Algorithmic Game Theory
Introduction to Strategic Games - Problems 9

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Problem 1. *Consider the game description below.*

role(white)
role(black)

base(p)
base(q)
base(r)
base(s)

action(a)
action(b)
action(c)
action(d)

init(s)

legal(white,a)
legal(white,b)
legal(white,c)
legal(black,d)

next(p) :- does(white,a) & ∼true(p)
next(p) :- ∼does(white,a) & true(p)
next(q) :- does(white,b) & true(p)
next(q) :- does(white,c) & true(r)
next(q) :- ∼does(white,b) & ∼does(white,c) & true(q)
next(r) :- does(white,c) & true(q)
next(r) :- ∼does(white,c) & true(r)

terminal :- true(p) & true(q) & true(r)
goal(white,100) :- terminal
goal(white,0) :- ∼terminal
goal(black,100) :- terminal
goal(black,0) :- ∼terminal

Answer the following questions:

- *How many players are there?*

- *How many propositions are there?*

- *How many feasible actions are there?*

- *How many actions are legal for white in the inital state?*

- *How many propositions are true in the initial state?*

- *How many propositions are true in the state that results from white performing action a and black performing action d in the initial state?*

- *What is the minimum number of steps this game can take to terminate?*

Problem 2. *Define the predicate **line** from the lecture, possibly using auxiliary predicates.*

Problem 3. *As an example of how to describe imperfect-information games in GDL-II, let us look at a variation of Tic-Tac-Toe where the players don't get to see each others' moves. Of course it may then happen that a player intends to mark a cell that's already been occupied. In that case, the move shall have no effect but the player will be informed about it. To make the game fairer, both players mark concurrently. If they happen to choose the same cell at the same time, then the toss of a coin determines who is successful. To begin with, our new two-person game, which we call Blind Tic-Tac-Toe, features three roles, where the new random player is needed to simulate the coin toss.*

> *role(white)*
> *role(black)*
> *role(random)*

We also add a new feature, tried(P,M,N) that records every attempt by a player to mark a cell. Players will only be allowed to try each cell once, which will help to ensure that the game terminates.

- *How does the description of the initial state change compared to regular Tic-Tac-Toe?*

- *Next, we define legality in Blind Tic-Tac-Toe. In each round, both white and black may choose any cell that they have not already tried. At the same time a coin toss is simulated, the result of which is used to break the tie in case both players attempt to mark the same cell. Accordingly, we call this random move a tiebreak. Complete the following first part of the description of legality:*

  input(white,mark(M,N)) :- index(M) & index(N)

  input(black,mark(M,N)) :- index(M) & index(N)

  input(random,tiebreak(x))

  input(random,tiebreak(o))

  legal(white,mark(X,Y)) :- index(X) & index(Y) & ∼true(tried(white,X,Y))

  ...

- *GDL-II is based on the assumption that players are no longer automatically informed about any other players' moves. Thus, without additional hints our Blind Tic-Tac-Toe players would be completely oblivious as to whether any of their attempts to mark a cell was successful. Sketch a set of rules that defines the player's percepts and that provides the players with exactly that but no more information. For this, suppose a player gets ok as feedback when they mark a blank cell.*

- *How does the terminal condition for Blind Tic-Tac-Toe change compared to regular Tic-Tac-Toe?*