

Exercise 10: Datalog Evaluation

Database Theory

2022-06-21

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Exercise 1

Exercise. Consider the program

$$P = \{ T(x) \leftarrow e(x), T(x) \leftarrow a(x, y) \wedge T(y) \wedge b(x, z) \wedge T(z) \}.$$

1. Describe, in your own words, the kind of structures that the query $\langle T, P \rangle$ recognises.
2. Compute the semi-naive evaluation of P for the database D with the following facts:

e(1) e(2) e(6) a(3, 1) a(4, 3) a(5, 3) a(7, 5) b(3, 2) b(5, 3) b(7, 6)

Specify for each newly derived fact which of the rule(s) of P will produce it at the given point in the derivation.

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1. $\langle T, P \rangle$ recognises binary forests with leafs labelled e and branches labelled a and b.

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Solution.

1. $\langle T, P \rangle$ recognises binary forests with leafs labelled e and branches labelled a and b.
2. Transformed program:

$$\hat{P} = \left\{ \underbrace{T(x) \leftarrow e(x)}_{(R1)}, \underbrace{T(x) \leftarrow a(x, y) \wedge \Delta_T^i(y) \wedge b(x, z) \wedge T^i(z)}_{(R2.1)}, \underbrace{T(x) \leftarrow a(x, y) \wedge T^{i-1}(y) \wedge b(x, z) \wedge \Delta_T^i(z)}_{(R2.2)} \right\}$$

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$$T_{\hat{P}}^0 = \emptyset$$

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$$T_{\hat{P}}^4 = T_{\hat{P}}^3 \cup \{ T(7) \} = T_{\hat{P}}^5 = T_{\hat{P}}^\infty \quad (R2.1)$$

Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge S(v, w) \wedge p(y, v)$$

and the adorned version for query $S(1, x)$:

$$r_1 = \text{Query}^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge S^{fb}(v, w) \wedge p(y, v)$$

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together with the database that contains the following facts for predicate p :

$h(1)$ $h(2)$ $h(3)$ $h(4)$ $h(5)$ $h(6)$ $h(7)$ $p(1, 2)$ $p(2, 3)$ $p(4, 3)$ $p(5, 4)$ $p(6, 1)$ $p(7, 1)$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

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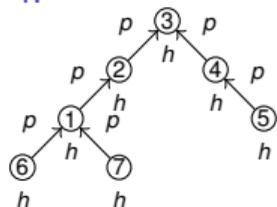
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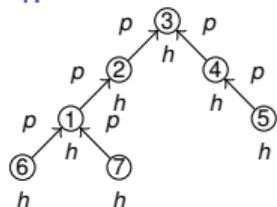
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Solution.

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Expected answers:

$\{ \text{Query}(1), \text{Query}(5) \}$

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Solution.

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$$\text{sup}_1^{f_1}[x] = \emptyset$$

$$\text{output}_{\text{Query}}^f[x] = \emptyset$$

$$\text{input}_S^{bf}[x] = \emptyset$$

$$\text{sup}_0^{b_2}[x] = \emptyset$$

$$\text{sup}_1^{b_2}[x] = \emptyset$$

$$\text{output}_S^{bf}[x, y] = \emptyset$$

$$\text{input}_S^{bf}[x] = \emptyset$$

$$\text{sup}_0^{b_3}[x] = \emptyset$$

$$\text{sup}_1^{b_3}[x, w] = \emptyset$$

$$\text{sup}_2^{b_3}[x, v] = \emptyset$$

$$\text{sup}_3^{b_3}[x, y] = \emptyset$$

$$\text{output}_S^{bf}[x, y] = \emptyset$$

$$\text{input}_S^{fb}[x] = \emptyset$$

$$\text{sup}_0^{f_4}[x] = \emptyset$$

$$\text{sup}_1^{f_4}[x] = \emptyset$$

$$\text{output}_S^{fb}[x, y] = \emptyset$$

$$\text{input}_S^{fb}[y] = \emptyset$$

$$\text{sup}_0^{f_5}[y] = \emptyset$$

$$\text{sup}_1^{f_5}[x, w, y] = \emptyset$$

$$\text{sup}_2^{f_5}[x, v, y] = \emptyset$$

$$\text{sup}_3^{f_5}[x, y] = \emptyset$$

$$\text{output}_S^{fb}[x, y] = \emptyset$$

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Solution.

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$$\text{input}_{\text{Query}}^f[] = \{1, 2, \dots, 7\}$$

$$\text{sup}_0^{r_1}[] = \emptyset$$

$$\text{sup}_1^{r_1}[x] = \emptyset$$

$$\text{output}_{\text{Query}}^f[x] = \emptyset$$

$$\text{input}_S^{bf}[x] = \emptyset$$

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$$\text{sup}_0^{r_4}[x] = \emptyset$$

$$\text{sup}_1^{r_4}[x] = \emptyset$$

$$\text{output}_S^{fb}[x, y] = \emptyset$$

$$\text{input}_S^{fb}[y] = \emptyset$$

$$\text{sup}_0^{r_5}[y] = \emptyset$$

$$\text{sup}_1^{r_5}[x, w, y] = \emptyset$$

$$\text{sup}_2^{r_5}[x, v, y] = \emptyset$$

$$\text{sup}_3^{r_5}[x, y] = \emptyset$$

$$\text{output}_S^{fb}[x, y] = \emptyset$$

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$$\text{sup}_1^f[x] = \emptyset$$

$$\text{output}_{\text{Query}}^f[x] = \emptyset$$

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Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \emptyset \\ \text{sup}_1^{bf}[x] &= \emptyset \\ \text{output}_S^{bf}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \emptyset \\ \text{sup}_1^{fb}[x, w] &= \emptyset \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \emptyset \\ \text{sup}_0^{fb}[x] &= \emptyset \\ \text{sup}_1^{fb}[x] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \emptyset \\ \text{sup}_0^{fb}[y] &= \emptyset \\ \text{sup}_1^{fb}[x, w, y] &= \emptyset \\ \text{sup}_2^{fb}[x, v, y] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge S(v, w) \wedge p(y, v)$$

and the adorned version for query $S(1, x)$:

$$r_1 = \text{Query}^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge S^{fb}(v, w) \wedge p(y, v)$$

$$r_4 = S^{fb}(x, x) \leftarrow h(x)$$

$$r_5 = S^{fb}(x, y) \leftarrow p(x, w) \wedge S^{fb}(v, w) \wedge p(y, v)$$

together with the database that contains the following facts for predicate p :

$$h(1) \quad h(2) \quad h(3) \quad h(4) \quad h(5) \quad h(6) \quad h(7) \quad p(1, 2) \quad p(2, 3) \quad p(4, 3) \quad p(5, 4) \quad p(6, 1) \quad p(7, 1)$$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^2[x] &= \{1\} \\ \text{sup}_1^2[x] &= \emptyset \\ \text{output}_S^{bf}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^3[x] &= \emptyset \\ \text{sup}_1^3[x, w] &= \emptyset \\ \text{sup}_2^3[x, v] &= \emptyset \\ \text{sup}_3^3[x, y] &= \emptyset \\ \text{output}_S^{bf}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \emptyset \\ \text{sup}_0^4[x] &= \emptyset \\ \text{sup}_1^4[x] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \emptyset \\ \text{sup}_0^5[y] &= \emptyset \\ \text{sup}_1^5[x, w, y] &= \emptyset \\ \text{sup}_2^5[x, v, y] &= \emptyset \\ \text{sup}_3^5[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{fb}[x] &= \emptyset \\ \text{sup}_0^4[x] &= \emptyset \\ \text{sup}_1^4[x] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^2[x] &= \{1\} \\ \text{sup}_1^2[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^3[x] &= \emptyset \\ \text{sup}_1^3[x, w] &= \emptyset \\ \text{sup}_2^3[x, v] &= \emptyset \\ \text{sup}_3^3[x, y] &= \emptyset \\ \text{output}_S^{bf}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \emptyset \\ \text{sup}_0^4[x] &= \emptyset \\ \text{sup}_1^4[x] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{fb}[x] &= \emptyset \\ \text{sup}_0^4[x] &= \emptyset \\ \text{sup}_1^4[x] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{fb}[x] &= \emptyset \\ \text{sup}_0^4[x] &= \emptyset \\ \text{sup}_1^4[x] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{fb}[x] &= \{2\} \\ \text{sup}_0^4[x] &= \emptyset \\ \text{sup}_1^4[x] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{2\} \\ \text{sup}_0^5[y] &= \emptyset \\ \text{sup}_1^5[x, w, y] &= \emptyset \\ \text{sup}_2^5[x, v, y] &= \emptyset \\ \text{sup}_3^5[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{fb}[x] &= \{2\} \\ \text{sup}_0^4[x] &= \{2\} \\ \text{sup}_1^4[x] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{fb}[x] &= \{2\} \\ \text{sup}_0^4[x] &= \{2\} \\ \text{sup}_1^4[x] &= \{2\} \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^2[x] &= \{1\} \\ \text{sup}_1^2[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^3[x] &= \{1\} \\ \text{sup}_1^3[x, w] &= \{(1, 2)\} \\ \text{sup}_2^3[x, v] &= \emptyset \\ \text{sup}_3^3[x, y] &= \emptyset \\ \text{output}_S^{bf}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{2\} \\ \text{sup}_0^4[x] &= \{2\} \\ \text{sup}_1^4[x] &= \{2\} \\ \text{output}_S^{fb}[x, y] &= \{(2, 2)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{2\} \\ \text{sup}_0^5[y] &= \emptyset \\ \text{sup}_1^5[x, w, y] &= \emptyset \\ \text{sup}_2^5[x, v, y] &= \emptyset \\ \text{sup}_3^5[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge S(v, w) \wedge p(y, v)$$

and the adorned version for query $S(1, x)$:

$$r_1 = \text{Query}^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

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together with the database that contains the following facts for predicate p :

$h(1)$ $h(2)$ $h(3)$ $h(4)$ $h(5)$ $h(6)$ $h(7)$ $p(1, 2)$ $p(2, 3)$ $p(4, 3)$ $p(5, 4)$ $p(6, 1)$ $p(7, 1)$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^2[x] &= \{1\} \\ \text{sup}_1^2[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^3[x] &= \{1\} \\ \text{sup}_1^3[x, w] &= \{(1, 2)\} \\ \text{sup}_2^3[x, v] &= \emptyset \\ \text{sup}_3^3[x, y] &= \emptyset \\ \text{output}_S^{bf}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{2\} \\ \text{sup}_0^4[x] &= \{2\} \\ \text{sup}_1^4[x] &= \{2\} \\ \text{output}_S^{fb}[x, y] &= \{(2, 2)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{2\} \\ \text{sup}_0^5[y] &= \{2\} \\ \text{sup}_1^5[x, w, y] &= \emptyset \\ \text{sup}_2^5[x, v, y] &= \emptyset \\ \text{sup}_3^5[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

Exercise 2

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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

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$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \{1\} \\ \text{sup}_1^{bf}[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \{1\} \\ \text{sup}_1^{fb}[x, w] &= \{(1, 2)\} \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{2\} \\ \text{sup}_0^{fb}[x] &= \{2\} \\ \text{sup}_1^{fb}[x] &= \{2\} \\ \text{output}_S^{fb}[x, y] &= \{(2, 2)\} \end{aligned}$$

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Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \{1\} \\ \text{sup}_1^{bf}[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \{1\} \\ \text{sup}_1^{fb}[x, w] &= \{(1, 2)\} \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[x] &= \{2\} \\ \text{sup}_1^{fb}[x] &= \{2\} \\ \text{output}_S^{fb}[x, y] &= \{(2, 2)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[y] &= \{2\} \\ \text{sup}_1^{fb}[x, w, y] &= \{(1, 2, 2), (2, 3, 2), \dots, (7, 1, 2)\} \\ \text{sup}_2^{fb}[x, v, y] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

Exercise 2

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Solution.

2.

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$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \{1\} \\ \text{sup}_1^{bf}[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \{1\} \\ \text{sup}_1^{fb}[x, w] &= \{(1, 2)\} \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x] &= \{2\} \\ \text{output}_S^{fb}[x, y] &= \{(2, 2)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[y] &= \{2\} \\ \text{sup}_1^{fb}[x, w, y] &= \{(1, 2, 2), (2, 3, 2), \dots, (7, 1, 2)\} \\ \text{sup}_2^{fb}[x, v, y] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f[] &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f[] &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \{1\} \\ \text{sup}_1^{bf}[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \{1\} \\ \text{sup}_1^{fb}[x, w] &= \{(1, 2)\} \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{output}_S^{fb}[x, y] &= \{(2, 2)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[y] &= \{2\} \\ \text{sup}_1^{fb}[x, w, y] &= \{(1, 2, 2), (2, 3, 2), \dots, (7, 1, 2)\} \\ \text{sup}_2^{fb}[x, v, y] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

Exercise 2

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1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \{1\} \\ \text{sup}_1^{bf}[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \langle 1, 1 \rangle \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \{1\} \\ \text{sup}_1^{bf}[x, w] &= \langle 1, 2 \rangle \\ \text{sup}_2^{bf}[x, v] &= \emptyset \\ \text{sup}_3^{bf}[x, y] &= \emptyset \\ \text{output}_S^{bf}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{output}_S^{fb}[x, y] &= \langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 4 \rangle \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[y] &= \{2\} \\ \text{sup}_1^{fb}[x, w, y] &= \langle 1, 2, 2 \rangle, \langle 2, 3, 2 \rangle, \dots, \langle 7, 1, 2 \rangle \\ \text{sup}_2^{fb}[x, v, y] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \{1\} \\ \text{sup}_1^{fb}[x, w] &= \langle 1, 2 \rangle \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{output}_S^{fb}[x, y] &= \langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 4 \rangle \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x, w, y] &= \langle 1, 2, 2 \rangle, \langle 2, 3, 2 \rangle, \dots, \langle 7, 1, 2 \rangle \\ \text{sup}_2^{fb}[x, v, y] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

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h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

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Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \{1\} \\ \text{sup}_1^{bf}[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{\langle 1, 1 \rangle\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \{1\} \\ \text{sup}_1^{fb}[x, w] &= \{\langle 1, 2 \rangle\} \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{output}_S^{fb}[x, y] &= \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 4 \rangle\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x, w, y] &= \{\langle 1, 2, 1 \rangle, \langle 2, 3, 1 \rangle, \dots, \langle 7, 1, 4 \rangle\} \\ \text{sup}_2^{fb}[x, v, y] &= \{\langle 1, 2, 1 \rangle, \langle 2, 3, 1 \rangle, \dots, \langle 7, 1, 4 \rangle\} \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge S(v, w) \wedge p(y, v)$$

and the adorned version for query $S(1, x)$:

$$r_1 = \text{Query}^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge S^{fb}(v, w) \wedge p(y, v)$$

$$r_4 = S^{fb}(x, x) \leftarrow h(x)$$

$$r_5 = S^{fb}(x, y) \leftarrow p(x, w) \wedge S^{fb}(v, w) \wedge p(y, v)$$

together with the database that contains the following facts for predicate p :

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{bf}[x] &= \{1\} \\ \text{sup}_1^{bf}[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{\langle 1, 1 \rangle\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \{1\} \\ \text{sup}_1^{fb}[x, w] &= \{\langle 1, 2 \rangle\} \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{output}_S^{fb}[x, y] &= \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 4 \rangle\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x, w, y] &= \{\langle 1, 2, 1 \rangle, \langle 2, 3, 1 \rangle, \dots, \langle 7, 1, 4 \rangle\} \\ \text{sup}_2^{fb}[x, v, y] &= \{\langle 1, 2, 1 \rangle, \langle 2, 3, 1 \rangle, \dots, \langle 7, 1, 4 \rangle\} \\ \text{sup}_3^{fb}[x, y] &= \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 4 \rangle, \langle 4, 2 \rangle, \langle 4, 4 \rangle\} \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge S(v, w) \wedge p(y, v)$$

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together with the database that contains the following facts for predicate p :

$h(1)$ $h(2)$ $h(3)$ $h(4)$ $h(5)$ $h(6)$ $h(7)$ $p(1, 2)$ $p(2, 3)$ $p(4, 3)$ $p(5, 4)$ $p(6, 1)$ $p(7, 1)$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f &= \{1, 2, \dots, 7\} \\ \text{sup}_0^f &= \{1, 2, \dots, 7\} \\ \text{sup}_1^f[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

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$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1\} \\ \text{sup}_0^{fb}[x] &= \{1\} \\ \text{sup}_1^{fb}[x, w] &= \{\langle 1, 2 \rangle\} \\ \text{sup}_2^{fb}[x, v] &= \emptyset \\ \text{sup}_3^{fb}[x, y] &= \emptyset \\ \text{output}_S^{fb}[x, y] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{output}_S^{fb}[x, y] &= \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 4 \rangle\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{fb}[x, w, y] &= \{\langle 1, 2, 1 \rangle, \langle 2, 3, 1 \rangle, \dots, \langle 7, 1, 4 \rangle\} \\ \text{sup}_2^{fb}[x, v, y] &= \{\langle 1, 2, 1 \rangle, \langle 2, 3, 1 \rangle, \dots, \langle 7, 1, 4 \rangle\} \\ \text{sup}_3^{fb}[x, y] &= \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 4 \rangle, \langle 4, 2 \rangle, \langle 4, 4 \rangle\} \\ \text{output}_S^{fb}[x, y] &= \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 4 \rangle, \langle 4, 2 \rangle, \langle 4, 4 \rangle\} \end{aligned}$$

Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

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together with the database that contains the following facts for predicate p :

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\text{input}_{\text{Query}}^f[] = \{1, 2, \dots, 7\}$	$\text{input}_S^{bf}[x] = \{1\}$	$\text{input}_S^{bf}[x] = \{1\}$	$\text{input}_S^{fb}[x] = \{1, 2, 3, 4\}$	$\text{input}_S^{fb}[y] = \{1, 2, 3, 4\}$
$\text{sup}_0^f[] = \{1, 2, \dots, 7\}$	$\text{sup}_0^{bf}[x] = \{1\}$	$\text{sup}_0^{bf}[x] = \{1\}$	$\text{sup}_0^{fb}[x] = \{1, 2, 3, 4\}$	$\text{sup}_0^{fb}[y] = \{1, 2, 3, 4\}$
$\text{sup}_1^f[x] = \emptyset$	$\text{sup}_1^{bf}[x] = \{1\}$	$\text{sup}_1^{bf}[x, w] = \{(1, 2)\}$	$\text{sup}_1^{fb}[x] = \{1, 2, 3, 4\}$	$\text{sup}_1^{fb}[x, w, y] = \{(1, 2, 1), (2, 3, 1), \dots, (7, 1, 4)\}$
$\text{output}_{\text{Query}}^f[x] = \emptyset$	$\text{output}_S^{bf}[x, y] = \{(1, 1)\}$	$\text{sup}_2^{bf}[x, v] = \{(1, 2), (1, 4)\}$	$\text{output}_S^{fb}[x, y] = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$	$\text{sup}_2^{fb}[x, v, y] = \{(1, 2, 1), (2, 3, 1), \dots, (7, 1, 4)\}$
		$\text{sup}_3^{bf}[x, y] = \emptyset$		$\text{sup}_3^{fb}[x, y] = \{(1, 1), (2, 2), (2, 4), (4, 2), (4, 4)\}$
		$\text{output}_S^{bf}[x, y] = \emptyset$		$\text{output}_S^{fb}[x, y] = \{(1, 1), (2, 2), (2, 4), (4, 2), (4, 4)\}$

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$h(1)$ $h(2)$ $h(3)$ $h(4)$ $h(5)$ $h(6)$ $h(7)$ $p(1, 2)$ $p(2, 3)$ $p(4, 3)$ $p(5, 4)$ $p(6, 1)$ $p(7, 1)$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\text{input}_{\text{Query}}^f[] = \{1, 2, \dots, 7\}$	$\text{input}_S^{bf}[x] = \{1\}$	$\text{input}_S^{bf}[x] = \{1\}$	$\text{input}_S^{fb}[x] = \{1, 2, 3, 4\}$	$\text{input}_S^{fb}[y] = \{1, 2, 3, 4\}$
$\text{sup}_0^f[] = \{1, 2, \dots, 7\}$	$\text{sup}_0^{bf}[x] = \{1\}$	$\text{sup}_0^{bf}[x] = \{1\}$	$\text{sup}_0^{fb}[x] = \{1, 2, 3, 4\}$	$\text{sup}_0^{fb}[y] = \{1, 2, 3, 4\}$
$\text{sup}_1^f[x] = \emptyset$	$\text{sup}_1^{bf}[x] = \{1\}$	$\text{sup}_1^{bf}[x, w] = \{\langle 1, 2 \rangle\}$	$\text{sup}_1^{fb}[x] = \{1, 2, 3, 4\}$	$\text{sup}_1^{fb}[x, w, y] = \{\langle 1, 2, 1 \rangle, \langle 2, 3, 1 \rangle, \dots, \langle 7, 1, 4 \rangle\}$
$\text{output}_{\text{Query}}^f[x] = \emptyset$	$\text{output}_S^{bf}[x, y] = \{\langle 1, 1 \rangle\}$	$\text{sup}_2^{bf}[x, v] = \{\langle 1, 2 \rangle, \langle 1, 4 \rangle\}$	$\text{output}_S^{fb}[x, y] = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 4 \rangle\}$	$\text{sup}_2^{fb}[x, v, y] = \{\langle 1, 2, 1 \rangle, \langle 2, 3, 1 \rangle, \dots, \langle 7, 1, 4 \rangle\}$
		$\text{sup}_3^{bf}[x, y] = \{\langle 1, 1 \rangle, \langle 1, 5 \rangle\}$		$\text{sup}_3^{fb}[x, y] = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 4 \rangle, \langle 4, 2 \rangle, \langle 4, 4 \rangle\}$
		$\text{output}_S^{bf}[x, y] = \emptyset$		$\text{output}_S^{fb}[x, y] = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 4 \rangle, \langle 4, 2 \rangle, \langle 4, 4 \rangle\}$

Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

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together with the database that contains the following facts for predicate p :

$h(1)$ $h(2)$ $h(3)$ $h(4)$ $h(5)$ $h(6)$ $h(7)$ $p(1, 2)$ $p(2, 3)$ $p(4, 3)$ $p(5, 4)$ $p(6, 1)$ $p(7, 1)$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$$\begin{aligned} \text{input}_{\text{Query}}^f[] &= \{1, 2, \dots, 7\} \\ \text{sup}_0^{r_1}[] &= \{1, 2, \dots, 7\} \\ \text{sup}_1^{r_1}[x] &= \emptyset \\ \text{output}_{\text{Query}}^f[x] &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{r_2}[x] &= \{1\} \\ \text{sup}_1^{r_2}[x] &= \{1\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{bf}[x] &= \{1\} \\ \text{sup}_0^{r_3}[x] &= \{1\} \\ \text{sup}_1^{r_3}[x, w] &= \{(1, 2)\} \\ \text{sup}_2^{r_3}[x, v] &= \{(1, 2), (1, 4)\} \\ \text{sup}_3^{r_3}[x, y] &= \{(1, 1), (1, 5)\} \\ \text{output}_S^{bf}[x, y] &= \{(1, 1), (1, 5)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{r_4}[x] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{r_4}[x] &= \{1, 2, 3, 4\} \\ \text{output}_S^{fb}[x, y] &= \{(1, 1), (2, 2), (3, 3), (4, 4)\} \end{aligned}$$

$$\begin{aligned} \text{input}_S^{fb}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_0^{r_5}[y] &= \{1, 2, 3, 4\} \\ \text{sup}_1^{r_5}[x, w, y] &= \{(1, 2, 1), (2, 3, 1), \dots, (7, 1, 4)\} \\ \text{sup}_2^{r_5}[x, v, y] &= \{(1, 2, 1), (2, 3, 1), \dots, (7, 1, 4)\} \\ \text{sup}_3^{r_5}[x, y] &= \{(1, 1), (2, 2), (2, 4), (4, 2), (4, 4)\} \\ \text{output}_S^{fb}[x, y] &= \{(1, 1), (2, 2), (2, 4), (4, 2), (4, 4)\} \end{aligned}$$

Exercise 2

Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

$$S(x, x) \leftarrow h(x)$$

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together with the database that contains the following facts for predicate p :

$$h(1) \quad h(2) \quad h(3) \quad h(4) \quad h(5) \quad h(6) \quad h(7) \quad p(1, 2) \quad p(2, 3) \quad p(4, 3) \quad p(5, 4) \quad p(6, 1) \quad p(7, 1)$$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

$$\begin{array}{llll}
 \text{input}_{\text{Query}}^f[] = \{1, 2, \dots, 7\} & \text{input}_S^{bf}[x] = \{1\} & \text{input}_S^{bf}[x] = \{1\} & \text{input}_S^{fb}[x] = \{1, 2, 3, 4\} \\
 \text{sup}_0^{r_1}[] = \{1, 2, \dots, 7\} & \text{sup}_0^{r_2}[x] = \{1\} & \text{sup}_0^{r_3}[x] = \{1\} & \text{sup}_0^{r_4}[x] = \{1, 2, 3, 4\} \\
 \text{sup}_1^{r_1}[x] = \{1, 5\} & \text{sup}_1^{r_2}[x] = \{1\} & \text{sup}_1^{r_3}[x, w] = \{(1, 2)\} & \text{sup}_1^{r_4}[x] = \{1, 2, 3, 4\} \\
 \text{output}_{\text{Query}}^f[x] = \emptyset & \text{output}_S^{bf}[x, y] = \{(1, 1)\} & \text{sup}_2^{r_3}[x, v] = \{(1, 2), (1, 4)\} & \text{output}_S^{fb}[x, y] = \{(1, 1), (2, 2), (3, 3), (4, 4)\} \\
 & & \text{sup}_3^{r_3}[x, y] = \{(1, 1), (1, 5)\} & \\
 & & \text{output}_S^{bf}[x, y] = \{(1, 1), (1, 5)\} & \\
 & & & \text{input}_S^{fb}[y] = \{1, 2, 3, 4\} \\
 & & & \text{sup}_0^{r_5}[y] = \{1, 2, 3, 4\} \\
 & & & \text{sup}_1^{r_5}[x, w, y] = \{(1, 2, 1), (2, 3, 1), \dots, (7, 1, 4)\} \\
 & & & \text{sup}_2^{r_5}[x, v, y] = \{(1, 2, 1), (2, 3, 1), \dots, (7, 1, 4)\} \\
 & & & \text{sup}_3^{r_5}[x, y] = \{(1, 1), (2, 2), (2, 4), (4, 2), (4, 4)\} \\
 & & & \text{output}_S^{fb}[x, y] = \{(1, 1), (2, 2), (2, 4), (4, 2), (4, 4)\}
 \end{array}$$

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Exercise. Consider the “Same generation” Datalog program given in the lecture (Lecture 15, Slide 15):

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together with the database that contains the following facts for predicate p :

$h(1)$ $h(2)$ $h(3)$ $h(4)$ $h(5)$ $h(6)$ $h(7)$ $p(1, 2)$ $p(2, 3)$ $p(4, 3)$ $p(5, 4)$ $p(6, 1)$ $p(7, 1)$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\text{input}_{\text{Query}}^f[] = \{1, 2, \dots, 7\}$	$\text{input}_S^{bf}[x] = \{1\}$	$\text{input}_S^{bf}[x] = \{1\}$	$\text{input}_S^{fb}[x] = \{1, 2, 3, 4\}$	$\text{input}_S^{fb}[y] = \{1, 2, 3, 4\}$
$\text{sup}_0^{r_1}[] = \{1, 2, \dots, 7\}$	$\text{sup}_0^{r_2}[x] = \{1\}$	$\text{sup}_0^{r_3}[x] = \{1\}$	$\text{sup}_0^{r_4}[x] = \{1, 2, 3, 4\}$	$\text{sup}_0^{r_5}[y] = \{1, 2, 3, 4\}$
$\text{sup}_1^{r_1}[x] = \{1, 5\}$	$\text{sup}_1^{r_2}[x] = \{1\}$	$\text{sup}_1^{r_3}[x, w] = \{(1, 2)\}$	$\text{sup}_1^{r_4}[x] = \{1, 2, 3, 4\}$	$\text{sup}_1^{r_5}[x, w, y] = \{(1, 2, 1), (2, 3, 1), \dots, (7, 1, 4)\}$
$\text{output}_{\text{Query}}^f[x] = \{1, 5\}$	$\text{output}_S^{bf}[x, y] = \{(1, 1)\}$	$\text{sup}_2^{r_3}[x, v] = \{(1, 2), (1, 4)\}$	$\text{output}_S^{fb}[x, y] = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$	$\text{sup}_2^{r_5}[x, v, y] = \{(1, 2, 1), (2, 3, 1), \dots, (7, 1, 4)\}$
		$\text{sup}_3^{r_3}[x, y] = \{(1, 1), (1, 5)\}$		$\text{sup}_3^{r_5}[x, y] = \{(1, 1), (2, 2), (2, 4), (4, 2), (4, 4)\}$
		$\text{output}_S^{bf}[x, y] = \{(1, 1), (1, 5)\}$		$\text{output}_S^{fb}[x, y] = \{(1, 1), (2, 2), (2, 4), (4, 2), (4, 4)\}$

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S(v, w)$$

What is the adorned version of this program for the query $S(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S(v, w)$$

What is the adorned version of this program for the query $S(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

Solution.

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S(v, w)$$

What is the adorned version of this program for the query $S(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

Solution.

$$r_1 = Q^f(x) \leftarrow S^{bf}(1, x)$$

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

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Solution.

$$r_1 = Q^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

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What is the adorned version of this program for the query $S(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

Solution.

$$r_1 = Q^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

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Solution.

$$r_1 = Q^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

$$r_4 = S^{bb}(x, x) \leftarrow h(x)$$

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S(v, w)$$

What is the adorned version of this program for the query $S(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

Solution.

$$r_1 = Q^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

$$r_4 = S^{bb}(x, x) \leftarrow h(x)$$

$$r_5 = S^{bb}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S(v, w)$$

What is the adorned version of this program for the query $S(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

Solution.

$$r_1 = Q^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

$$r_4 = S^{bb}(x, x) \leftarrow h(x)$$

$$r_5 = S^{bb}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

- ▶ Consider the auxiliary relations for r_4 : $\text{input}_S^{bb}[x, y]$ has arity two, since two variables are bound.

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S(v, w)$$

What is the adorned version of this program for the query $S(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

Solution.

$$r_1 = Q^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

$$r_4 = S^{bb}(x, x) \leftarrow h(x)$$

$$r_5 = S^{bb}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

- ▶ Consider the auxiliary relations for r_4 : $\text{input}_S^{bb}[x, y]$ has arity two, since two variables are bound.
- ▶ However, $\text{sup}_0^{r_4}[x]$ has arity one, since r_4 forces $x = y$.

Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S(v, w)$$

What is the adorned version of this program for the query $S(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

Solution.

$$r_1 = Q^f(x) \leftarrow S^{bf}(1, x)$$

$$r_2 = S^{bf}(x, x) \leftarrow h(x)$$

$$r_3 = S^{bf}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

$$r_4 = S^{bb}(x, x) \leftarrow h(x)$$

$$r_5 = S^{bb}(x, y) \leftarrow p(x, w) \wedge p(y, v) \wedge S^{bb}(v, w)$$

- ▶ Consider the auxiliary relations for r_4 : $\text{input}_S^{bb}[x, y]$ has arity two, since two variables are bound.
- ▶ However, $\text{sup}_0^{r_4}[x]$ has arity one, since r_4 forces $x = y$.
- ▶ Thus, only tuples $\langle x, y \rangle$ with $x = y$ are copied.

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$
$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow flat(x, y)$$

$$Sv(x, y) \leftarrow up(x, z_1) \wedge Sv(z_1, z_2) \wedge flat(z_2, z_3) \wedge Sv(z_3, z_4) \wedge down(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = Query^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow flat(x, y)$$

Exercise 4

Exercise. Consider the following program:

$$\text{Sv}(x, y) \leftarrow \text{flat}(x, y)$$

$$\text{Sv}(x, y) \leftarrow \text{up}(x, z_1) \wedge \text{Sv}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge \text{Sv}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $\text{Sv}(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow \text{Sv}^{bf}(a, y)$$

$$r_1 = \text{Sv}^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = \text{Sv}^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge \text{Sv}^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge \text{Sv}^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Magic Sets transformation:

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Magic Sets transformation:

$$\text{input}_{Sv}^{bf}(a) \leftarrow$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Magic Sets transformation:

$$\text{input}_{Sv}^{bf}(a) \leftarrow$$

$$\text{output}_{Sv}^{bf}(x, y) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{flat}(x, y)$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow flat(x, y)$$

$$Sv(x, y) \leftarrow up(x, z_1) \wedge Sv(z_1, z_2) \wedge flat(z_2, z_3) \wedge Sv(z_3, z_4) \wedge down(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = Query^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow flat(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow up(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge flat(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge down(z_4, y)$$

Magic Sets transformation:

$$input_{Sv}^{bf}(a) \leftarrow$$

$$output_{Sv}^{bf}(x, y) \leftarrow input_{Sv}^{bf}(x) \wedge flat(x, y)$$

$$sup_2^{r_2}(x, z_1) \leftarrow input_{Sv}^{bf}(x) \wedge up(x, z_1)$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Magic Sets transformation:

$$\text{input}_{Sv}^{bf}(a) \leftarrow$$

$$\text{output}_{Sv}^{bf}(x, y) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{flat}(x, y)$$

$$\text{sup}_2^{r_2}(x, z_1) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{up}(x, z_1)$$

$$\text{input}_{Sv}^{bf}(z_1) \leftarrow \text{sup}_2^{r_2}(x, z_1)$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Magic Sets transformation:

$$\text{input}_{Sv}^{bf}(a) \leftarrow$$

$$\text{output}_{Sv}^{bf}(x, y) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{flat}(x, y)$$

$$\text{sup}_2^{r_2}(x, z_1) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{up}(x, z_1)$$

$$\text{input}_{Sv}^{bf}(z_1) \leftarrow \text{sup}_2^{r_2}(x, z_1)$$

$$\text{sup}_3^{r_2}(x, z_2) \leftarrow \text{sup}_2^{r_2}(x, z_1) \wedge \text{output}_{Sv}^{bf}(z_1, z_2)$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Magic Sets transformation:

$$\text{input}_{Sv}^{bf}(a) \leftarrow$$

$$\text{output}_{Sv}^{bf}(x, y) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{flat}(x, y)$$

$$\text{sup}_2^{r_2}(x, z_1) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{up}(x, z_1)$$

$$\text{input}_{Sv}^{bf}(z_1) \leftarrow \text{sup}_2^{r_2}(x, z_1)$$

$$\text{sup}_3^{r_2}(x, z_2) \leftarrow \text{sup}_2^{r_2}(x, z_1) \wedge \text{output}_{Sv}^{bf}(z_1, z_2)$$

$$\text{sup}_4^{r_2}(x, z_3) \leftarrow \text{sup}_3^{r_2}(x, z_2) \wedge \text{flat}(z_2, z_3)$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Magic Sets transformation:

$$\text{input}_{Sv}^{bf}(a) \leftarrow$$

$$\text{sup}_2^{r_2}(x, z_1) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{up}(x, z_1)$$

$$\text{sup}_3^{r_2}(x, z_2) \leftarrow \text{sup}_2^{r_2}(x, z_1) \wedge \text{output}_{Sv}^{bf}(z_1, z_2)$$

$$\text{input}_{Sv}^{bf}(z_3) \leftarrow \text{sup}_4^{r_2}(x, z_3)$$

$$\text{output}_{Sv}^{bf}(x, y) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{flat}(x, y)$$

$$\text{input}_{Sv}^{bf}(z_1) \leftarrow \text{sup}_2^{r_2}(x, z_1)$$

$$\text{sup}_4^{r_2}(x, z_3) \leftarrow \text{sup}_3^{r_2}(x, z_2) \wedge \text{flat}(z_2, z_3)$$

Exercise 4

Exercise. Consider the following program:

$$Sv(x, y) \leftarrow \text{flat}(x, y)$$

$$Sv(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv(z_3, z_4) \wedge \text{down}(z_4, y)$$

Give the magic set transformation for this program and the query $Sv(a, y)$, where a is a constant.

Solution.

Adorned program:

$$r_0 = \text{Query}^f(y) \leftarrow Sv^{bf}(a, y)$$

$$r_1 = Sv^{bf}(x, y) \leftarrow \text{flat}(x, y)$$

$$r_2 = Sv^{bf}(x, y) \leftarrow \text{up}(x, z_1) \wedge Sv^{bf}(z_1, z_2) \wedge \text{flat}(z_2, z_3) \wedge Sv^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$

Magic Sets transformation:

$$\text{input}_{Sv}^{bf}(a) \leftarrow$$

$$\text{sup}_2^{r_2}(x, z_1) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{up}(x, z_1)$$

$$\text{sup}_3^{r_2}(x, z_2) \leftarrow \text{sup}_2^{r_2}(x, z_1) \wedge \text{output}_{Sv}^{bf}(z_1, z_2)$$

$$\text{input}_{Sv}^{bf}(z_3) \leftarrow \text{sup}_4^{r_2}(x, z_3)$$

$$\text{output}_{Sv}^{bf}(x, y) \leftarrow \text{input}_{Sv}^{bf}(x) \wedge \text{flat}(x, y)$$

$$\text{input}_{Sv}^{bf}(z_1) \leftarrow \text{sup}_2^{r_2}(x, z_1)$$

$$\text{sup}_4^{r_2}(x, z_3) \leftarrow \text{sup}_3^{r_2}(x, z_2) \wedge \text{flat}(z_2, z_3)$$

$$\text{output}_{Sv}^{bf}(x, y) \leftarrow \text{sup}_4^{r_2}(x, z_3) \wedge \text{output}_{Sv}^{bf}(z_3, z_4) \wedge \text{down}(z_4, y)$$