Do Repeat Yourself: Understanding Sufficient Conditions for Restricted Chase Non-Termination

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International Center
for Computational Logic

~~Running~~ Riding Example

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$x : \text{  } \rightarrow \Big(\exists y.\ y : \text{  } \wedge x \in y \Big) \vee x : \text{  }$

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$$x : \text{  } \rightarrow \left(\exists y.\ y : \text{  } \land x \in y \right) \lor x : \text{  }$$

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The **restricted chase** exhaustively applies *triggers* that are both *loaded* and not *obsolete*.

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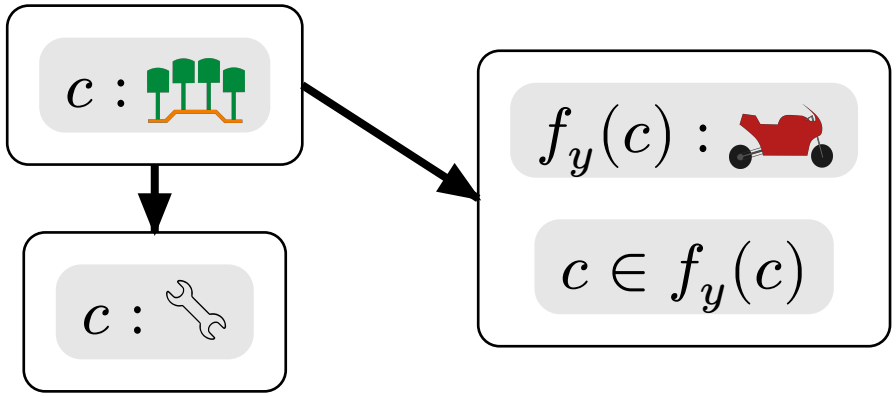

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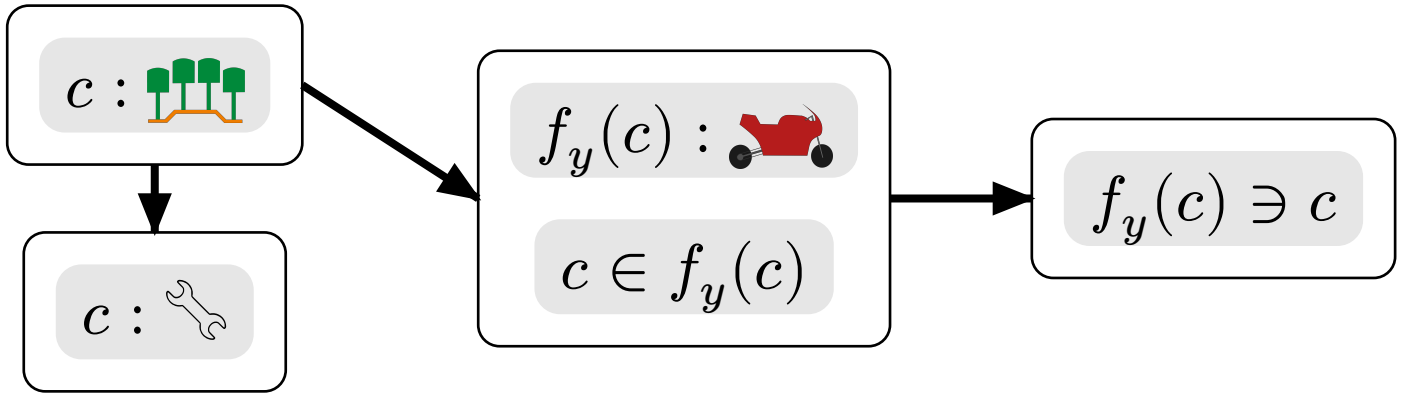

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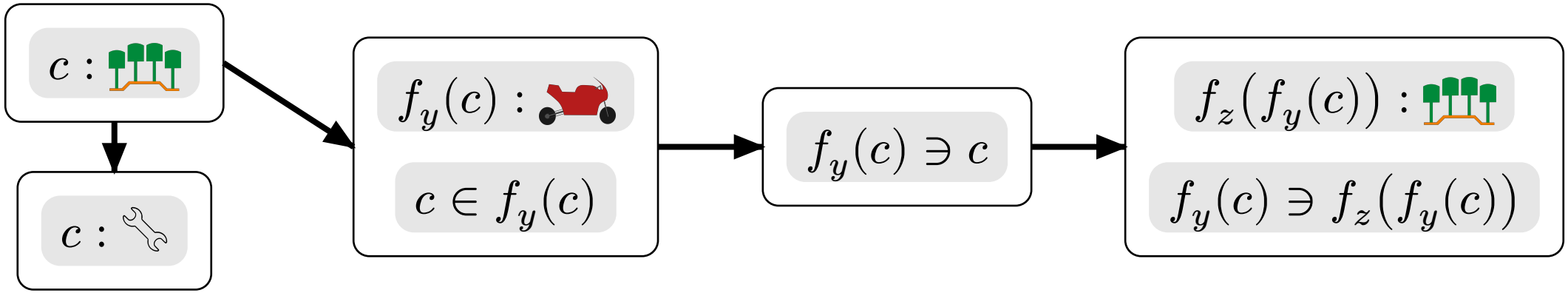

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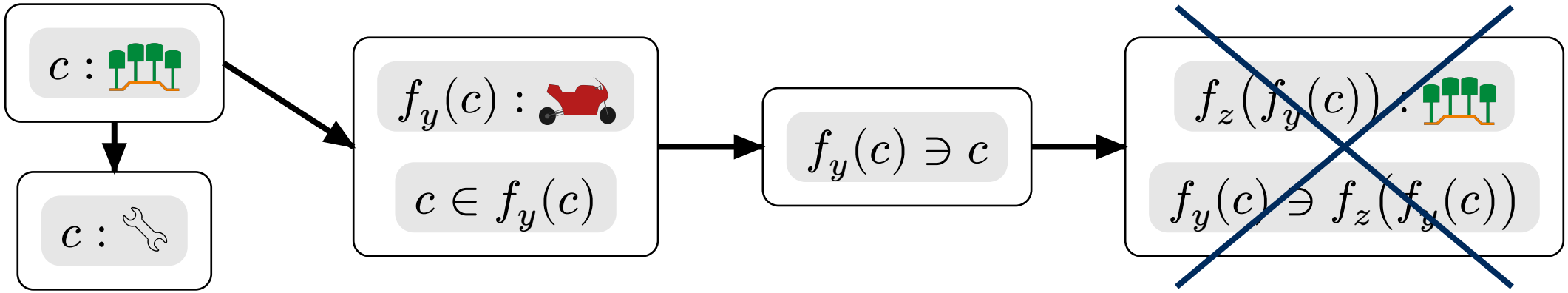

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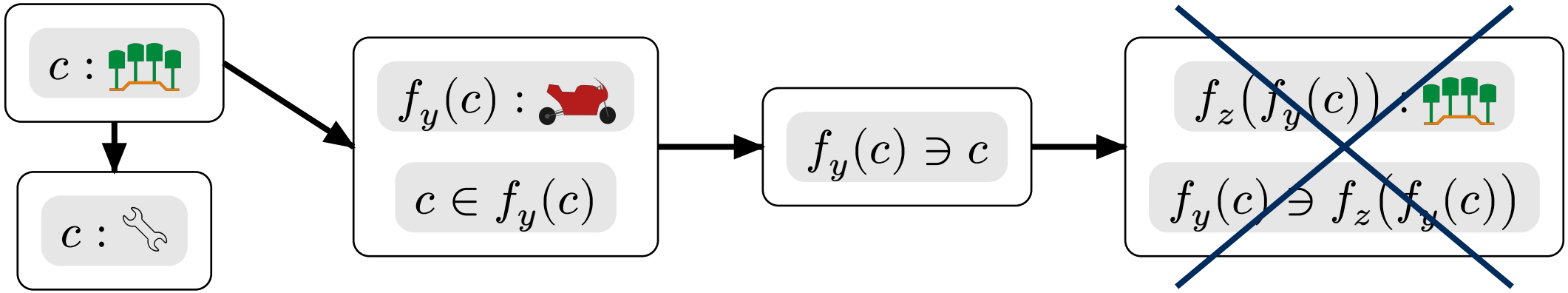

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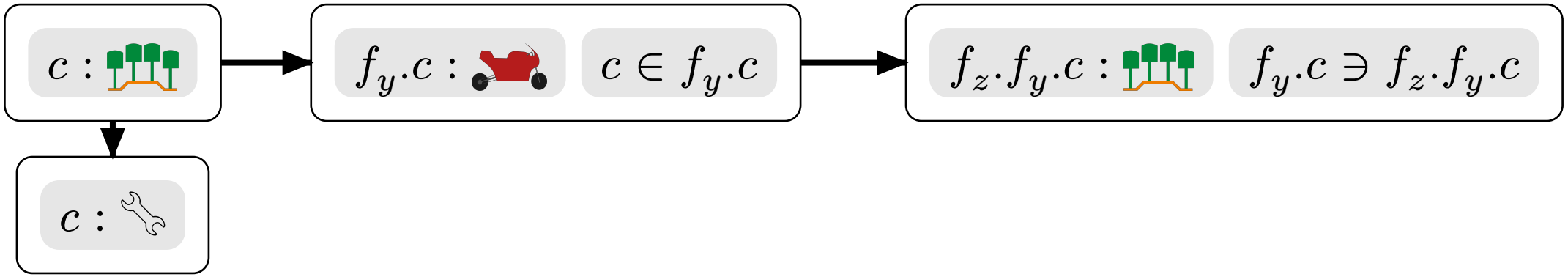

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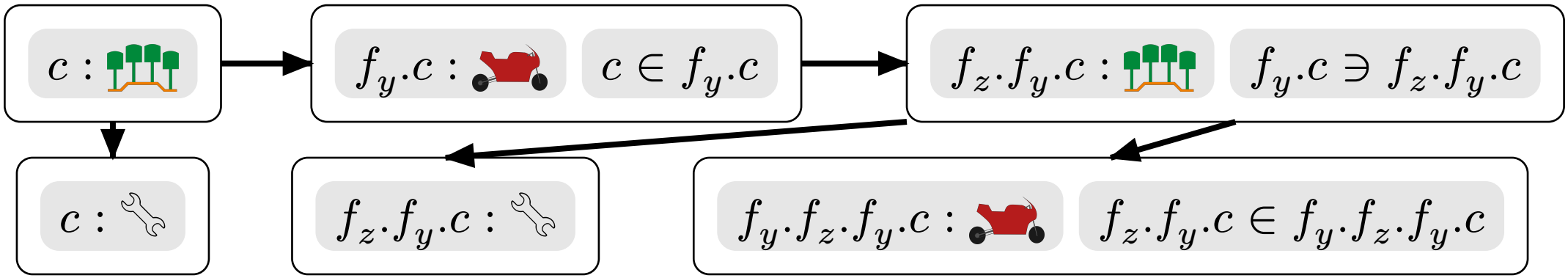

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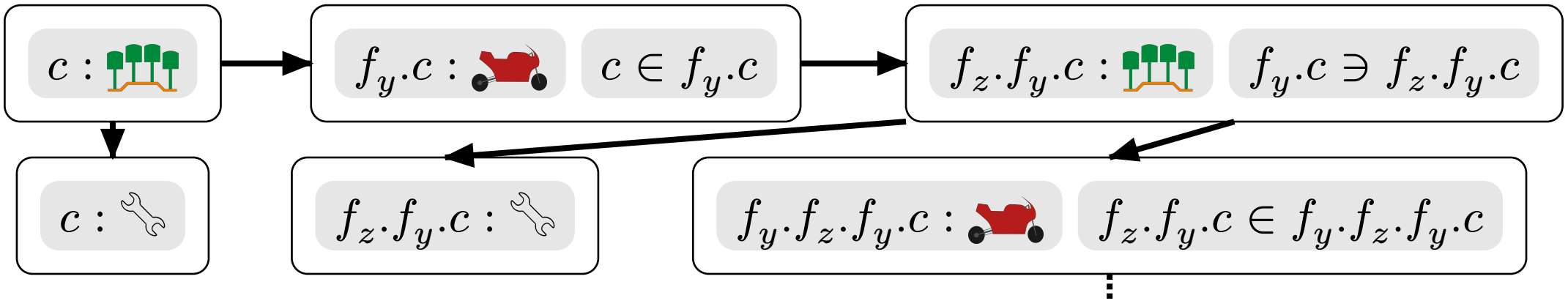
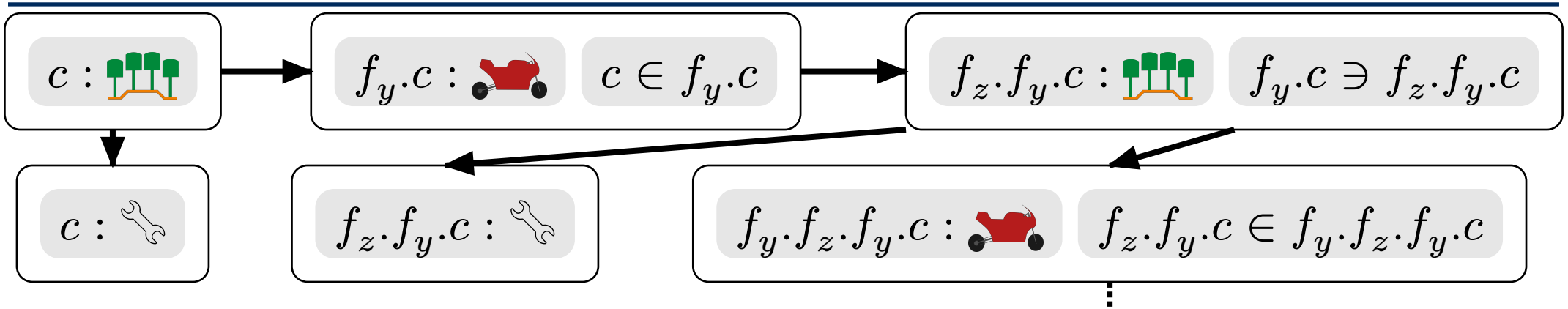


About that Chase...



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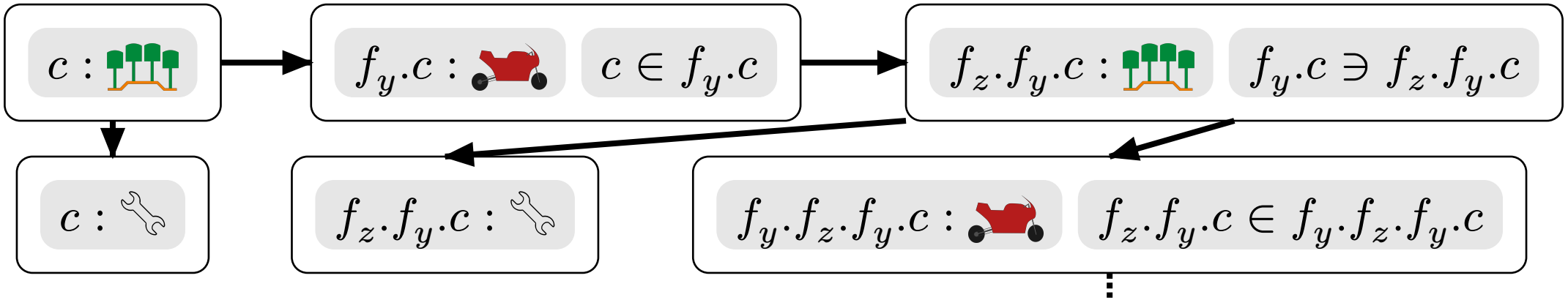

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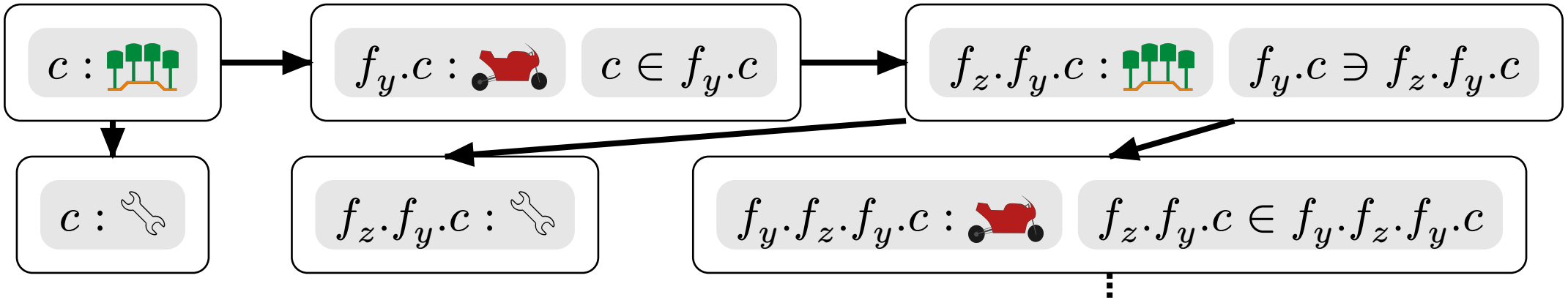

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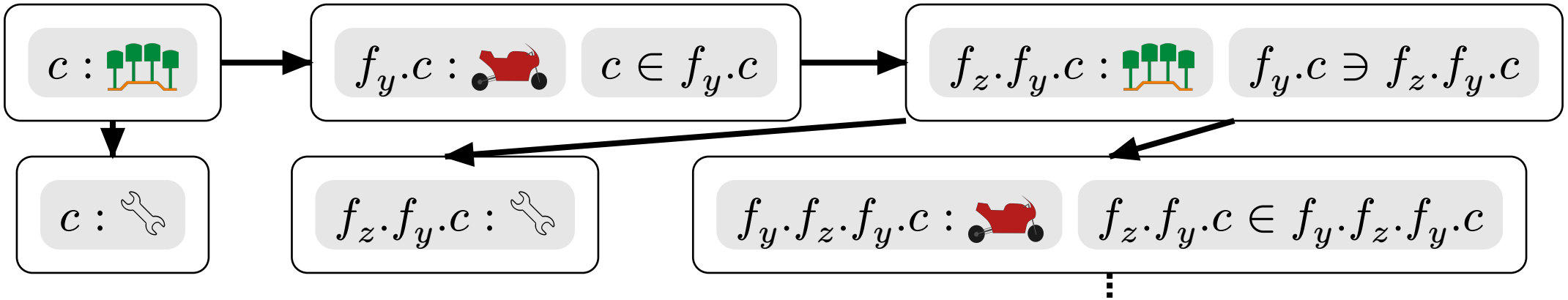

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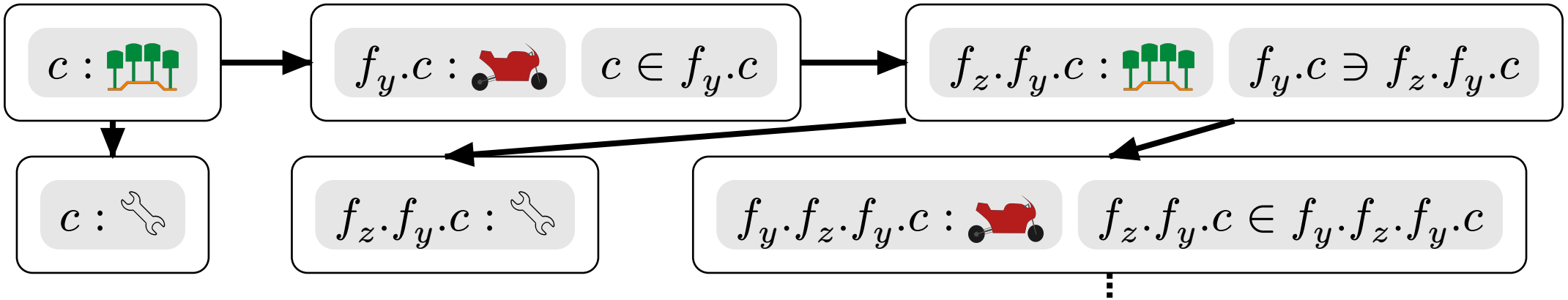

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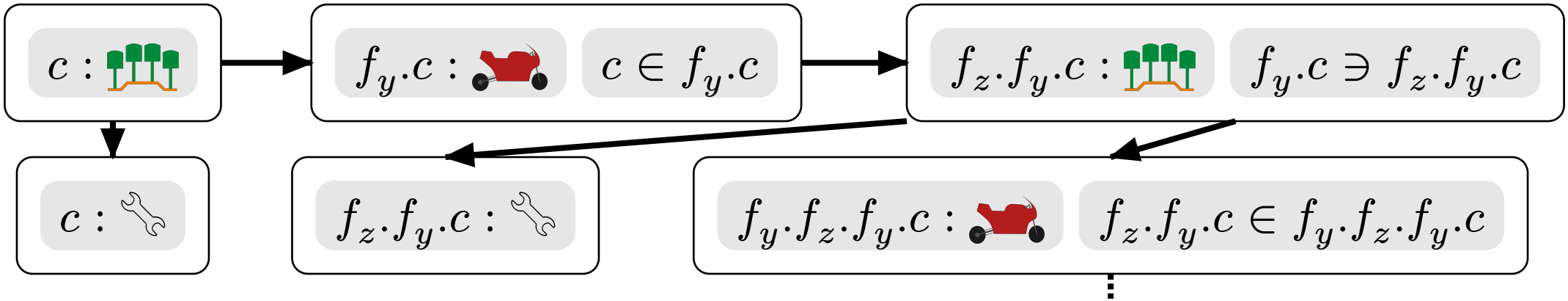

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- But query answering/entailment is undecidable [BV81]
- Chase termination is also undecidable [GM14, GO18]
- RMFA/RMFC can sometimes detect (non-)termination [CDK17]
- We reconsider RMFC with ideas from our previous work [GC23] introducing (D)RPC with a working proof and additional improvements.

Is it any good?

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cat my-ruleset | docker run --rm -i registry.gitlab.com/m0nstr/dmfa-checker -t [non_]termination -cv restricted -disj [-depth 2] ...
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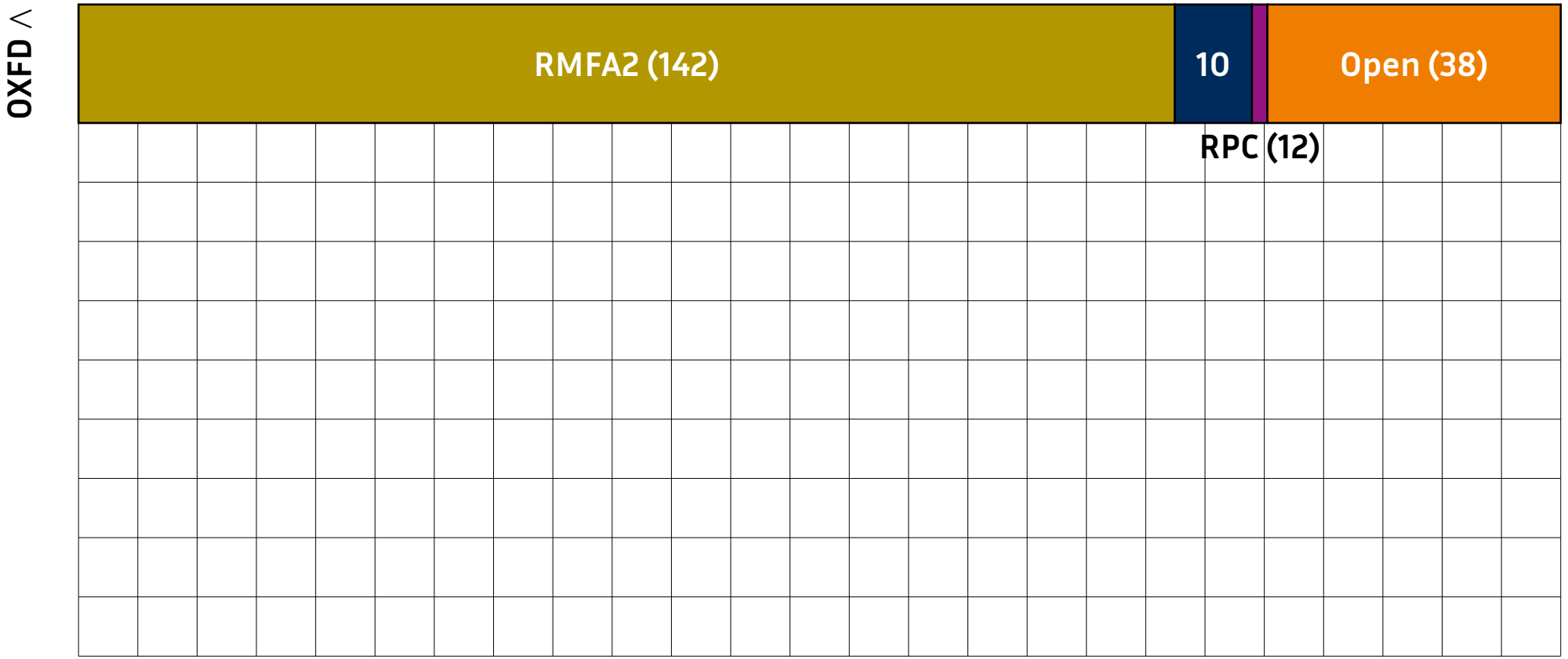

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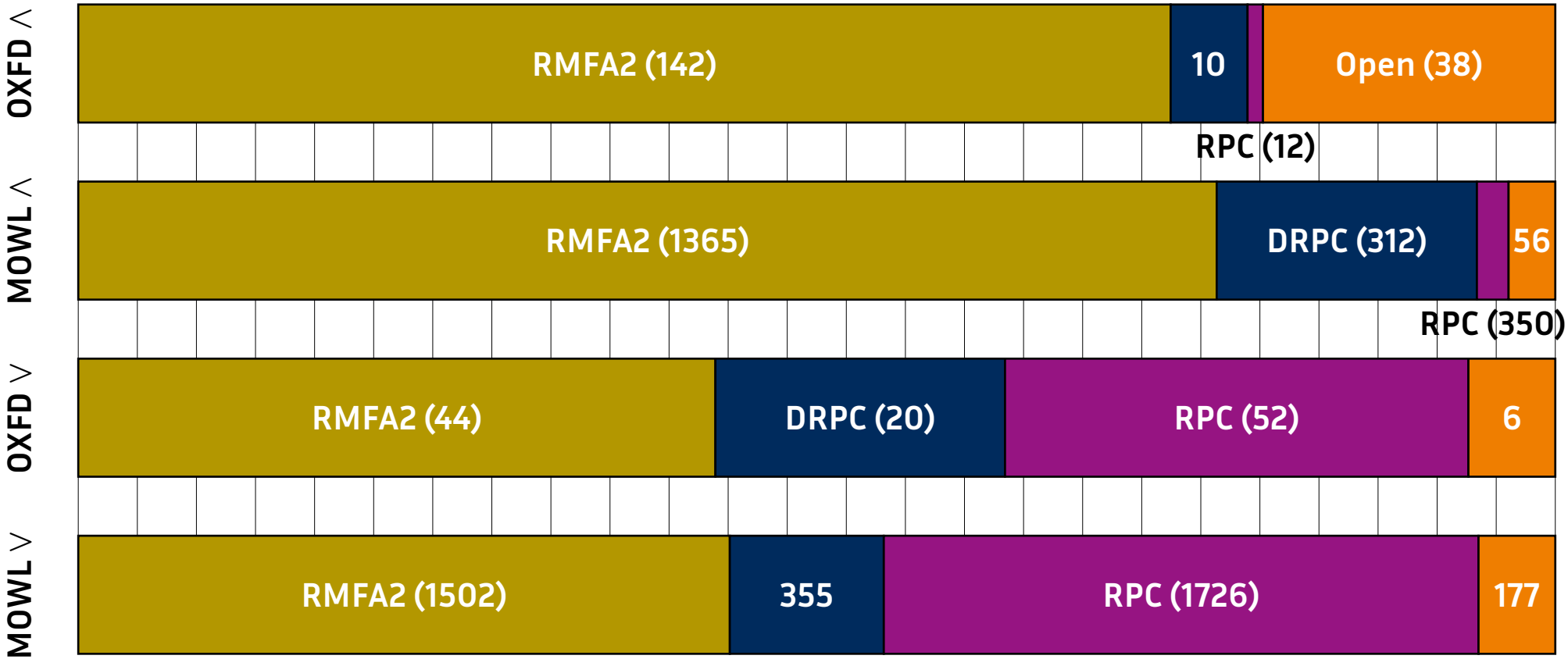

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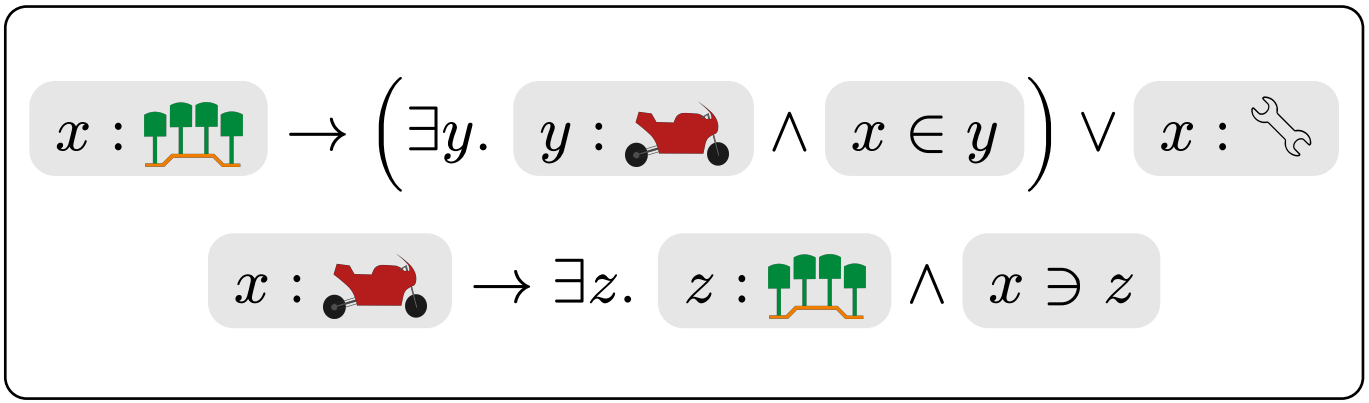

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$$x : \text{🌲} \rightarrow \Big(\exists y.\ y : \text{🏍} \wedge x \in y \Big) \vee x : \text{🔧}$$

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Plan of attack:

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3. Unblockability

$x : \text{  } \rightarrow \left(\exists y.\ y : \text{  } \land x \in y \right) \lor x : \text{  }$

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4. Cyclicity Prefixes

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5. (D)RPC

$$x : \text{trees} \rightarrow \Big(\exists y.\ y : \text{bike} \wedge x \in y \Big) \vee x : \text{wrench}$$

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Head Choice [GC23] fixes disjunct for each rule.

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2. Cyclicity Sequences

R is never-terminating if for some KB and HC there exists a *cyclicity sequence*, i.e. a *loaded*, *growing*, and *g-unblk.* trigger sequence.

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2. Cyclicity Sequences

Loaded:
The body of every trigger is satisfied by the previous outputs (and the database).

$$x : \text{  } \rightarrow \Big(\exists y.\ y : \text{  } \wedge x \in y\Big) \vee x : \text{  } \quad (1)$$

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Growing:
After finitely many steps a new term must be introduced.
This implies infinity.

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G-Unblockable:
If a trigger is loaded in the HC-branch of any CT, then its HC-output occurs in this branch.

$$x : \blacktriangle \rightarrow \Big(\exists y.\ y : \blacksquare \wedge x \in y\Big) \vee x : \blacktriangledown \quad (1)$$

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$$x : \text{bike} \rightarrow \exists z.\ z : \text{tree} \wedge x \ni z \quad (2)$$

Such a sequence is embeddable into every CT.

For the DB $c : \text{tree}$, we find the following cyclicity sequence:

$\langle (1), [x/c] \rangle$ $\langle (2), [x/f_y.c] \rangle$ $\langle (1), [x/f_z.f_y.c] \rangle$ $\langle (2), [x/f_y.f_z.f_y.c] \rangle$ ...

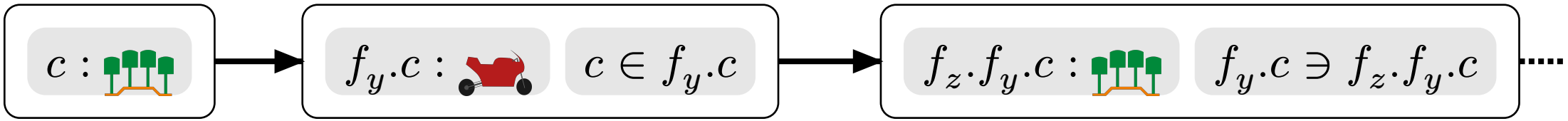
2. Cyclicity Sequences

Loadedness and growth are easily verified for a trigger.

$$x : \text{🌲} \rightarrow \left(\exists y.\ y : \text{🏍} \wedge x \in y \right) \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow \exists z.\ z : \text{🌲} \wedge x \ni z \quad (2)$$

Such a sequence is embeddable into every CT.



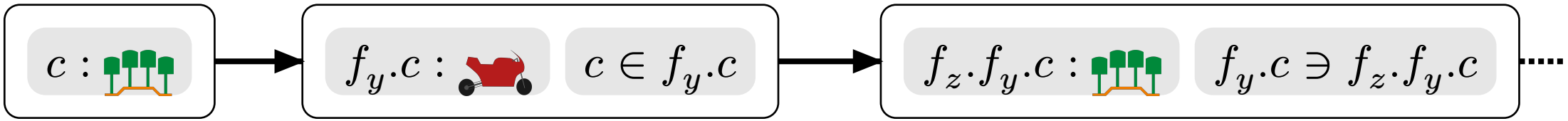
2. Cyclicity Sequences

Loadedness and growth are easily verified for a trigger. **How do we verify g-unblockability?**

$$x : \blacksquare \rightarrow \left(\exists y.\ y : \blacksquare \wedge x \in y\right) \vee x : \blacksquare \quad (1)$$

$$x : \blacksquare \rightarrow \exists z.\ z : \blacksquare \wedge x \ni z \quad (2)$$

Such a sequence is embeddable into every CT.



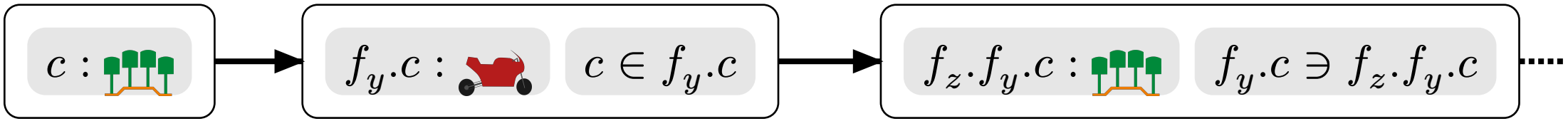
2. Cyclicity Sequences

Theorem.
Checking if a trigger is g-unblockable is undecidable!

$$x : \blacksquare \rightarrow \left(\exists y.\ y : \blacksquare \wedge x \in y\right) \vee x : \blacksquare \quad (1)$$

$$x : \blacksquare \rightarrow \exists z.\ z : \blacksquare \wedge x \ni z \quad (2)$$

Such a sequence is embeddable into every CT.



3. Unique Constant (UC) Unblockability

UC-Overapproximation
before a trigger λ

$$x : \text{🌲} \rightarrow \Big(\exists y.\ y : \text{🏍} \wedge x \in y\Big) \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow \exists z.\ z : \text{🌲} \wedge x \ni z \quad (2)$$

We show UC-Overapp. before $\langle (2), [x/f_y.c] \rangle$ *:*

3. Unique Constant (UC) Unblockability

UC-Overapproximation
before a trigger λ
1. Birth Facts of λ

$$x : \text{🌲} \rightarrow \Big(\exists y.\ y : \text{🏍} \wedge x \in y\Big) \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow \exists z.\ z : \text{🌲} \wedge x \ni z \quad (2)$$

We show UC-Overapp. before $\langle (2), [x/f_y.c] \rangle$ *:*

$f_y.c : \text{🏍}$ $\quad c \in f_y.c$

3. Unique Constant (UC) Unblockability

UC-Overapproximation
before a trigger λ
1. Birth Facts of λ
2. Critical Facts of λ

$$x : \text{🌲} \rightarrow \Big(\exists y.\ y : \text{🏍} \wedge x \in y\Big) \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow \exists z.\ z : \text{🌲} \wedge x \ni z \quad (2)$$

We show UC-Overapp. before $\langle (2), [x/f_y.c] \rangle$ *:*

$$f_y.c : \text{🏍} \qquad c \in f_y.c \qquad \star, c : \text{🏍} \qquad \star, c : \text{🌲} \qquad \star, c : \text{🔧}$$

$$\star \in \star \qquad \star \in c \qquad c \in \star \qquad c \in c \qquad \star \ni \star \qquad \star \ni c \qquad c \ni \star \qquad c \ni c$$

3. Unique Constant (UC) Unblockability

UC-Overapproximation
before a trigger λ
1. Birth Facts of λ
2. Critical Facts of λ
3. Chase with UC-Rules

$$x : \text{🌲} \rightarrow d_y : \text{🏍} \wedge x \in d_y \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow d_z : \text{🌲} \wedge x \ni d_z \quad (2)$$

We show UC-Overapp. before $\langle (2), [x/f_y.c] \rangle$ *:*

$f_y.c : \text{🏍}$ $\quad c \in f_y.c$ $\quad \star, c : \text{🏍}$ $\quad \star, c : \text{🌲}$ $\quad \star, c : \text{🔧}$

$\star \in \star$, $\star \in c$, $c \in \star$, $c \in c$ $\quad \star \ni \star$, $\star \ni c$, $c \ni \star$, $c \ni c$

$d_y : \text{🏍}$ $\quad \star, c \in d_y$ $\quad d_z : \text{🌲}$ $\quad \star, c \ni d_z$ $\quad d_z \in d_y$ $\quad d_y \ni d_z$

3. Unique Constant (UC) Unblockability

UC-Overapproximation
before a trigger λ
1. Birth Facts of λ
2. Critical Facts of λ
3. Chase with UC-Rules

$$x : \blacktriangle \rightarrow \left(\exists y.\ y : \blacksquare \wedge x \in y\right) \vee x : \blacktriangledown \quad (1)$$

$$x : \blacksquare \rightarrow \exists z.\ z : \blacktriangle \wedge x \ni z \quad (2)$$

$\langle (2), [x/f_y.c] \rangle$ is applicable to its overapp.

$f_y.c : \blacksquare$ $c \in f_y.c$ $\star, c : \blacksquare$ $\star, c : \blacktriangle$ $\star, c : \blacktriangledown$

$\star \in \star$, $\star \in c$, $c \in \star$, $c \in c$, $\star \ni \star$, $\star \ni c$, $c \ni \star$, $c \ni c$

$d_y : \blacksquare$ $\star, c \in d_y$ $d_z : \blacktriangle$ $\star, c \ni d_z$ $d_z \in d_y$ $d_y \ni d_z$

3. Unique Constant (UC) Unblockability

UC-Overapproximation
before a trigger λ
1. Birth Facts of λ
2. Critical Facts of λ
3. Chase with UC-Rules

$$x : \blacktriangle \rightarrow \left(\exists y.\ y : \blacksquare \wedge x \in y\right) \vee x : \blacktriangledown \quad (1)$$

$$x : \blacksquare \rightarrow \exists z.\ z : \blacktriangle \wedge x \ni z \quad (2)$$

$\langle (2), [x/f_y.c] \rangle$ is uc-unblockable!

$$f_y.c : \blacksquare \quad c \in f_y.c \quad \star, c : \blacksquare \quad \star, c : \blacktriangle \quad \star, c : \blacktriangledown$$

$$\star \in \star \quad \star \in c \quad c \in \star \quad c \in c \quad \star \ni \star \quad \star \ni c \quad c \ni \star \quad c \ni c$$

$$d_y : \blacksquare \quad \star, c \in d_y \quad d_z : \blacktriangle \quad \star, c \ni d_z \quad d_z \in d_y \quad d_y \ni d_z$$

3. Unique Constant (UC) Unblockability

UC-Overapproximation
before a trigger λ
1. Birth Facts of λ
2. Critical Facts of λ
3. Chase with UC-Rules

$$x : \text{🌲} \rightarrow \left(\exists y.\ y : \text{🏍} \wedge x \in y \right) \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow \exists z.\ z : \text{🌲} \wedge x \ni z \quad (2)$$

Would not work with $x \in y \rightarrow y \ni x$

$f_y.c : \text{🏍}$ $\quad c \in f_y.c$ $\quad \star, c : \text{🏍}$ $\quad \star, c : \text{🌲}$ $\quad \star, c : \text{🔧}$

$\star \in \star$, $\star \in c$, $c \in \star$, $c \in c$ $\quad \star \ni \star$, $\star \ni c$, $c \ni \star$, $c \ni c$

$d_y : \text{🏍}$ $\quad \star, c \in d_y$ $\quad d_z : \text{🌲}$ $\quad \star, c \ni d_z$ $\quad d_z \in d_y$ $\quad d_y \ni d_z$

3. Unique Constant (UC) Unblockability

UC-Overapproximation
before a trigger λ
1. Birth Facts of λ
2. Critical Facts of λ
3. Chase with UC-Rules

Lemma.
If a trigger is uc-unblockable, it is g-unblk.
Every possible fact that may occur in any CT where the trigger has not been applied can be embedded into the overapprox.

$f_y.c :$  $\quad c \in f_y.c \quad \star, c :$  $\quad \star, c :$  $\quad \star, c :$ 

$\star \in \star$, $\star \in c$, $c \in \star$, $c \in c$ $\quad \star \ni \star$, $\star \ni c$, $c \ni \star$, $c \ni c$

$d_y :$  $\quad \star, c \in d_y \quad d_z :$  $\quad \star, c \ni d_z \quad d_z \in d_y \quad d_y \ni d_z$

3.' Unblockability Propagates

We only want to check uc-unblockability for finitely many triggers.

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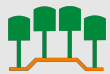
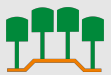
Reversible Constant Mappings [GC23] preserve uc-unblk.

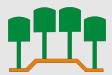
$$x : \text{  } \rightarrow \left(\exists y.\ y : \text{  } \wedge x \in y \right) \vee x : \text{  } \quad (1)$$

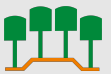
$$x : \text{  } \rightarrow \exists z.\ z : \text{  } \wedge x \ni z \quad (2)$$

3.' Unblockability Propagates

We only want to check uc-unblockability for finitely many triggers.

Reversible Constant Mappings [GC23] preserve uc-unblk.

$$x : \text{  } \rightarrow \Big(\exists y.\ y : \text{  } \wedge x \in y\Big) \vee x : \text{  } \quad (1)$$

$$x : \text{  } \rightarrow \exists z.\ z : \text{  } \wedge x \ni z \quad (2)$$

$\lambda' = \langle (2), [x/f_y.f_z.f_y.c] \rangle$ results from $\lambda = \langle (2), [x/f_y.c] \rangle$ with the reversible constant mapping $g : c \mapsto f_z.f_y.c$.

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We only want to check uc-unblockability for finitely many triggers.

Reversible Constant Mappings [GC23] preserve uc-unblk.

$$x : \text{🌲} \rightarrow \left(\exists y.\ y : \text{🏍} \wedge x \in y\right) \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow \exists z.\ z : \text{🌲} \wedge x \ni z \quad (2)$$

$\lambda' = \langle (2), [x/f_y.f_z.f_y.c] \rangle$ results from $\lambda = \langle (2), [x/f_y.c] \rangle$ with the reversible constant mapping $g : c \mapsto f_z.f_y.c$. The UC-overapproximation of λ' can be embedded into the uc-overapproximation of λ by mapping all "superterms" of $f_z.f_y.c$ by the inverse of g and all other terms to $\star$.

3.' Unblockability Propagates

We only want to check uc-unblockability for finitely many triggers.

Reversible Constant Mappings [GC23] preserve uc-unblk.

$$x : \text{tree} \rightarrow \left(\exists y.\ y : \text{bike} \wedge x \in y \right) \vee x : \text{wrench} \quad (1)$$

$$x : \text{bike} \rightarrow \exists z.\ z : \text{tree} \wedge x \ni z \quad (2)$$

$$f_y.c : \text{bike} \quad c \in f_y.c \quad \star, c : \text{bike} \quad \star, c : \text{tree} \quad \star, c : \text{wrench}$$

$$\star \in \star \quad \star \in c \quad c \in \star \quad c \in c \quad \star \ni \star \quad \star \ni c \quad c \ni \star \quad c \ni c$$

$$d_y : \text{bike} \quad \star, c \in d_y \quad d_z : \text{tree} \quad \star, c \ni d_z \quad d_z \in d_y \quad d_y \ni d_z$$

3.' Unblockability Propagates

We only want to check uc-unblockability for finitely many triggers.

Reversible Constant Mappings [GC23] preserve uc-unblk.

$$x : \text{🌲} \rightarrow \left(\exists y.\ y : \text{🏍} \wedge x \in y\right) \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow \exists z.\ z : \text{🌲} \wedge x \ni z \quad (2)$$

$\lambda' = \langle (2), [x/f_y.f_z.f_y.c] \rangle$ results from $\lambda = \langle (2), [x/f_y.c] \rangle$ with the reversible constant mapping $g : c \mapsto f_z.f_y.c$. The UC-overapproximation of λ' can be embedded into the uc-overapproximation of λ by mapping all "superterms" of $f_z.f_y.c$ by the inverse of g and all other terms to $\star$.
In fact, we can show that this is always possible!

3.' Unblockability Propagates

We only want to check uc-unblockability for finitely many triggers.

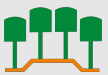
Reversible Constant Mappings [GC23] preserve uc-unblk.

Lemma.
If a trigger $\langle \rho, \sigma \rangle$ is uc-unblk., then $\langle \rho, g \circ \sigma \rangle$ is uc-unblk. for every rev. constant mapping g.

$\lambda' = \langle (2), [x/f_y.f_z.f_y.c] \rangle$ results from $\lambda = \langle (2), [x/f_y.c] \rangle$ with the reversible constant mapping $g : c \mapsto f_z.f_y.c$. The UC-overapproximation of λ' can be embedded into the uc-overapproximation of λ by mapping all "superterms" of $f_z.f_y.c$ by the inverse of g and all other terms to $\star$.
In fact, we can show that this is always possible!

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

$$x : \text{  } \rightarrow \left(\exists y.\ y : \text{  } \wedge x \in y \right) \vee x : \text{  } \quad (1)$$

$$x : \text{  } \rightarrow \exists z.\ z : \text{  } \wedge x \ni z \quad (2)$$

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

1. Start: Rule DB Trg

$$x : \text{🌲} \rightarrow \left(\exists y.\ y : \text{🏍} \wedge x \in y\right) \vee x : \text{🔧} \quad (1)$$

$$x : \text{🏍} \rightarrow \exists z.\ z : \text{🌲} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{🌲}$

Prefix: $\langle (1), [x/c_x] \rangle$

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

1. Start: Rule DB Trg
2. DB Trg is g-unblk.

$$x : \text{tree} \rightarrow \left(\exists y.\ y : \text{bike} \wedge x \in y\right) \vee x : \text{wrench} \quad (1)$$

$$x : \text{bike} \rightarrow \exists z.\ z : \text{tree} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{tree}$

Prefix: $\langle (1), [x/c_x] \rangle$

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

1. Start: Rule DB Trg
2. DB Trg is g-unblk.
3. Ldd, UC-unblk trgs

$$x : \text{trees} \rightarrow \left(\exists y.\ y : \text{motorcycle} \wedge x \in y\right) \vee x : \text{wrench} \quad (1)$$

$$x : \text{motorcycle} \rightarrow \exists z.\ z : \text{trees} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{trees}$

Prefix: $\langle (1), [x/c_x] \rangle$ $\langle (2), [x/f_y.c_x] \rangle$

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

1. Start: Rule DB Trg
2. DB Trg is g-unblk.
3. Ldd, UC-unblk trgs
4. Repeat with cycl. t.

$$x : \text{trees} \rightarrow \left(\exists y.\ y : \text{motorcycle} \wedge x \in y\right) \vee x : \text{wrench} \quad (1)$$

$$x : \text{motorcycle} \rightarrow \exists z.\ z : \text{trees} \wedge x \ni z \quad (2)$$

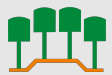
Consider the rule DB for (1): $c_x : \text{trees}$

Prefix: $\langle (1), [x/c_x] \rangle$ $\langle (2), [x/f_y.c_x] \rangle$ $\langle (1), [x/f_z.f_y.c_x] \rangle$

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

1. Start: Rule DB Trg
2. DB Trg is g-unblk.
3. Ldd, UC-unblk trgs
4. Repeat with cycl. t.
5. Induced CM is rev.

$$x : \text{  } \rightarrow \left(\exists y.\ y : \text{  } \wedge\ x \in y\right) \vee x : \text{  } \quad (1)$$

$$x : \text{  } \rightarrow \exists z.\ z : \text{  } \wedge\ x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{  }$

Prefix: $\langle (1), [x/c_x] \rangle$ $\langle (2), [x/f_y.c_x] \rangle$ $\langle (1), [x/f_z.f_y.c_x] \rangle$

with constant mapping $g : c_x \mapsto f_z.f_y.c_x$ (which is reversible).

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

1. Start: Rule DB Trg
2. DB Trg is g-unblk.
3. Ldd, UC-unblk trgs
4. Repeat with cycl. t.
5. Induced CM is rev.

$$x : \text{trees} \rightarrow \left(\exists y.\ y : \text{bike} \wedge x \in y \right) \vee x : \text{wrench} \quad (1)$$

$$x : \text{bike} \rightarrow \exists z.\ z : \text{trees} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{trees}$

Rule Database

$c_x : \text{trees}$

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

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$$x : \text{tree} \rightarrow \left(\exists y.\ y : \text{bike} \wedge x \in y\right) \vee x : \text{wrench} \quad (1)$$

$$x : \text{bike} \rightarrow \exists z.\ z : \text{tree} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{tree}$

Rule Database

$c_x : \text{tree}$

$f_y.c : \text{bike}$

$c \in f_y.c$

4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

1. Start: Rule DB Trg
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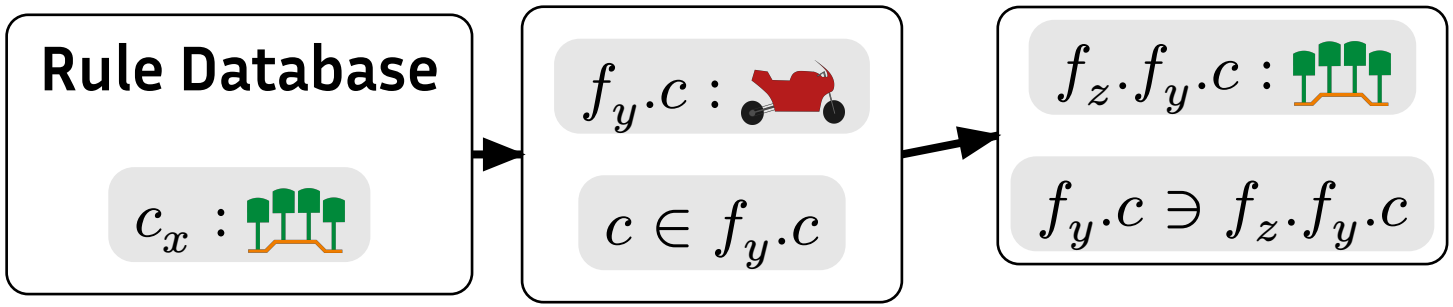
$$x : \text{bike} \rightarrow \exists z.\ z : \text{tree} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{tree}$



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$$x : \text{bike} \rightarrow \exists z.\ z : \text{tree} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{tree}$

Rule Database

$c_x : \text{tree}$

$f_y.c : \text{bike}$

$c \in f_y.c$

$f_z.f_y.c : \text{tree}$

$f_y.c \ni f_z.f_y.c$

$f_y.f_z.f_y.c : \text{bike}$

$f_z.f_y.c \in f_y.f_z.f_y.c$

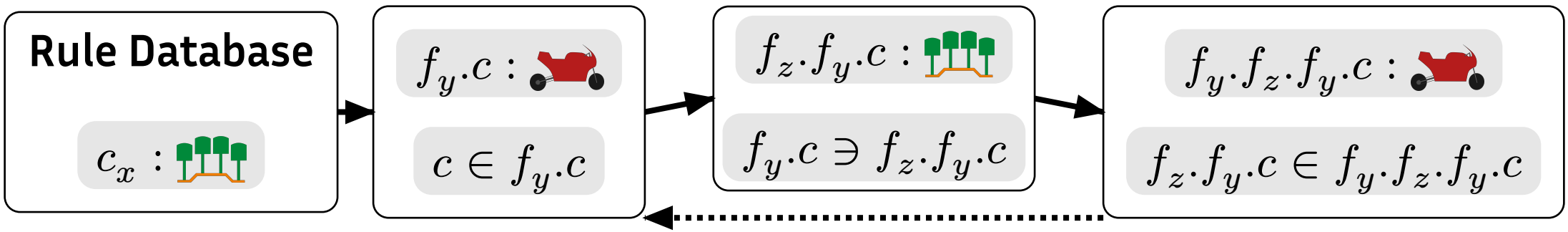
4. Cyclicity Prefixes

Finite Trigger Sequence extendable into an (infinite) Cyclicity Sequence

1. Start: Rule DB Trg
2. DB Trg is g-unblk.
3. Ldd, UC-unblk trgs
4. Repeat with cycl. t.
5. Induced CM is rev.

Theorem.
Every cyclicity prefix can be extended into a cyclicity sequence implying never-term.

Consider the rule DB for (1): c_x : 



5. Restricted Prefix Cyclicity (RPC)

RPC is a computational procedure for (possibly) finding a Cyclicity Prefix:

$$x : \text{  } \rightarrow \Big(\exists y.\ y : \text{  } \wedge x \in y \Big) \vee x : \text{  } \quad (1)$$

$$x : \text{  } \rightarrow \exists z.\ z : \text{  } \wedge x \ni z \quad (2)$$

5. Restricted Prefix Cyclicity (RPC)

RPC is a computational procedure for (possibly) finding a Cyclicity Prefix:

Each rule ρ and HC

$$x : \text{tree} \rightarrow \Big(\exists y.\ y : \text{bike} \wedge x \in y \Big) \vee x : \text{wrench} \quad (1)$$

$$x : \text{bike} \rightarrow \exists z.\ z : \text{tree} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{tree}$

Rule Database

$c_x : \text{tree}$

5. Restricted Prefix Cyclicity (RPC)

RPC is a computational procedure for (possibly) finding a Cyclicity Prefix:

Each rule ρ and HC

1. Apply Rule-DB Trg

$$x : \text{tree} \rightarrow \left(\exists y.\ y : \text{bike} \wedge x \in y \right) \vee x : \text{wrench} \quad (1)$$

$$x : \text{bike} \rightarrow \exists z.\ z : \text{tree} \wedge x \ni z \quad (2)$$

Consider the rule DB for (1): $c_x : \text{tree}$

Rule Database

$c_x : \text{tree}$

$f_y.c : \text{bike}$

$c \in f_y.c$

5. Restricted Prefix Cyclicity (RPC)

RPC is a computational procedure for (possibly) finding a Cyclicity Prefix:

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1. Apply Rule-DB Trg
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$$x : \blacktriangle \rightarrow \left(\exists y.\ y : \blacktriangleright \wedge x \in y\right) \vee x : \blacktriangleleft \quad (1)$$

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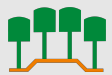
$$x : \text{  } \rightarrow \Big(\exists y.\ y : \text{  } \wedge\ x \in y\Big) \vee x : \text{  } \quad (1)$$

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Theorem.
If a rule set is RPC, it never-terminates.

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***2** follows from uc-unblk of last trigger.*

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5 *follows from injectivity of substitutions in trgs with* ρ *and the observation that the first functional term is a subterm of every other.*

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(If you are fine with not having colored emojis in PDF exports yet. 😉*)*

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If $\langle \rho, \sigma \rangle$ is uc-unblk and g rev for its skeleton, then $\langle \rho, g \circ \sigma \rangle$ is uc-unblk.

Why condition 2 for reversibility?

Rev. Cond. 2: We have $g(s) \neq g(t)$ for every $s, t \in T$ with $s \neq t$.

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$$x \xrightarrow{\rho} y \rightarrow \exists u.\ x \xrightarrow{\gamma} u \wedge y \xrightarrow{\beta} u \tag{1}$$

$$x \xrightarrow{\gamma} y \rightarrow \exists v.\ y \xrightarrow{\alpha} v \tag{2}$$

$$x \xrightarrow{\gamma} y \wedge x \xrightarrow{\beta} y \rightarrow y \xrightarrow{\alpha} x \tag{3}$$

$$x \xrightarrow{\alpha} y \rightarrow y \xrightarrow{\rho} y \tag{4}$$

Why condition 2 for reversibility?

Rev. Cond. 2: We have $g(s) \neq g(t)$ for every $s, t \in T$ with $s \neq t$.

$$x \xrightarrow{R} y \rightarrow \exists u.\ x \xrightarrow{G} u \wedge y \xrightarrow{B} u \tag{1}$$

$$x \xrightarrow{G} y \rightarrow \exists v.\ y \xrightarrow{O} v \tag{2}$$

$$x \xrightarrow{G} y \wedge x \xrightarrow{B} y \rightarrow y \xrightarrow{O} x \tag{3}$$

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The trigger $\langle (2), [x/c, y/f_1(c,d)] \rangle$ is uc-unblockable.
The trigger $\langle (2), [x/f_2(f_1(c,d)), y/f_1(f_2(\ldots), f_2(\ldots))] \rangle$ is not.

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$$x \xrightarrow{G} y \rightarrow \exists v.\ y \xrightarrow{O} v \quad (2)$$

$$x \xrightarrow{G} y \wedge x \xrightarrow{B} y \rightarrow y \xrightarrow{O} x \quad (3)$$

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The trigger $\langle (2), [x/c, y/f_1(c,d)] \rangle$ is uc-unblockable.
The trigger $\langle (2), [x/f_2(f_1(c,d)), y/f_1(f_2(\ldots), f_2(\ldots))] \rangle$ is not.
Note that the second results from the first with the constant mapping that maps both c and d to $f_2(f_1(c,d))$ and thus violates cond. 2. ⚡

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$$x : A \to \exists u.\ x \to u \qquad x \to y \to y : T \vee \exists z.\ R(x,y,z) \qquad (\dagger)$$

$$x : B \to \exists v.\ x \to v \qquad x \to y \to x : T$$

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$$x : A \to \exists u.\ x \mathrel{\color{red}{\longrightarrow}} u \qquad x \mathrel{\color{red}{\longrightarrow}} y \to y : T \lor \exists z.\ R(x,y,z) \qquad (\dagger)$$

$$x : B \to \exists v.\ x \mathrel{\color{green}{\longrightarrow}} v \qquad x \mathrel{\color{green}{\longrightarrow}} y \to x : T$$

$$x : C \to \exists w.\ x \mathrel{\color{blue}{\longrightarrow}} w$$

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The trigger $\langle(\dagger), [x/c, y/f_u(d)]\rangle$ is uc-unblockable. Mapping c to $f_w(f_v(f_u(d)))$ and d to itself via g fulfills conditions 1 and 2 of reversibility but not 3 since $g(f_u(d)) = f_u(d)$ is a subterm of $g(c)$. Indeed, $\langle(\dagger), [x/f_w(f_v(f_u(d))), y/f_u(d)]\rangle$ is not uc-unblockable! ⚡

Problematic RMFC Example

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u \quad (1)$$

$$x : \square \wedge x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \wedge z \rightarrow v \quad (2)$$

$$\not\square\ y : \bigcirc \wedge y \rightarrow w \rightarrow w : \bigcirc \quad (3)$$

Problematic RMFC Example

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \xrightarrow{r} u \wedge y \xrightarrow{r} u \quad (1)$$

$$x : \square \wedge x \xrightarrow{r} z \rightarrow \exists v.\ x \xrightarrow{g} v \wedge z \xrightarrow{b} v \quad (2)$$

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$$y \xrightarrow{r} z \wedge z \xrightarrow{b} w \wedge x \xrightarrow{g} w \rightarrow y \xrightarrow{g} y \wedge z \xrightarrow{b} y \wedge y : \square \quad (4)$$

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$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \land y \rightarrow u \quad (1)$$

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$$\not\square\ y : \bigcirc \land y \rightarrow w \rightarrow w : \bigcirc \quad (3)$$

$$y \rightarrow z \land z \rightarrow w \land x \rightarrow w \rightarrow y \rightarrow y \land z \rightarrow y \land y : \square \quad (4)$$

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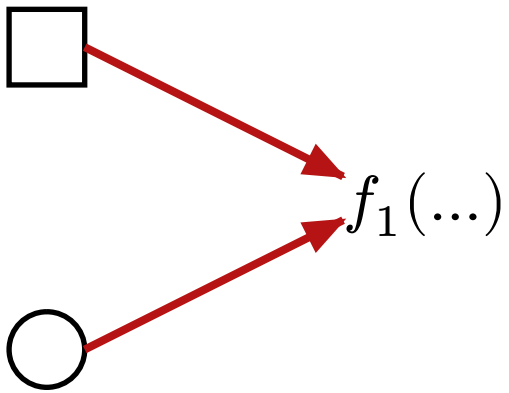

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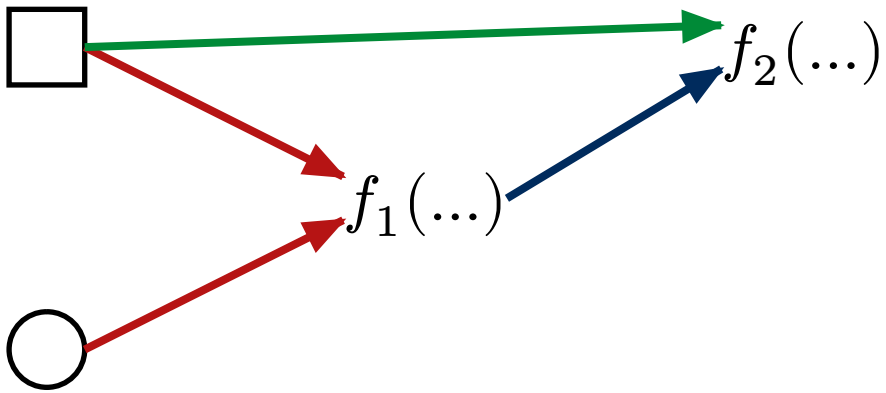

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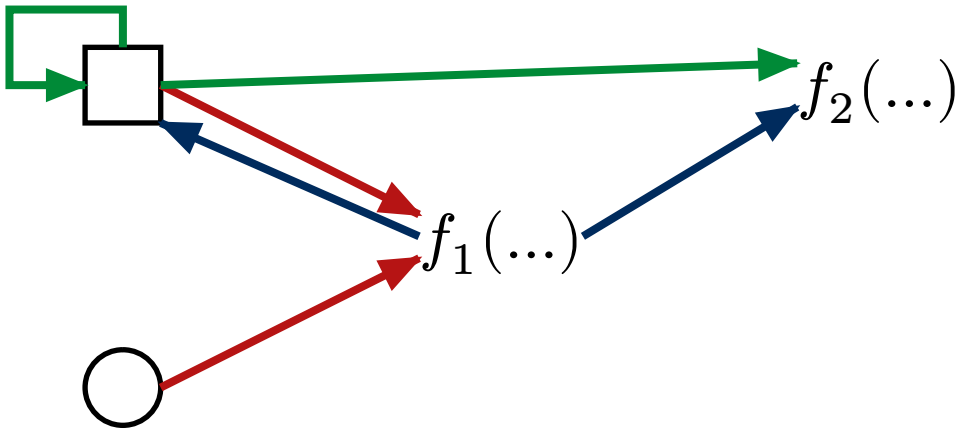

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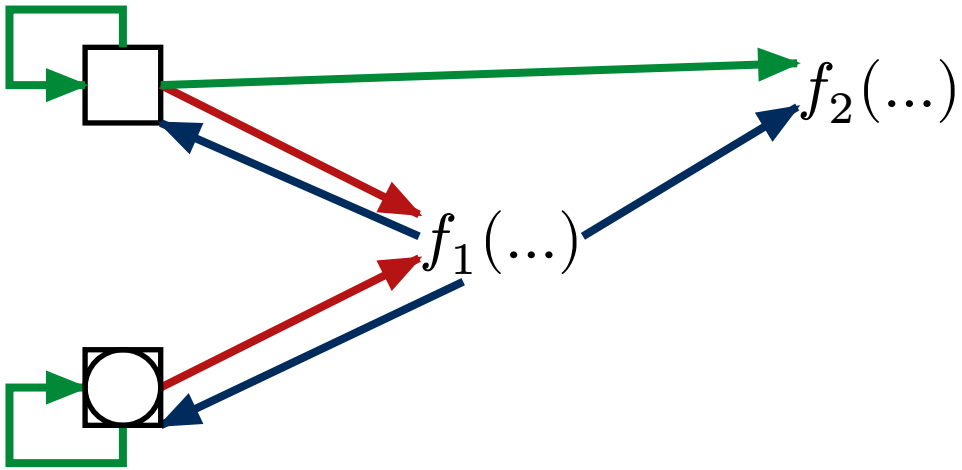


$\langle (1), [x/c, y/d] \rangle$
$\langle (2), [x/c, y/f_1(c,d)] \rangle$
$\langle (4), [x/c, y/d, z/f_1(\ldots), w/f_2(\ldots)] \rangle$
$\langle (4), [x/d, y/d, z/f_1(\ldots), w/f_2(\ldots)] \rangle$

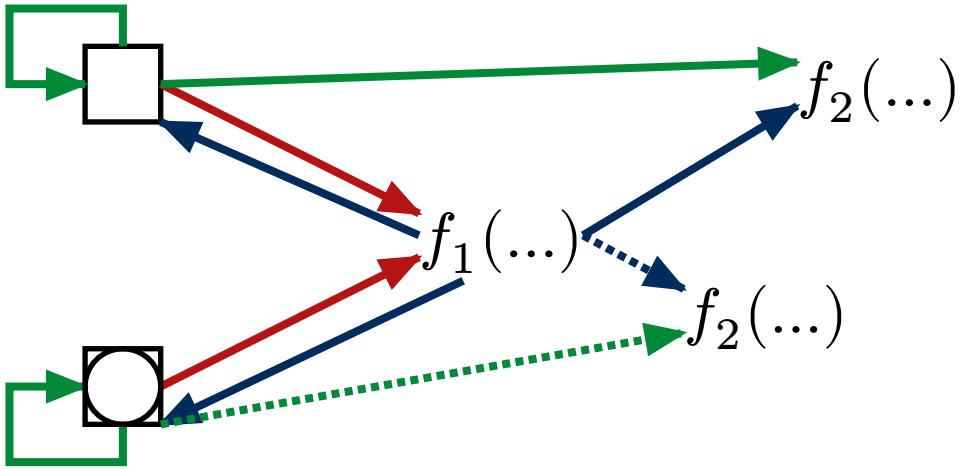
Problematic RMFC Example

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u \qquad (1)$$

$$x : \square \wedge x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \wedge z \rightarrow v \qquad (2)$$

$$\not\square\ y : \bigcirc \wedge y \rightarrow w \rightarrow w : \bigcirc \qquad (3)$$

$$y \rightarrow z \wedge z \rightarrow w \wedge x \rightarrow w \rightarrow y \rightarrow y \wedge z \rightarrow y \wedge y : \square \qquad (4)$$



$\langle (1), [x/c, y/d] \rangle$
$\langle (2), [x/c, y/f_1(c,d)] \rangle$
$\langle (4), [x/c, y/d, z/f_1(\ldots), w/f_2(\ldots)] \rangle$
$\langle (4), [x/d, y/d, z/f_1(\ldots), w/f_2(\ldots)] \rangle$
$\langle (2), [x/d, y/f_1(c,d)] \rangle$**?**

Problematic RMFC Example

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u \qquad (1)$$

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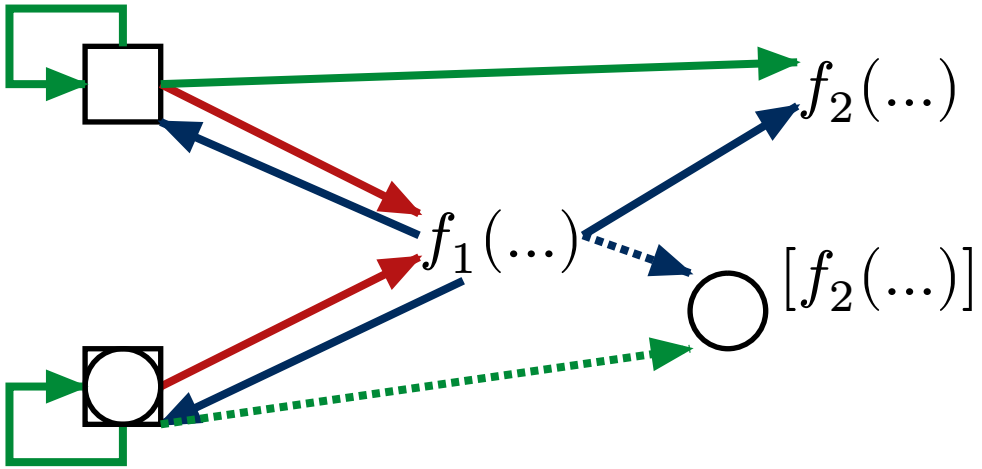

$\langle (1), [x/c, y/d] \rangle$
$\langle (2), [x/c, y/f_1(c,d)] \rangle$
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$\langle (4), [x/d, y/d, z/f_1(\ldots), w/f_2(\ldots)] \rangle$
$\langle (2), [x/d, y/f_1(c,d)] \rangle$**?**
$\langle (3), [y/d, w/f_2(\ldots)] \rangle$**??**

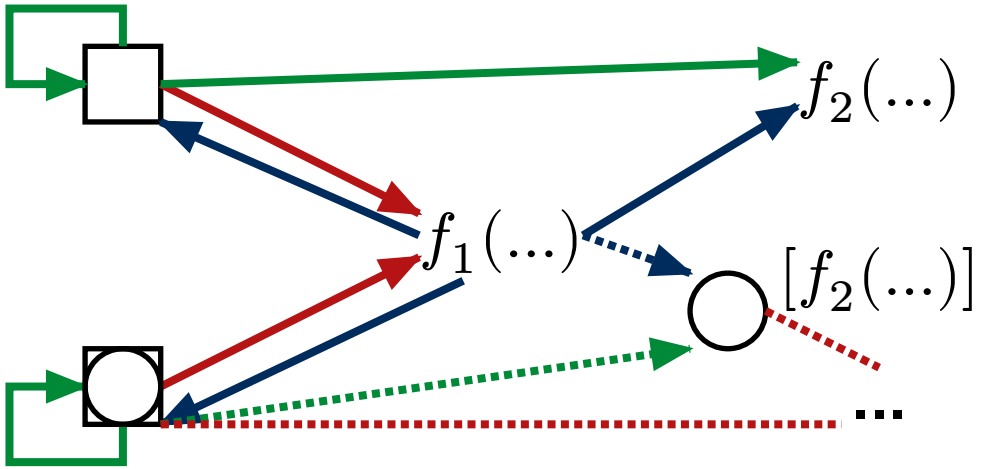
Problematic RMFC Example

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u \qquad (1)$$

$$x : \square \wedge x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \wedge z \rightarrow v \qquad (2)$$

$$\not\square\ y : \bigcirc \wedge y \rightarrow w \rightarrow w : \bigcirc \qquad (3)$$

$$y \rightarrow z \wedge z \rightarrow w \wedge x \rightarrow w \rightarrow y \rightarrow y \wedge z \rightarrow y \wedge y : \square \qquad (4)$$



$\langle (1), [x/c, y/d] \rangle$
$\langle (2), [x/c, y/f_1(c,d)] \rangle$
$\langle (4), [x/c, y/d, z/f_1(\ldots), w/f_2(\ldots)] \rangle$
$\langle (4), [x/d, y/d, z/f_1(\ldots), w/f_2(\ldots)] \rangle$
$\langle (2), [x/d, y/f_1(c,d)] \rangle$**?**
$\langle (3), [y/d, w/f_2(\ldots)] \rangle$**??**
$\langle (1), [x/d, y/f_2(\ldots)] \rangle$**???**

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \land y \rightarrow u$$

$$x : \square \land x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \land z \rightarrow v$$

$$y : \bigcirc \land y \rightarrow w \rightarrow w : \bigcirc$$

$$y \rightarrow z \land z \rightarrow w \land x \rightarrow w \rightarrow y \rightarrow y \land z \rightarrow y \land y : \square$$

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \xrightarrow{R} u \land y \xrightarrow{R} u$$

$$x : \square \land x \xrightarrow{R} z \rightarrow \exists v.\ x \xrightarrow{G} v \land z \xrightarrow{B} v$$

$$y : \bigcirc \land y \xrightarrow{G} w \rightarrow w : \bigcirc$$

$$y \xrightarrow{R} z \land z \xrightarrow{B} w \land x \xrightarrow{G} w \rightarrow y \xrightarrow{G} y \land z \xrightarrow{B} y \land y : \square$$

RPC ✗;

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \xrightarrow{r} u \land y \xrightarrow{r} u$$

$$x : \square \land x \xrightarrow{r} z \rightarrow \exists v.\ x \xrightarrow{g} v \land z \xrightarrow{b} v$$

$$y : \bigcirc \land y \xrightarrow{g} w \rightarrow w : \bigcirc$$

$$y \xrightarrow{r} z \land z \xrightarrow{b} w \land x \xrightarrow{g} w \rightarrow y \xrightarrow{g} y \land z \xrightarrow{b} y \land y : \square$$

RPC ☒; RMFC ☑;

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \land y \rightarrow u$$

$$x : \square \land x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \land z \rightarrow v$$

$$y : \bigcirc \land y \rightarrow w \rightarrow w : \bigcirc$$

$$y \rightarrow z \land z \rightarrow w \land x \rightarrow w \rightarrow y \rightarrow y \land z \rightarrow y \land y : \square$$

RPC ☒ ; RMFC ☑ ; Cycl. Sequence with $c : \square$, $d : \bigcirc$ ☒ ;

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \land y \rightarrow u$$

$$x : \square \land x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \land z \rightarrow v$$

$$y : \bigcirc \land y \rightarrow w \rightarrow w : \bigcirc$$

$$y \rightarrow z \land z \rightarrow w \land x \rightarrow w \rightarrow y \rightarrow y \land z \rightarrow y \land y : \square$$

RPC ✗ ; RMFC ☑ ; Cycl. Sequence with $c : \square$, $d : \bigcirc$ ✗ ; never-term. 🤔

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \land y \rightarrow u$$

$$x : \square \land x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \land z \rightarrow v$$

$$y : \bigcirc \land y \rightarrow w \rightarrow w : \bigcirc$$

$$y \rightarrow z \land z \rightarrow w \land x \rightarrow w \rightarrow y \rightarrow y \land z \rightarrow y \land y : \square$$

RPC ✗; RMFC ☑; Cycl. Sequence with $c : \square$, $d : \bigcirc$ ✗; never-term. 🤔

Cycl. Sequence with $d : \square$, $d : \bigcirc$ ☑;

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \to u \land y \to u$$

$$x : \square \land x \to z \rightarrow \exists v.\ x \to v \land z \to v$$

$$y : \bigcirc \land y \to w \rightarrow w : \bigcirc$$

$$y \to z \land z \to w \land x \to w \rightarrow y \to y \land z \to y \land y : \square$$

RPC ☒; RMFC ☑; Cycl. Sequence with $c : \square$, $d : \bigcirc$ ☒; never-term. 🤔

Cycl. Sequence with $d : \square$, $d : \bigcirc$ ☑; never-term. ☑ 🐕

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \land y \rightarrow u$$

$$x : \square \land x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \land z \rightarrow v$$

$$y : \bigcirc \land y \rightarrow w \rightarrow w : \bigcirc$$

$$y \rightarrow z \land z \rightarrow w \land x \rightarrow w \rightarrow y \rightarrow y \land z \rightarrow y \land y : \square$$

RPC ✗ ; RMFC ☑ ; Cycl. Sequence with $c : \square$, $d : \bigcirc$ ✗ ; never-term. 🤔

Cycl. Sequence with $d : \square$, $d : \bigcirc$ ☑ ; never-term. ☑ 🐕

Was RMFC right all along?

So does RPC fix our problem? (cont.)

$$x : \square \land y : \bigcirc \rightarrow \exists u.\ x \xrightarrow{R} u \land y \xrightarrow{R} u$$

$$x : \square \land x \xrightarrow{R} z \rightarrow \exists v.\ x \xrightarrow{G} v \land z \xrightarrow{B} v$$

$$y : \bigcirc \land y \xrightarrow{G} w \rightarrow w : \bigcirc$$

$$y \xrightarrow{R} z \land z \xrightarrow{B} w \land x \xrightarrow{G} w \rightarrow y \xrightarrow{G} y \land z \xrightarrow{B} y \land y : \square$$

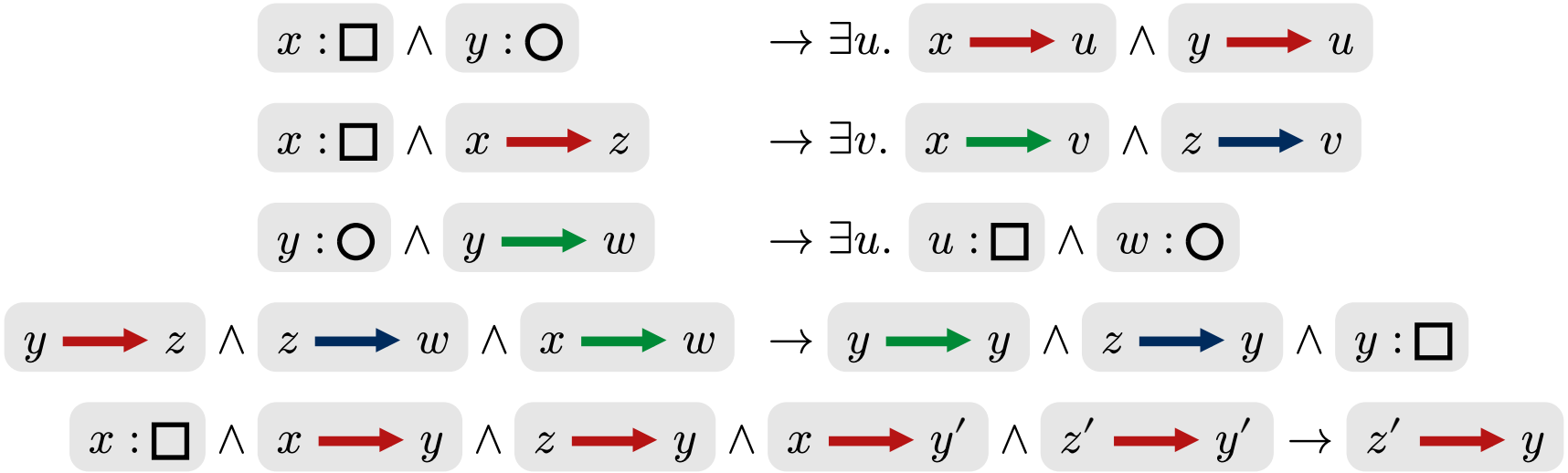
RPC ✗ ; RMFC ☑ ; Cycl. Sequence with $c : \square$, $d : \bigcirc$ ✗ ; never-term. 🤔

Cycl. Sequence with $d : \square$, $d : \bigcirc$ ☑ ; never-term. ☑ 🐕

Was RMFC right all along?

Frankly, we cannot say for sure; what we can say is that its correctness has not been proven.

Potential Counterexample for Never Termination



Potential Counterexample for Never Termination

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u$$

$$x : \square \wedge x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \wedge z \rightarrow v$$

$$y : \bigcirc \wedge y \rightarrow w \rightarrow \exists u.\ u : \square \wedge w : \bigcirc$$

$$y \rightarrow z \wedge z \rightarrow w \wedge x \rightarrow w \rightarrow y \rightarrow y \wedge z \rightarrow y \wedge y : \square$$

$$x : \square \wedge x \rightarrow y \wedge z \rightarrow y \wedge x \rightarrow y' \wedge z' \rightarrow y' \rightarrow z' \rightarrow y$$

RPC ✗;

Potential Counterexample for Never Termination

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u$$

$$x : \square \wedge x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \wedge z \rightarrow v$$

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$$x : \square \wedge x \rightarrow y \wedge z \rightarrow y \wedge x \rightarrow y' \wedge z' \rightarrow y' \rightarrow z' \rightarrow y$$

RPC ☒; RMFC ☑;

Potential Counterexample for Never Termination

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u$$

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$$y \rightarrow z \wedge z \rightarrow w \wedge x \rightarrow w \rightarrow y \rightarrow y \wedge z \rightarrow y \wedge y : \square$$

$$x : \square \wedge x \rightarrow y \wedge z \rightarrow y \wedge x \rightarrow y' \wedge z' \rightarrow y' \rightarrow z' \rightarrow y$$

RPC ☒; RMFC ☑; Cycl. Sequence with $d : \square$, $d : \bigcirc$ ☒;

Potential Counterexample for Never Termination

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u$$

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$$y \rightarrow z \wedge z \rightarrow w \wedge x \rightarrow w \rightarrow y \rightarrow y \wedge z \rightarrow y \wedge y : \square$$

$$x : \square \wedge x \rightarrow y \wedge z \rightarrow y \wedge x \rightarrow y' \wedge z' \rightarrow y' \rightarrow z' \rightarrow y$$

RPC ✗ ; RMFC ☑ ; Cycl. Sequence with $d : \square$, $d : \bigcirc$ ✗ ; never-term. 🤔

Potential Counterexample for Never Termination

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$$y \rightarrow z \wedge z \rightarrow w \wedge x \rightarrow w \rightarrow y \rightarrow y \wedge z \rightarrow y \wedge y : \square$$

$$x : \square \wedge x \rightarrow y \wedge z \rightarrow y \wedge x \rightarrow y' \wedge z' \rightarrow y' \rightarrow z' \rightarrow y$$

RPC ✗; RMFC ☑; Cycl. Sequence with $d : \square$, $d : \bigcirc$ ✗; never-term. 🤔

We think that every DB admits a terminating chase tree.

Potential Counterexample for Never Termination

$$x : \square \wedge y : \bigcirc \rightarrow \exists u.\ x \rightarrow u \wedge y \rightarrow u$$

$$x : \square \wedge x \rightarrow z \rightarrow \exists v.\ x \rightarrow v \wedge z \rightarrow v$$

$$y : \bigcirc \wedge y \rightarrow w \rightarrow \exists u.\ u : \square \wedge w : \bigcirc$$

$$y \rightarrow z \wedge z \rightarrow w \wedge x \rightarrow w \rightarrow y \rightarrow y \wedge z \rightarrow y \wedge y : \square$$

$$x : \square \wedge x \rightarrow y \wedge z \rightarrow y \wedge x \rightarrow y' \wedge z' \rightarrow y' \rightarrow z' \rightarrow y$$

RPC ✗; RMFC ☑; Cycl. Sequence with $d : \square$, $d : \bigcirc$ ✗; never-term. 🤔

We think that every DB admits a terminating chase tree. (It could still be that RMFC guarantees "sometimes non-termination" for exotic DBs.)