Exercise 10: Datalog Evaluation

Database Theory

2022-06-21

Maximilian Marx, Markus Krötzsch

Exercise 1

Exercise. Consider the program

$$P = \{\, \mathsf{T}(x) \leftarrow \mathsf{e}(x), \mathsf{T}(x) \leftarrow \mathsf{a}(x, y) \wedge \mathsf{T}(y) \wedge \mathsf{b}(x, z) \wedge \mathsf{T}(z) \,\}.$$

1. Describe, in your own words, the kind of structures that the query $\langle \mathsf{T}, P \rangle$ recognises.
2. Compute the semi-naive evaluation of P for the database D with the following facts:

   $\mathsf{e}(1)$ $\mathsf{e}(2)$ $\mathsf{e}(6)$ $\mathsf{a}(3,1)$ $\mathsf{a}(4,3)$ $\mathsf{a}(5,3)$ $\mathsf{a}(7,5)$ $\mathsf{b}(3,2)$ $\mathsf{b}(5,3)$ $\mathsf{b}(7,6)$

   Specify for each newly derived fact which of the rule(s) of P will produce it at the given point in the derivation.

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Solution.

1. $\langle \mathsf{T}, P \rangle$ recognises binary forests with leafs labelled e and branches labelled a and b.

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1. $\langle \mathsf{T}, P \rangle$ recognises binary forests with leafs labelled e and branches labelled a and b.
2. Transformed program:

$$\hat{P} = \Big\{ \underbrace{\mathsf{T}(x) \leftarrow \mathsf{e}(x)}_{\text{(R1)}}, \underbrace{\mathsf{T}(x) \leftarrow \mathsf{a}(x,y) \wedge \Delta_T^i(y) \wedge \mathsf{b}(x,z) \wedge \mathsf{T}^i(z)}_{\text{(R2.1)}}, \underbrace{\mathsf{T}(x) \leftarrow \mathsf{a}(x,y) \wedge \mathsf{T}^{i-1}(y) \wedge \mathsf{b}(x,z) \wedge \Delta_T^i(z)}_{\text{(R2.2)}} \Big\}$$

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$$T^0_{\hat{P}} = \varnothing$$

$$T^1_{\hat{P}} = D \cup \{\, \mathsf{T}(1), \mathsf{T}(2), \mathsf{T}(6) \,\} \qquad \text{(R1)}$$

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$$\begin{aligned}
T_{\hat{P}}^0 &= \varnothing \\
T_{\hat{P}}^1 &= D \cup \{\, \mathsf{T}(1), \mathsf{T}(2), \mathsf{T}(6) \,\} && \text{(R1)} \\
T_{\hat{P}}^2 &= T_{\hat{P}}^1 \cup \{\, \mathsf{T}(3) \,\} && \text{(R2.1)} \\
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T^3_{\hat{P}} &= T^2_{\hat{P}} \cup \{\, \mathsf{T}(5) \,\} && \text{(R2.1)} \\
T^4_{\hat{P}} &= T^3_{\hat{P}} \cup \{\, \mathsf{T}(7) \,\} = T^5_{\hat{P}} = T^\infty_{\hat{P}} && \text{(R2.1)}
\end{aligned}$$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
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together with the database that contains the following facts for predicate p:

$\mathrm{h}(1)$ $\mathrm{h}(2)$ $\mathrm{h}(3)$ $\mathrm{h}(4)$ $\mathrm{h}(5)$ $\mathrm{h}(6)$ $\mathrm{h}(7)$ $\quad$ $\mathrm{p}(1,2)$ $\mathrm{p}(2,3)$ $\mathrm{p}(4,3)$ $\mathrm{p}(5,4)$ $\mathrm{p}(6,1)$ $\mathrm{p}(7,1)$

1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

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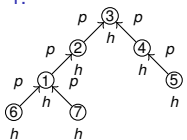

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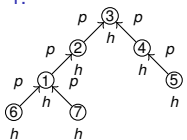

Expected answers:

$$\big\{\mathrm{Query}(1), \mathrm{Query}(5)\big\}$$

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1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \varnothing$
$\mathrm{sup}^{r_1}_0[] = \varnothing$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_2}_0[x] = \varnothing$
$\mathrm{sup}^{r_2}_1[x] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_3}_0[x] = \varnothing$
$\mathrm{sup}^{r_3}_1[x,w] = \varnothing$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_4}_0[x] = \varnothing$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \varnothing$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

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together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_0[] = \varnothing$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_3}_0[x] = \varnothing$
$\mathrm{sup}^{r_3}_1[x] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

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$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_4}_0[x] = \varnothing$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \varnothing$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad\qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_2}_0[x] = \varnothing$
$\mathrm{sup}^{r_2}_1[x] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_3}_0[x] = \varnothing$
$\mathrm{sup}^{r_3}_1[x,w] = \varnothing$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_4}_0[x] = \varnothing$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \varnothing$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
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Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

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1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

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$\mathrm{sup}^{r_1}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \varnothing$
$\mathrm{sup}^{r_2}_1[x] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_3}_0[x] = \varnothing$
$\mathrm{sup}^{r_3}_1[x,w] = \varnothing$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_4}_0[x] = \varnothing$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \varnothing$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

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$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \{1\}$
$\mathrm{sup}^{r_2}_1[x] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

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$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_4}_0[x] = \varnothing$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \varnothing$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
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Exercise 2

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Solution.

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$\mathrm{sup}^{r_1}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \{1\}$
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$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \varnothing$
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$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \varnothing$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
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Exercise 2

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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

2.

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$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \{1\}$
$\mathrm{sup}^{r_2}_1[x] = \{1\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_3}_0[x] = \varnothing$
$\mathrm{sup}^{r_3}_1[x,w] = \varnothing$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_4}_0[x] = \varnothing$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \varnothing$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
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$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

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and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

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$\mathrm{sup}^{r_1}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \{1\}$
$\mathrm{sup}^{r_2}_1[x] = \{1\}$
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$\mathrm{sup}^{r_3}_1[x,w] = \varnothing$
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$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \varnothing$
$\mathrm{sup}^{r_4}_0[x] = \varnothing$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \varnothing$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
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Solution.

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$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

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$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
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$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{2\}$
$\mathrm{sup}^{r_4}_0[x] = \varnothing$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{2\}$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
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Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \{1\}$
$\mathrm{sup}^{r_2}_1[x] = \{1\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_3}_0[x] = \{1\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\langle 1,2\rangle\}$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{2\}$
$\mathrm{sup}^{r_4}_0[x] = \{2\}$
$\mathrm{sup}^{r_4}_1[x] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{2\}$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad\qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \{1\}$
$\mathrm{sup}^{r_2}_1[x] = \{1\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_3}_0[x] = \{1\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\langle 1,2\rangle\}$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{2\}$
$\mathrm{sup}^{r_4}_0[x] = \{2\}$
$\mathrm{sup}^{r_4}_1[x] = \{2\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{2\}$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
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and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
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$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad\qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

$\mathrm{h}(1)$ $\mathrm{h}(2)$ $\mathrm{h}(3)$ $\mathrm{h}(4)$ $\mathrm{h}(5)$ $\mathrm{h}(6)$ $\mathrm{h}(7)$ $\qquad$ $\mathrm{p}(1,2)$ $\mathrm{p}(2,3)$ $\mathrm{p}(4,3)$ $\mathrm{p}(5,4)$ $\mathrm{p}(6,1)$ $\mathrm{p}(7,1)$

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_0[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\,\langle 1,2\rangle\,\}$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,2\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,2\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,2\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 2,2\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,2\,\}$
$\mathrm{sup}^{r_5}_0[y] = \varnothing$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_0[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
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$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,2\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,2\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,2\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 2,2\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,2\,\}$
$\mathrm{sup}^{r_5}_0[y] = \{\,2\,\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \varnothing$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad\qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_0}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \{1\}$
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$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{2\}$
$\mathrm{sup}^{r_4}_0[x] = \{2\}$
$\mathrm{sup}^{r_4}_1[x] = \{2\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\langle 2,2\rangle\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{2\}$
$\mathrm{sup}^{r_5}_0[y] = \{2\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\langle 1,2,2\rangle, \langle 2,3,2\rangle, \ldots, \langle 7,1,2\rangle\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
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Exercise 2

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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

2.

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$\mathrm{sup}^{r_1}_0[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_2}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_2}_1[x,w] = \{\,\langle 1,2\rangle\,\}$
$\mathrm{sup}^{r_2}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_2}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,2\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,2\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 2,2\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_0[y] = \{\,2\,\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\,\langle 1,2,2\rangle, \langle 2,3,2\rangle, \ldots, \langle 7,1,2\rangle\,\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
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Exercise 2

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1. Sketch the database as a tree. What are the expected answers to the query?
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Solution.

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$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
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$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_0[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_1[x] = \{2\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\langle 2,2\rangle\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_0[y] = \{2\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\langle 1,2,2\rangle, \langle 2,3,2\rangle, \ldots, \langle 7,1,2\rangle\}$
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$\mathrm{sup}^{r_3}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

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$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,1,2,3,4\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 2,2\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_0[y] = \{\,2\,\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\,\langle 1,2,2\rangle,\langle 2,3,2\rangle,\ldots,\langle 7,1,2\rangle\,\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
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$\mathrm{sup}^{r_1}_1[x] = \varnothing$
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$\mathrm{sup}^{r_2}_0[x] = \{1\}$
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$\mathrm{sup}^{r_3}_1[x,w] = \{\langle 1,2\rangle\}$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_0[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_1[x] = \{1,2,3,4\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle,\langle 4,4\rangle\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_0[y] = \{2\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\langle 1,2,2\rangle,\langle 2,3,2\rangle,\ldots,\langle 7,1,2\rangle\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_3}_0[x] = \{1\}$
$\mathrm{sup}^{r_3}_1[x] = \{1\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_3}_0[x] = \{1\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\langle 1,2\rangle\}$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_0[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_1[x] = \{1,2,3,4\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle,\langle 4,4\rangle\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_0[y] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\langle 1,2,2\rangle,\langle 2,3,2\rangle,\ldots,\langle 7,1,2\rangle\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_0[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_2}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_2}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\,\langle 1,2\rangle\,\}$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,1,2,3,4\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle,\langle 4,4\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_0[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_0[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\,\langle 1,2\rangle\,\}$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,1,2,3,4\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle,\langle 4,4\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_0[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_3}_0[x] = \{1\}$
$\mathrm{sup}^{r_3}_1[x] = \{1\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_5}_0[x] = \{1\}$
$\mathrm{sup}^{r_5}_1[x,w] = \{\langle 1,2\rangle\}$
$\mathrm{sup}^{r_5}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_5}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_0[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_1[x] = \{1,2,3,4\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle,\langle 4,4\rangle\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_0[y] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \{\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\}$
$\mathrm{sup}^{r_5}_3[x,y] = \{\langle 1,1\rangle,\langle 2,2\rangle,\langle 2,4\rangle,\langle 4,2\rangle,\langle 4,4\rangle\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \varnothing$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad\qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_0[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\,\langle 1,2\rangle\,\}$
$\mathrm{sup}^{r_3}_2[x,v] = \varnothing$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,1,2,3,4\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle,\langle 4,4\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_0[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_3[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 2,4\rangle,\langle 4,2\rangle,\langle 4,4\rangle\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 2,4\rangle,\langle 4,2\rangle,\langle 4,4\rangle\,\}$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad\qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_0[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\,\langle 1,2\rangle\,\}$
$\mathrm{sup}^{r_3}_2[x,v] = \{\,\langle 1,2\rangle,\langle 1,4\rangle\,\}$
$\mathrm{sup}^{r_3}_3[x,y] = \varnothing$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,1,2,3,4\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle,\langle 4,4\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_0[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_3[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 2,4\rangle,\langle 4,2\rangle,\langle 4,4\rangle\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 2,4\rangle,\langle 4,2\rangle,\langle 4,4\rangle\,\}$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad\qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^r_0[] = \{\,1,2,\ldots,7\,\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_2}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_2}_1[x] = \{\,1\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle\,\}$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_0[x] = \{\,1\,\}$
$\mathrm{sup}^{r_3}_1[x,w] = \{\,\langle 1,2\rangle\,\}$
$\mathrm{sup}^{r_3}_2[x,v] = \{\,\langle 1,2\rangle,\langle 1,4\rangle\,\}$
$\mathrm{sup}^{r_3}_3[x,y] = \{\,\langle 1,1\rangle,\langle 1,5\rangle\,\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \varnothing$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_0[x] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_4}_1[x] = \{\,1,2,3,4\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle,\langle 4,4\rangle\,\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_0[y] = \{\,1,2,3,4\,\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \{\,\langle 1,2,1\rangle,\langle 2,3,1\rangle,\ldots,\langle 7,1,4\rangle\,\}$
$\mathrm{sup}^{r_5}_3[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 2,4\rangle,\langle 4,2\rangle,\langle 4,4\rangle\,\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\,\langle 1,1\rangle,\langle 2,2\rangle,\langle 2,4\rangle,\langle 4,2\rangle,\langle 4,4\rangle\,\}$

Exercise 2

Exercise. Consider the "Same generation" Datalog program given in the lecture (Lecture 15, Slide 15):

$$\mathrm{S}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad \mathrm{S}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}(v,w) \wedge \mathrm{p}(y,v)$$

and the adorned version for query $\mathrm{S}(1,x)$:

$$r_1 = \mathrm{Query}^f(x) \leftarrow \mathrm{S}^{bf}(1,x)$$
$$r_2 = \mathrm{S}^{bf}(x,x) \leftarrow \mathrm{h}(x) \qquad\qquad r_4 = \mathrm{S}^{fb}(x,x) \leftarrow \mathrm{h}(x)$$
$$r_3 = \mathrm{S}^{bf}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v) \qquad\qquad r_5 = \mathrm{S}^{fb}(x,y) \leftarrow \mathrm{p}(x,w) \wedge \mathrm{S}^{fb}(v,w) \wedge \mathrm{p}(y,v)$$

together with the database that contains the following facts for predicate p:

h(1) h(2) h(3) h(4) h(5) h(6) h(7) p(1,2) p(2,3) p(4,3) p(5,4) p(6,1) p(7,1)

1. Sketch the database as a tree. What are the expected answers to the query?
2. Apply the QSQR algorithm to compute the answer to the query.

Solution.

2.

$\mathrm{input}^f_{\mathrm{Query}}[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^r_0[] = \{1,2,\ldots,7\}$
$\mathrm{sup}^{r_1}_1[x] = \varnothing$
$\mathrm{output}^f_{\mathrm{Query}}[x] = \varnothing$

$\mathrm{input}^{bf}_{\mathrm{S}}[x] = \{1\}$
$\mathrm{sup}^{r_2}_0[x] = \{1\}$
$\mathrm{sup}^{r_3}_1[x] = \{1\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle\}$

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$\mathrm{sup}^{r_3}_0[x] = \{1\}$
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$\mathrm{sup}^{r_3}_2[x,v] = \{\langle 1,2\rangle, \langle 1,4\rangle\}$
$\mathrm{sup}^{r_3}_3[x,y] = \{\langle 1,1\rangle, \langle 1,5\rangle\}$
$\mathrm{output}^{bf}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle, \langle 1,5\rangle\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_4}_0[x] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_1[x] = \{1,2,3,4\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle\}$

$\mathrm{input}^{fb}_{\mathrm{S}}[y] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_0[y] = \{1,2,3,4\}$
$\mathrm{sup}^{r_5}_1[x,w,y] = \{\langle 1,2,1\rangle, \langle 2,3,1\rangle, \ldots, \langle 7,1,4\rangle\}$
$\mathrm{sup}^{r_5}_2[x,v,y] = \{\langle 1,2,1\rangle, \langle 2,3,1\rangle, \ldots, \langle 7,1,4\rangle\}$
$\mathrm{sup}^{r_5}_3[x,y] = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 2,4\rangle, \langle 4,2\rangle, \langle 4,4\rangle\}$
$\mathrm{output}^{fb}_{\mathrm{S}}[x,y] = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 2,4\rangle, \langle 4,2\rangle, \langle 4,4\rangle\}$

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$\mathrm{sup}^{r_2}_0[x] = \{\,1\,\}$
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Exercise 3

Exercise. Consider the following modified version of the same generation program:

$$\mathrm{S}(x, x) \leftarrow \mathrm{h}(x)$$
$$\mathrm{S}(x, y) \leftarrow \mathrm{p}(x, w) \wedge \mathrm{p}(y, v) \wedge \mathrm{S}(v, w)$$

What is the adorned version of this program for the query $\mathrm{S}(1, x)$? Use this program to show that it is possible that some tuples in an input-relation are not copied to the sup_0 relation of a rule during the execution of the QSQR algorithm.

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Solution.

$$r_1 = \mathrm{Q}^f(x) \leftarrow \mathrm{S}^{bf}(1, x)$$

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$$r_5 = \mathrm{S}^{bb}(x, y) \leftarrow \mathrm{p}(x, w) \wedge \mathrm{p}(y, v) \wedge \mathrm{S}^{bb}(v, w)$$

- Consider the auxiliary relations for r_4: $\mathrm{input}_S^{bb}[x, y]$ has arity two, since two variables are bound.

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- Consider the auxiliary relations for r_4: $\mathrm{input}_S^{bb}[x, y]$ has arity two, since two variables are bound.
- However, $\mathrm{sup}_0^{r_4}[x]$ has arity one, since r_4 forces $x = y$.

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Exercise. Consider the following modified version of the same generation program:

$$\mathrm{S}(x, x) \leftarrow \mathrm{h}(x)$$
$$\mathrm{S}(x, y) \leftarrow \mathrm{p}(x, w) \wedge \mathrm{p}(y, v) \wedge \mathrm{S}(v, w)$$

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- Consider the auxiliary relations for r_4: $\mathrm{input}_S^{bb}[x, y]$ has arity two, since two variables are bound.
- However, $\mathrm{sup}_0^{r_4}[x]$ has arity one, since r_4 forces $x = y$.
- Thus, only tuples $\langle x, y \rangle$ with $x = y$ are copied.

Exercise 4

Exercise. Consider the following program:

$$\mathrm{Sv}(x, y) \leftarrow \mathrm{flat}(x, y)$$
$$\mathrm{Sv}(x, y) \leftarrow \mathrm{up}(x, z_1) \wedge \mathrm{Sv}(z_1, z_2) \wedge \mathrm{flat}(z_2, z_3) \wedge \mathrm{Sv}(z_3, z_4) \wedge \mathrm{down}(z_4, y)$$

Give the magic set transformation for this program and the query $\mathrm{Sv}(a, y)$, where a is a constant.

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$$\mathrm{Sv}(x, y) \leftarrow \mathrm{flat}(x, y)$$
$$\mathrm{Sv}(x, y) \leftarrow \mathrm{up}(x, z_1) \wedge \mathrm{Sv}(z_1, z_2) \wedge \mathrm{flat}(z_2, z_3) \wedge \mathrm{Sv}(z_3, z_4) \wedge \mathrm{down}(z_4, y)$$

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Adorned program:

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Solution.
Adorned program:

$$r_0 = \mathrm{Query}^f(y) \leftarrow \mathrm{Sv}^{bf}(a, y)$$

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Give the magic set transformation for this program and the query $\mathrm{Sv}(a, y)$, where a is a constant.
Solution.
Adorned program:

$$r_0 = \mathrm{Query}^f(y) \leftarrow \mathrm{Sv}^{bf}(a, y)$$
$$r_1 = \mathrm{Sv}^{bf}(x, y) \leftarrow \mathrm{flat}(x, y)$$

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$$\mathrm{Sv}(x, y) \leftarrow \mathrm{up}(x, z_1) \wedge \mathrm{Sv}(z_1, z_2) \wedge \mathrm{flat}(z_2, z_3) \wedge \mathrm{Sv}(z_3, z_4) \wedge \mathrm{down}(z_4, y)$$

Give the magic set transformation for this program and the query $\mathrm{Sv}(a, y)$, where a is a constant.
Solution.
Adorned program:

$$r_0 = \mathrm{Query}^{f}(y) \leftarrow \mathrm{Sv}^{bf}(a, y)$$
$$r_1 = \mathrm{Sv}^{bf}(x, y) \leftarrow \mathrm{flat}(x, y)$$
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$$r_0 = \text{Query}^f(y) \leftarrow \text{Sv}^{bf}(a, y)$$
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Magic Sets transformation:

Exercise 4

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$$\mathrm{Sv}(x, y) \leftarrow \mathrm{flat}(x, y)$$
$$\mathrm{Sv}(x, y) \leftarrow \mathrm{up}(x, z_1) \wedge \mathrm{Sv}(z_1, z_2) \wedge \mathrm{flat}(z_2, z_3) \wedge \mathrm{Sv}(z_3, z_4) \wedge \mathrm{down}(z_4, y)$$

Give the magic set transformation for this program and the query $\mathrm{Sv}(a, y)$, where a is a constant.
Solution.
Adorned program:

$$r_0 = \mathrm{Query}^{f}(y) \leftarrow \mathrm{Sv}^{bf}(a, y)$$
$$r_1 = \mathrm{Sv}^{bf}(x, y) \leftarrow \mathrm{flat}(x, y)$$
$$r_2 = \mathrm{Sv}^{bf}(x, y) \leftarrow \mathrm{up}(x, z_1) \wedge \mathrm{Sv}^{bf}(z_1, z_2) \wedge \mathrm{flat}(z_2, z_3) \wedge \mathrm{Sv}^{bf}(z_3, z_4) \wedge \mathrm{down}(z_4, y)$$

Magic Sets transformation:

$$\mathrm{input}^{bf}_{\mathrm{Sv}}(a) \leftarrow$$

Exercise 4

Exercise. Consider the following program:

$$\mathrm{Sv}(x, y) \leftarrow \mathrm{flat}(x, y)$$
$$\mathrm{Sv}(x, y) \leftarrow \mathrm{up}(x, z_1) \wedge \mathrm{Sv}(z_1, z_2) \wedge \mathrm{flat}(z_2, z_3) \wedge \mathrm{Sv}(z_3, z_4) \wedge \mathrm{down}(z_4, y)$$

Give the magic set transformation for this program and the query $\mathrm{Sv}(a, y)$, where a is a constant.
Solution.
Adorned program:

$$r_0 = \mathrm{Query}^f(y) \leftarrow \mathrm{Sv}^{bf}(a, y)$$
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Magic Sets transformation:

$$\mathrm{input}^{bf}_{\mathrm{Sv}}(a) \leftarrow \qquad\qquad \mathrm{output}^{bf}_{\mathrm{Sv}}(x, y) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{flat}(x, y)$$

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$$\mathrm{Sv}(x, y) \leftarrow \mathrm{flat}(x, y)$$
$$\mathrm{Sv}(x, y) \leftarrow \mathrm{up}(x, z_1) \wedge \mathrm{Sv}(z_1, z_2) \wedge \mathrm{flat}(z_2, z_3) \wedge \mathrm{Sv}(z_3, z_4) \wedge \mathrm{down}(z_4, y)$$

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Magic Sets transformation:

$$\mathrm{input}^{bf}_{\mathrm{Sv}}(a) \leftarrow$$
$$\mathrm{sup}^{r_2}_2(x, z_1) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{up}(x, z_1)$$
$$\mathrm{output}^{bf}_{\mathrm{Sv}}(x, y) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{flat}(x, y)$$

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Magic Sets transformation:

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$$\text{sup}^{r_2}_2(x, z_1) \leftarrow \text{input}^{bf}_{\text{Sv}}(x) \wedge \text{up}(x, z_1)$$
$$\text{output}^{bf}_{\text{Sv}}(x, y) \leftarrow \text{input}^{bf}_{\text{Sv}}(x) \wedge \text{flat}(x, y)$$
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Magic Sets transformation:

$$\mathrm{input}^{bf}_{\mathrm{Sv}}(a) \leftarrow$$
$$\mathrm{sup}^{r_2}_{2}(x,z_1) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{up}(x,z_1)$$
$$\mathrm{sup}^{r_2}_{3}(x,z_2) \leftarrow \mathrm{sup}^{r_2}_{2}(x,z_1) \wedge \mathrm{output}^{bf}_{\mathrm{Sv}}(z_1,z_2)$$
$$\mathrm{output}^{bf}_{\mathrm{Sv}}(x,y) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{flat}(x,y)$$
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Magic Sets transformation:

$$\mathrm{input}^{bf}_{\mathrm{Sv}}(a) \leftarrow$$
$$\mathrm{sup}^{r_2}_{2}(x,z_1) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{up}(x,z_1)$$
$$\mathrm{sup}^{r_2}_{3}(x,z_2) \leftarrow \mathrm{sup}^{r_2}_{2}(x,z_1) \wedge \mathrm{output}^{bf}_{\mathrm{Sv}}(z_1,z_2)$$
$$\mathrm{output}^{bf}_{\mathrm{Sv}}(x,y) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{flat}(x,y)$$
$$\mathrm{input}^{bf}_{\mathrm{Sv}}(z_1) \leftarrow \mathrm{sup}^{r_2}_{2}(x,z_1)$$
$$\mathrm{sup}^{r_2}_{4}(x,z_3) \leftarrow \mathrm{sup}^{r_2}_{3}(x,z_2) \wedge \mathrm{flat}(z_2,z_3)$$

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Magic Sets transformation:

$$\mathrm{input}^{bf}_{\mathrm{Sv}}(a) \leftarrow$$
$$\mathrm{sup}^{r_2}_2(x,z_1) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{up}(x,z_1)$$
$$\mathrm{sup}^{r_2}_3(x,z_2) \leftarrow \mathrm{sup}^{r_2}_2(x,z_1) \wedge \mathrm{output}^{bf}_{\mathrm{Sv}}(z_1,z_2)$$
$$\mathrm{input}^{bf}_{\mathrm{Sv}}(z_3) \leftarrow \mathrm{sup}^{r_2}_4(x,z_3)$$
$$\mathrm{output}^{bf}_{\mathrm{Sv}}(x,y) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{flat}(x,y)$$
$$\mathrm{input}^{bf}_{\mathrm{Sv}}(z_1) \leftarrow \mathrm{sup}^{r_2}_2(x,z_1)$$
$$\mathrm{sup}^{r_2}_4(x,z_3) \leftarrow \mathrm{sup}^{r_2}_3(x,z_2) \wedge \mathrm{flat}(z_2,z_3)$$

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$$\mathrm{sup}^{r_2}_{3}(x, z_2) \leftarrow \mathrm{sup}^{r_2}_{2}(x, z_1) \wedge \mathrm{output}^{bf}_{\mathrm{Sv}}(z_1, z_2)$$
$$\mathrm{input}^{bf}_{\mathrm{Sv}}(z_3) \leftarrow \mathrm{sup}^{r_2}_{4}(x, z_3)$$
$$\mathrm{output}^{bf}_{\mathrm{Sv}}(x, y) \leftarrow \mathrm{input}^{bf}_{\mathrm{Sv}}(x) \wedge \mathrm{flat}(x, y)$$
$$\mathrm{input}^{bf}_{\mathrm{Sv}}(z_1) \leftarrow \mathrm{sup}^{r_2}_{2}(x, z_1)$$
$$\mathrm{sup}^{r_2}_{4}(x, z_3) \leftarrow \mathrm{sup}^{r_2}_{3}(x, z_2) \wedge \mathrm{flat}(z_2, z_3)$$
$$\mathrm{output}^{bf}_{\mathrm{Sv}}(x, y) \leftarrow \mathrm{sup}^{r_2}_{4}(x, z_3) \wedge \mathrm{output}^{bf}_{\mathrm{Sv}}(z_3, z_4) \wedge \mathrm{down}(z_4, y)$$