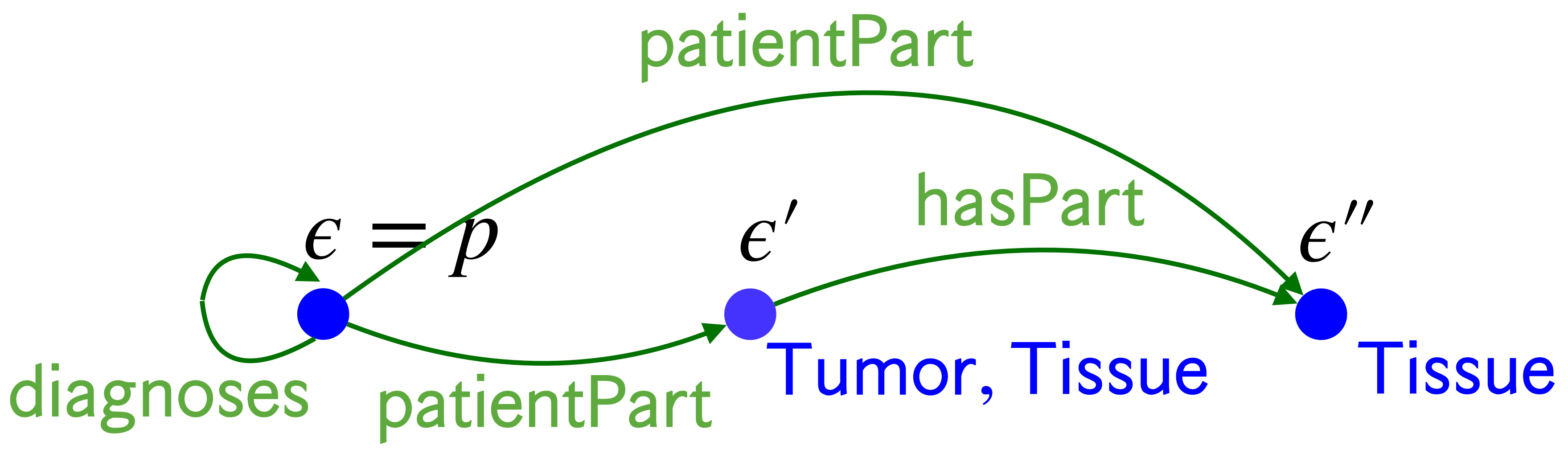
Tissue $\quad \exists$diagnoses . Self
Process $\sqcap$ Tissue $\quad \exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) $\quad$ (patientPart $\circ$ hasPart $\sqsubseteq$ patientPart)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



Towards Standpoint-$\mathscr{EL}$+

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists r \,.\, Self$$

With $A \in N_C, r \in N_R$

Tissue $\quad \exists \text{diagnoses} \,.\, \text{Self}$

$\text{Process} \sqcap \text{Tissue} \quad \exists \text{patientPart} \,.\, \text{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue}) \qquad (\exists \text{patientPart} \,.\, \text{Tumor})(p)$

Towards Standpoint-$\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S}\rangle$ of concept, role, individual and standpoint names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,Self$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue $\exists \mathsf{diagnoses}\,.\,\mathsf{Self}$
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor})(p)$

Towards Standpoint-$\mathscr{EL}$+

Vocabulary $\langle N_C, N_R, N_I, N_S \rangle$ of concept, role, individual and standpoint names, $* \in N_S$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . Self$

With $A \in N_C, r \in N_R$

Tissue $\quad \exists$diagnoses . Self

Process $\sqcap$ Tissue $\quad \exists$patientPart . Tumor

The **set of axioms** includes:

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(Tumor $\sqsubseteq$ Tissue) $\quad$ ($\exists$patientPart . Tumor)(p)

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Process ⊓ Tissue ∃patientPart . Tumor

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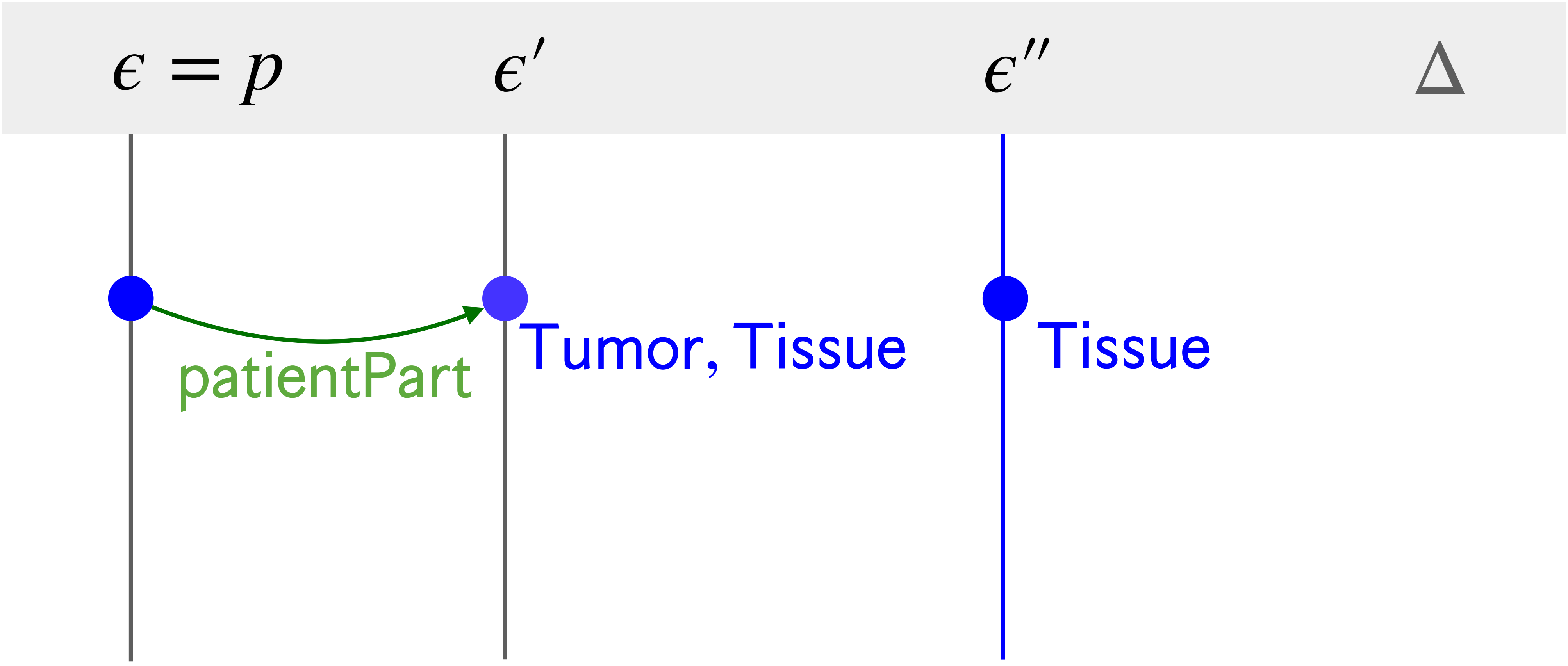

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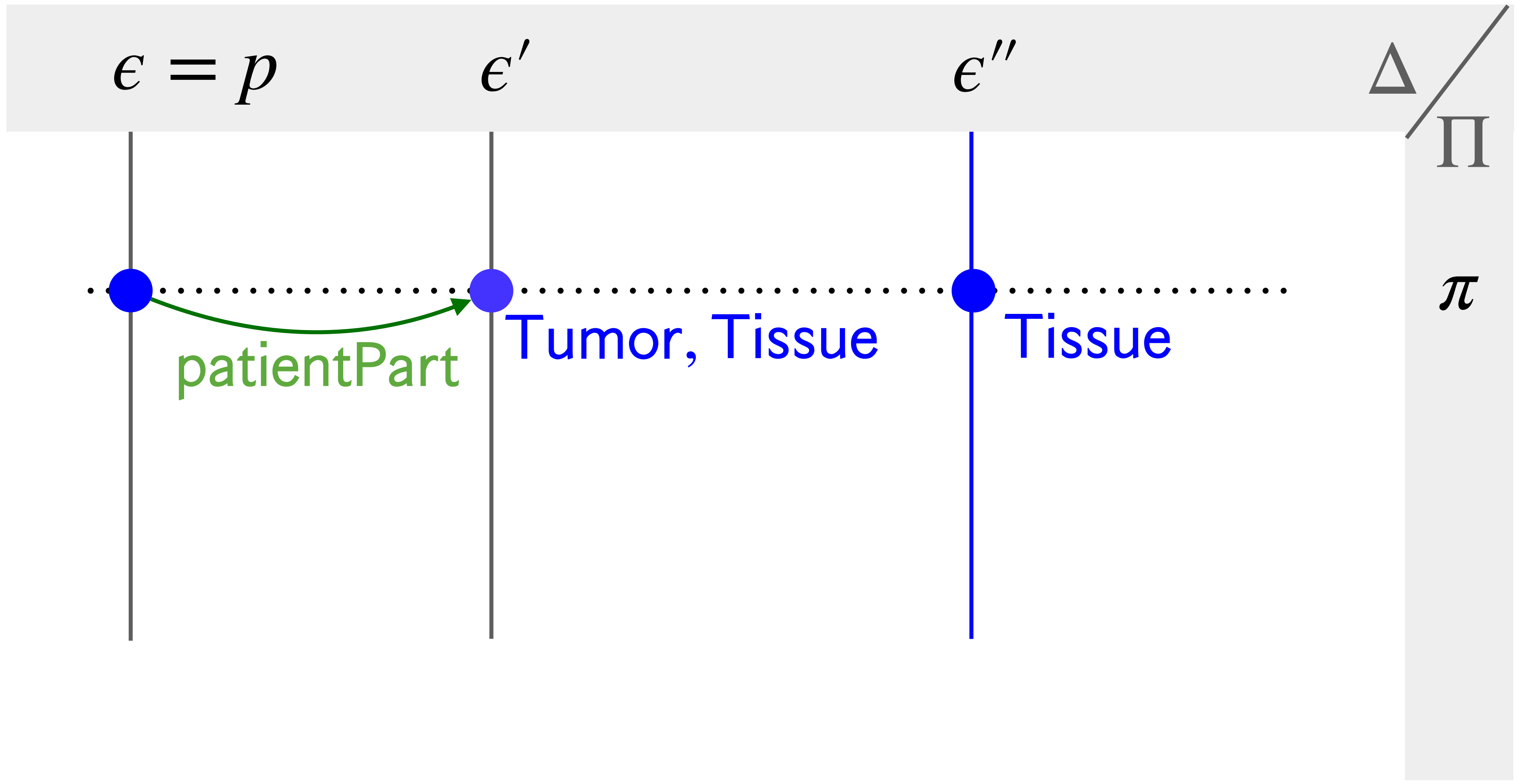

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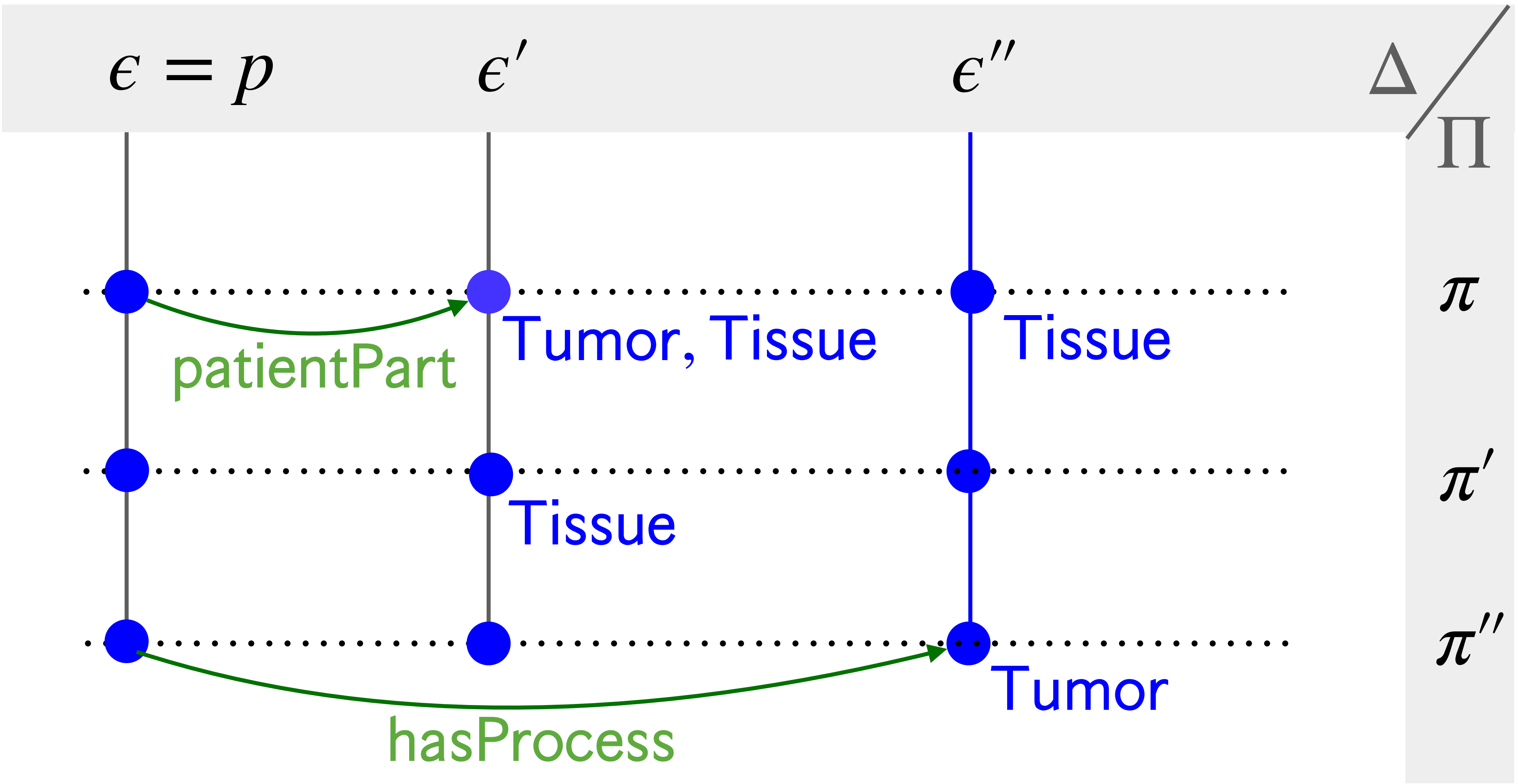

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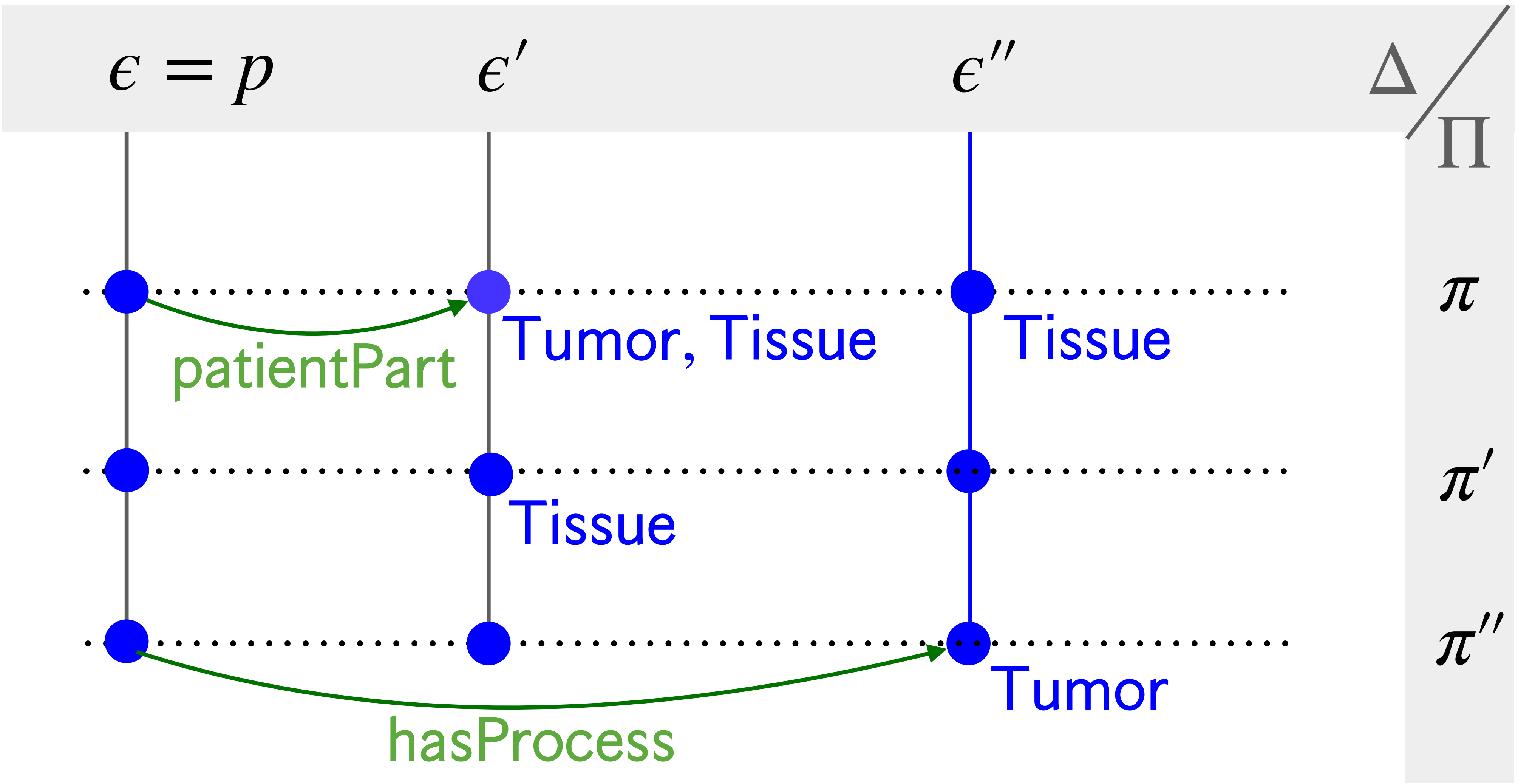

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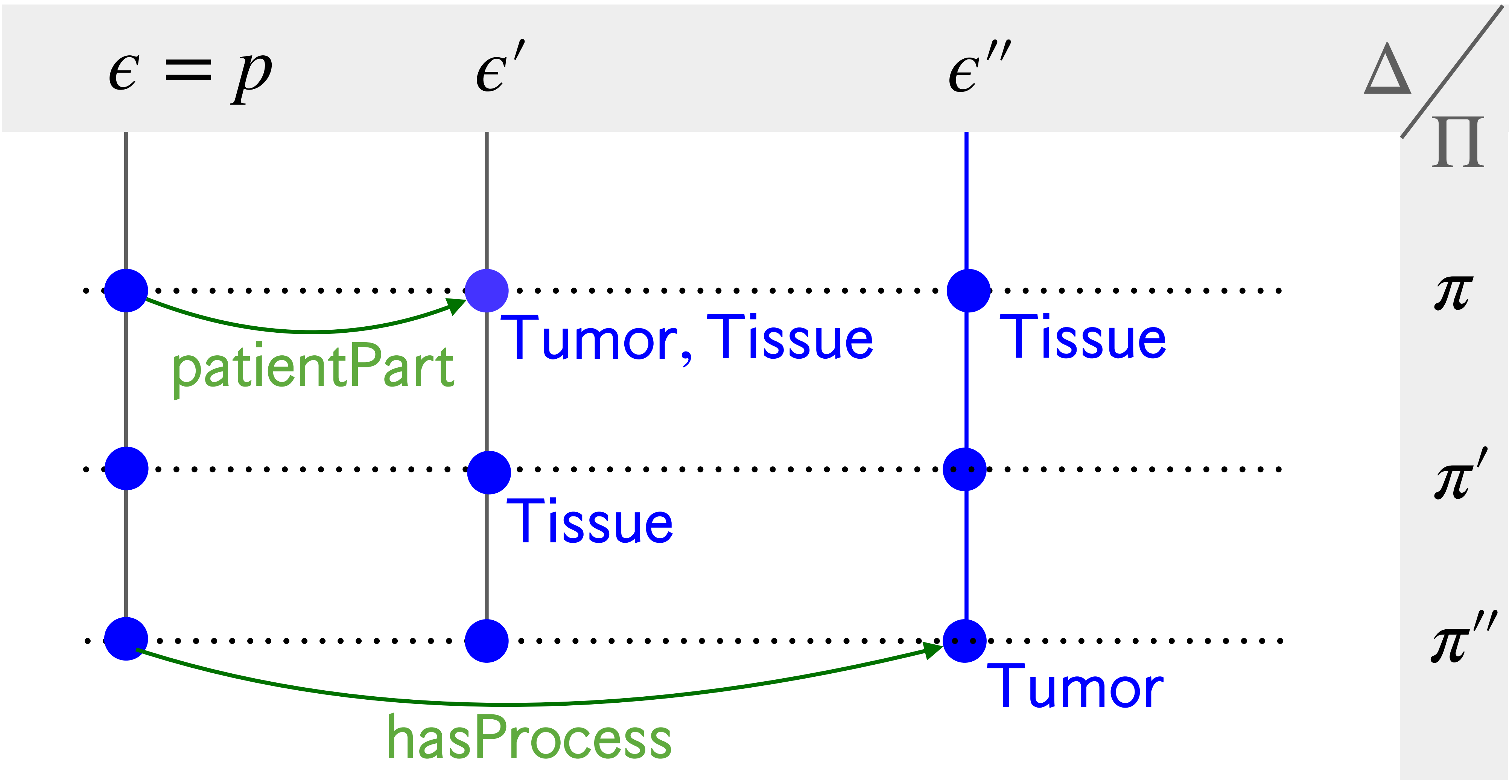

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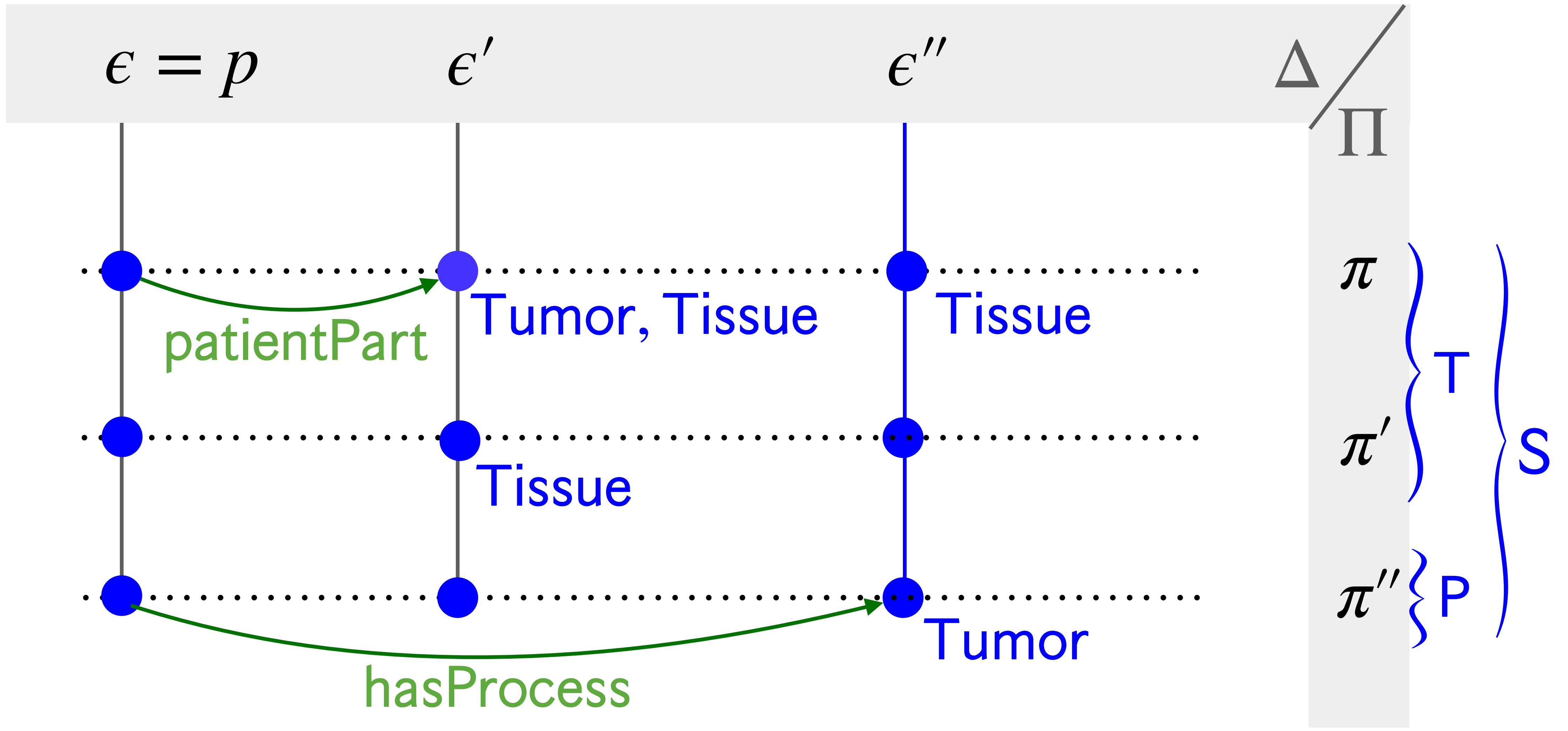

Complexity and Automated Reasoning

Tractable Reasoning in $\mathbb{S}_{\mathscr{EL}+}$

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- Nominal Concepts → ExpTime-hard

Tractable Reasoning in $\mathbb{S}_{\mathscr{EL}+}$

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  and a prototype implementation based in Datalog

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$

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(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\qquad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
- GCIs:
  - $\square_\mathsf{s} (C \sqsubseteq D)$ $\qquad$ $\square_\mathsf{s} (C_1 \sqcap C_2 \sqsubseteq D)$
  - $\square_\mathsf{s} (\exists r . C \sqsubseteq D)$ $\qquad$ $\square_\mathsf{s} (C \sqsubseteq \exists r . D)$
  - $\square_\mathsf{s} (C \sqsubseteq \square_\mathsf{u} D)$ $\qquad$ $\square_\mathsf{s} (C \sqsubseteq \lozenge_\mathsf{u} D)$
- RIAs:
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- Concept and role assertions:
  - $\square_\mathsf{s} C(a)$ $\qquad$ $\square_\mathsf{s} r(a, b)$

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(2) Extended modalised GCIs:

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* can be rewritten (with a fresh concept D) to
$\Box_{\mathsf{t}}[A \sqsubseteq \Box_{\mathsf{s}} D]$ and $\Box_{\mathsf{s}}[D \sqcap B \sqsubseteq C]]$

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  - $\Box_{\mathsf{s}}(C \sqsubseteq D)$ $\quad$ $\Box_{\mathsf{s}}(C_1 \sqcap C_2 \sqsubseteq D)$
  - $\Box_{\mathsf{s}}(\exists r\,.\,C \sqsubseteq D)$ $\quad$ $\Box_{\mathsf{s}}(C \sqsubseteq \exists r\,.\,D)$
  - $\Box_{\mathsf{s}}(C \sqsubseteq \Box_{\mathsf{u}} D)$ $\quad$ $\Box_{\mathsf{s}}(C \sqsubseteq \Diamond_{\mathsf{u}} D)$
- RIAs:
  - $\Box_{\mathsf{s}}(R' \sqsubseteq R)$ $\quad$ $\Box_{\mathsf{s}}(R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\Box_{\mathsf{s}}\, C(a)$ $\quad$ $\Box_{\mathsf{s}}\, r(a, b)$

(2) Extended modalised GCIs:

$$\Box_{\mathsf{t}}[A \sqsubseteq \Box_{\mathsf{s}}[B \Rightarrow C]]$$

* can be rewritten (with a fresh concept D) to
  $\Box_{\mathsf{t}}[A \sqsubseteq \Box_{\mathsf{s}} D]$ and $\Box_{\mathsf{s}}[D \sqcap B \sqsubseteq C]]$

Then replace:

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$

(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\quad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
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  - $\square_\mathsf{s}(C \sqsubseteq D)$ $\quad$ $\square_\mathsf{s}(C_1 \sqcap C_2 \sqsubseteq D)$
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- $\Box_s C(a)$ by $\Box_s[\{a\} \sqsubseteq \Box_s[\top \Rightarrow C]]$

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$

Tautologies

$$(T.1)\ \frac{}{\mathsf{s} \preceq *} \qquad (T.2)\ \frac{}{\mathsf{s} \preceq \mathsf{s}} \qquad (T.3)\ \frac{}{\Box_*[\top \sqsubseteq \Box_*[C \Rightarrow C]]} \qquad (T.4)\ \frac{}{\Box_*[\top \sqsubseteq \Box_*[C \Rightarrow \top]]} \qquad (T.5)\ \frac{}{\Box_*[R \sqsubseteq R]}$$

Standpoint hierarchy rules (for all $\mathsf{s} \in N_S$, ξ being any extended GCI, RIA, or role assertion)

$$(S.1)\ \frac{\mathsf{s} \preceq \mathsf{s}' \quad \mathsf{s}' \preceq \mathsf{s}''}{\mathsf{s} \preceq \mathsf{s}''} \qquad (S.2)\ \frac{\mathsf{s} \preceq \mathsf{s}_1 \quad \mathsf{s} \preceq \mathsf{s}_2 \quad \mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}'}{\mathsf{s} \preceq \mathsf{s}'} \qquad (S.3)\ \frac{\Box_{\mathsf{s}'}\xi \quad \mathsf{s} \preceq \mathsf{s}'}{\Box_{\mathsf{s}}\xi} \qquad (S.4)\ \frac{\Box_{\mathsf{t}}[C \sqsubseteq \Box_{\mathsf{s}'}[D \Rightarrow E]] \quad \mathsf{s} \preceq \mathsf{s}'}{\Box_{\mathsf{t}}[C \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}$$

Internal inferences for extended GCIs

$$(I.1)\ \frac{\Box_{\mathsf{s}}[C \sqsubseteq \Box_{\mathsf{s}}[\top \Rightarrow D]]}{\Box_*[\top \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow D]]} \qquad (I.2)\ \frac{\Box_{\mathsf{u}}[\top \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow D]]}{\Box_*[\top \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow D]]}$$

Role subsumptions

$$(R.1)\ \frac{\Box_{\mathsf{s}}[R \sqsubseteq R''] \quad \Box_{\mathsf{s}}[R'' \sqsubseteq R']}{\Box_{\mathsf{s}}[R \sqsubseteq R']}$$

Forward chaining

$$(C.1)\ \frac{\Box_{\mathsf{t}}[B \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow D]] \quad \Box_{\mathsf{t}}[B \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}{\Box_{\mathsf{t}}[B \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow E]]} \qquad (C.2)\ \frac{\Box_{\mathsf{u}}[\top \sqsubseteq \Box_{\mathsf{t}}[B \Rightarrow C]] \quad \Box_{\mathsf{t}}[C \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}{\Box_{\mathsf{t}}[B \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}$$

$$(C.3)\ \frac{\Box_{\mathsf{u}}[\top \sqsubseteq \Box_{\mathsf{t}}[C \Rightarrow D]] \quad \Box_{\mathsf{t}}[D \sqsubseteq \Diamond_{\mathsf{s}}E]}{\Box_{\mathsf{t}}[C \sqsubseteq \Diamond_{\mathsf{s}}E]} \qquad (C.4)\ \frac{\Box_{\mathsf{t}}[C \sqsubseteq \Diamond_{\mathsf{s}}D] \quad \Box_{\mathsf{t}}[C \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}{\Box_{\mathsf{t}}[C \sqsubseteq \Diamond_{\mathsf{s}}E]}$$

. . . (26 more rules)

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$

Tautologies

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Forward chaining

If $\Box_*[\top \sqsubseteq \Box_*[\top \Rightarrow \bot]] \notin \mathscr{K}^{\vdash}$, then $\mathscr{K}$ is satisfiable

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$ (Proofs)

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Theorem 4 (Termination). *The closure of $\mathbb{S}_{\mathscr{EL}+}$ knowledge bases under the deduction calculus can be computed in PTIME.*

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- We prove the existence of a model whenever $\square_*[\top \sqsubseteq \square_*[\top \Rightarrow \bot]] \notin \mathscr{K}^{\vdash}$.

- This model is canonical in a sense but it will typically be infinite.

Conclusions and Future Work

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➡ Managing perspectives is interesting in knowledge integration scenarios

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Future Work:

- ➡ Calculus optimisation and efficient implementations
- ➡ Reasoning with more expressive languages (eg. $\mathcal{SHIQ}$)
- ➡ Towards conceptual modelling with standpoints for knowledge integration challenges

The end.

Labels example

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$\Box_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\Diamond_L [\text{Tumor}] \sqsubseteq \Box_L [\text{Tissue}]$

$\Diamond_H [\text{Tumor}] \sqsubseteq \Box_H [\text{Process}]$

$\Diamond_S \neg\text{Tumor}(a)$

$(L \cup H) \preceq S$

(It could be a Tumor according to someone else)

$\Box_L \text{ Tumor}(a)$

Labels example

$\Box_S$ [Process $\sqcap$ Tissue $\sqsubseteq \bot$]

$\Diamond_L$ [Tumor] $\sqsubseteq \Box_L$ [Tissue]

$\Diamond_H$ [Tumor] $\sqsubseteq \Box_H$ [Process]

$\Diamond_S$ $\neg$Tumor(a)

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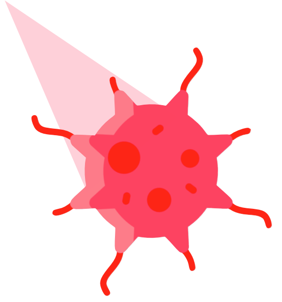

(It could be a Tumor according to someone else)

$\Box_L$ Tumor(a)

[Process $\sqcap$ Tissue $\sqsubseteq \bot$]

Tumor$_L$ $\sqsubseteq$ Tissue , Tumor$_L$ $\sqsubseteq$ Tumor

Tumor$_H$ $\sqsubseteq$ Process , Tumor$_H$ $\sqsubseteq$ Tumor

$\neg$Tumor(a)

Infer: (It cannot be a Tumor according to anyone)

$\neg$Tumor$_L(a)$

$\neg$Tumor$_H(a)$