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Complexity Theory

Exercise 10: Randomised Computation

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Exercise 10.1. Show that **MajSat** is in PP.

$$\textbf{MajSat} = \{\varphi \mid \begin{array}{l}\varphi \text{ is some propositional logic formula that} \\ \text{is satisfied by more than half of its assignments}\}\end{array}$$

Exercise 10.2. Show $\mathrm{BPP} = \mathrm{coBPP}$.

$*$ **Exercise 10.3.** Show $\mathrm{BPP}^{\mathrm{BPP}} = \mathrm{BPP}$.

Exercise 10.4. Find the error in the following proof that shows $\mathrm{PP} = \mathrm{BPP}$: *Let $\boldsymbol{L} \in \mathrm{PP}$. Then there exists a poly-time bounded PTM accepting $\boldsymbol{L}$ with error probability smaller than $\frac{1}{2}$. Using error amplification, we can make this error arbitrarily small, and in particular smaller than $\frac{1}{3}$. Hence, $\boldsymbol{L} \in \mathrm{BPP}$.*

Exercise 10.5. Let $\mathcal{M}$ be a polynomial-time probabilistic Turing machine. We say that $\mathcal{M}$ has *error probability smaller than* $\frac{1}{3}$ if and only if

$$Pr[\mathcal{M} \text{ accepts } w] < \frac{1}{3} \quad \text{or} \quad Pr[\mathcal{M} \text{ accepts } w] \geq \frac{2}{3}$$

for all inputs w. Show that deciding whether a polynomial-time probabilistic TM has error probability smaller than $\frac{1}{3}$ is undecidable.