Technische Universität Dresden

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Introduction to Formal Concept Analysis

Exercise Sheet 5, Winter Semester 2017/18

Exercise 1 (multi-valued context)

a) Write down the derived contexts for the given multi-valued context about winter tyres using the

- nominal scale
- ordinal scale
- interordinal scale
- biordinal scale

(++ very good, + good, o satisfactory, − unsatisfactory)

b) Draw the line diagrams of the corresponding concept lattices.

c) Which tyre would you buy for your car? Why?

	Dry	Wet	Snow
Conti	+	+	++
Firestone	+	o	+
Fulda	+	+	+
Gislaved	+	o	++
Goodyear 2	+	+	+
Goodyear 3	+	o	++
Michelin	o	o	++
Pirelli	+	+	+
Semperit	o	+	+
Uniroyal	+	o	+
Vredestein	+	−	+

Solution:

(a) Conceptual Scaling

- Nominal Scales: $N_n := (n, n, =)$ Used for scaling attributes whose values exclude each other.

	−	o	+	++
−	x			
o		x		
+			x	
++				x

	Dry		Wet			Snow	
	o	+	−	o	+	+	++
Conti		x			x		x
Firestone		x		x		x	
Fulda		x			x	x	
Gislaved		x		x			x
Goodyear 2		x			x	x	
Goodyear 3		x		x			x
Michelin	x			x			x
Pirelli		x			x	x	
Semperit	x				x	x	
Uniroyal		x		x		x	
Vredestein		x	x			x	

- Ordinal Scales: $O_n := (n, n, \leq)$ Used with attributes with ordered values, where each value implies the smaller value.

	-	o	+	++
-	x			
o	x	x		
+	x	x	x	
++	x	x	x	x

	Dry		Wet			Snow	
	o	+	−	o	+	+	++
Conti	x	x	x	x	x	x	x
Firestone	x	x	x	x		x	
Fulda	x	x	x	x	x	x	
Gislaved	x	x	x	x		x	x
Goodyear 2	x	x	x	x	x	x	
Goodyear 3	x	x	x	x		x	x
Michelin	x		x	x		x	x
Pirelli	x	x	x	x	x	x	
Semperit	x		x	x	x	x	
Uniroyal	x	x	x	x		x	
Vredestein	x	x	x			x	

- Inter-ordinal Scales: $I_n := (n, n, \leq) \mid (n, n, \geq)$ Used in questionnaires for example. The concept intents are exactly the intervals of scale.

	≤ -	≤ o	≤ +	≤ ++	≥ -	≥ o	≥ +	≥ ++
-	x	x	x	x	x			
o		x	x	x	x	x		
+			x	x	x	x	x	
++				x	x	x	x	x

	Dry				Wet					Snow			
	≤ o	≤ +	≥ o	≥ +	≤ -	≤ o	≤ +	≥ -	≥ o	≥ +	≤ +	≤ ++	≥ +	≥ ++
Conti		x	x	x			x	x	x	x		x	x	x
Firestone		x	x	x		x	x	x	x		x	x	x	
Fulda		x	x	x			x	x	x	x	x	x	x	
Gislaved		x	x	x		x	x	x	x			x	x	x
Goodyear 2		x	x	x			x	x	x	x	x	x	x	
Goodyear 3		x	x	x		x	x	x	x			x	x	x
Michelin	x	x	x			x	x	x	x			x	x	x
Pirelli		x	x	x			x	x	x	x	x	x	x	
Semperit	x	x	x				x	x	x	x	x	x	x	
Uniroyal		x	x	x		x	x	x	x		x	x	x	
Vredestein		x	x	x	x	x	x	x			x	x	x	

- Bi-ordinal Scales: $B_{n,m} := (n, n, \leq) \mid (m, m, \geq)$ Used when objects are assigned to one of two groups, and in each group with a different degree. The result is a partition with ranking.

	-	o	+	++
-	x			
o		x		
+		x	x	
++		x	x	x

	Dry		Wet			Snow	
	o	+	−	o	+	+	++
Conti	x	x		x	x	x	x
Firestone	x	x		x		x	
Fulda	x	x		x	x	x	
Gislaved	x	x		x		x	x
Goodyear 2	x	x		x	x	x	
Goodyear 3	x	x		x		x	x
Michelin	x			x		x	x
Pirelli	x	x		x	x	x	
Semperit	x			x	x	x	
Uniroyal	x	x		x		x	
Vredestein	x	x	x			x	

(b) To simplify the line diagrams, the following abbreviations for attributes and objects are used:

Conti	C	Dry 0	D0
Firestone	F1	Dry +	D+
Fulda	F2	Wet -	W-
Gislaved	G1	Wet +	W0
Goodyear 2	G2	Wet ++	W+
Goodyear 3	G3	Snow +	S+
Michelin	M	Snow ++	S++
Pirelli	P		
Semperit	S		
Uniroyal	U		
Vredestein	V		

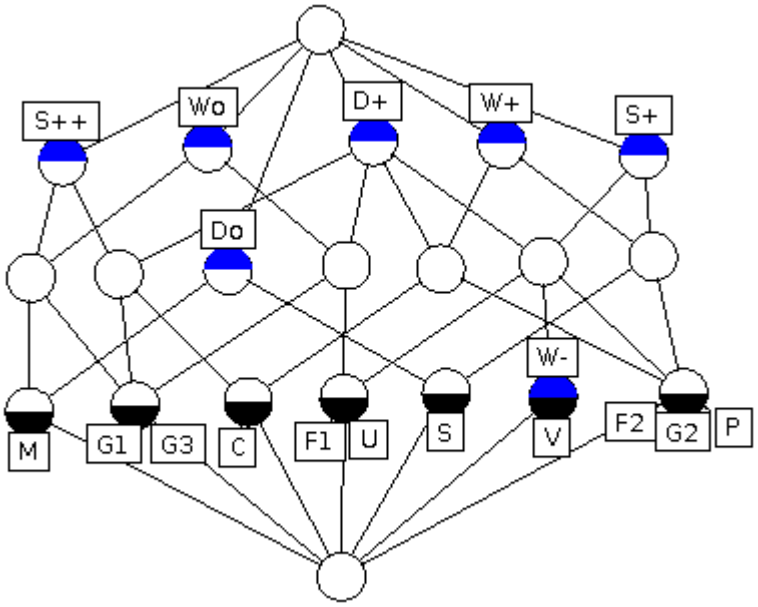
- 

Abbildung 1: Applying nominal scaling

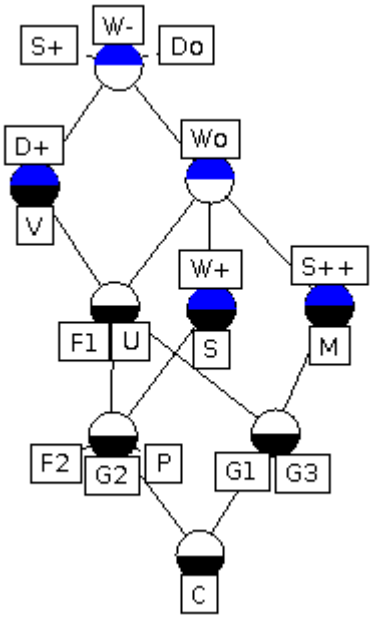
- 

Abbildung 2: Applying ordinal scaling

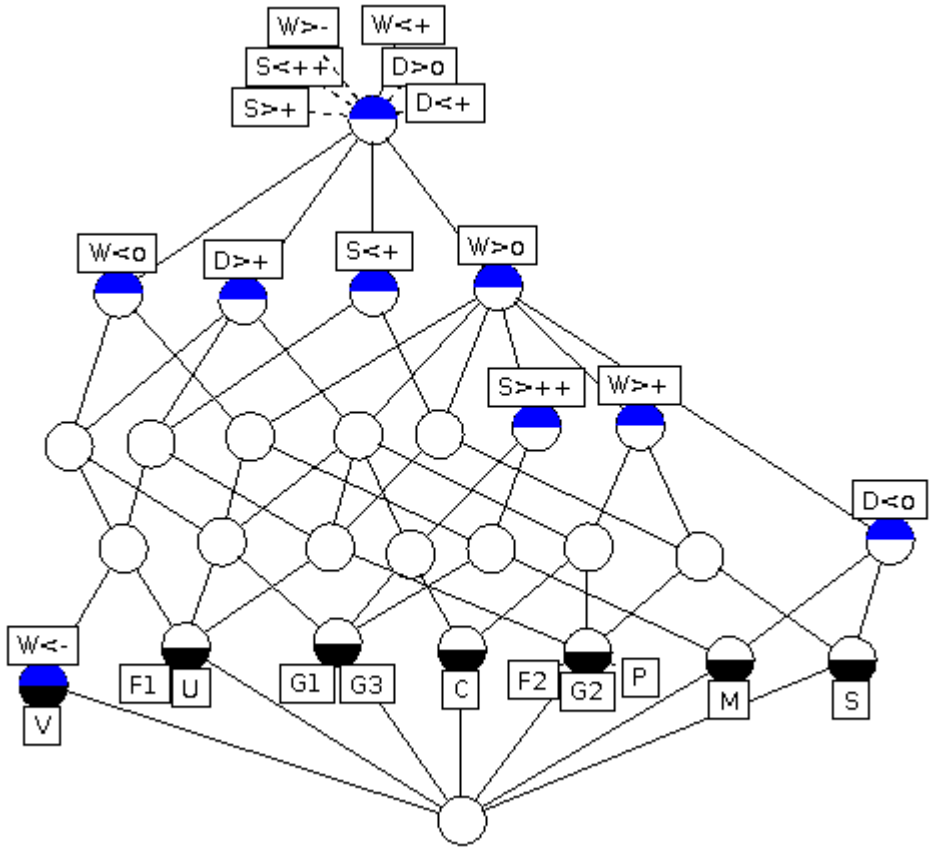


Abbildung 3: Applying interordinal scaling

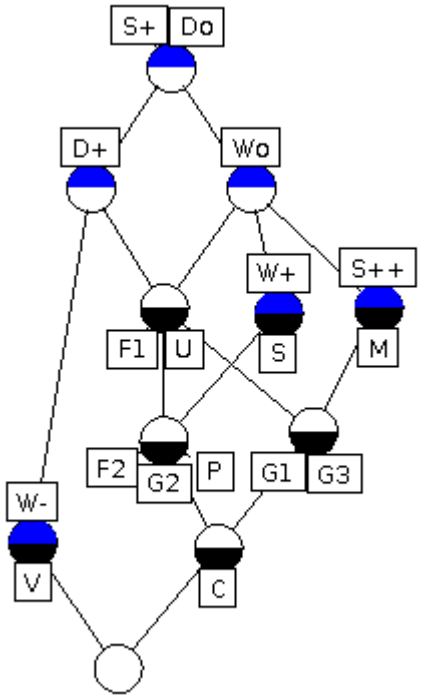


Abbildung 4: Applying biordinal scaling

(c) Which tire would you buy? Bi-ordinal scaling seems natural in this situation. In which case, Conti has the best characteristics for all weather types.