Reasoning over Existential Rules with Acyclicity Notions and the Datalog-first Restricted Chase

David Carral

TECHNISCHE UNIVERSITÄT DRESDEN

Slides available at https://iccl.inf.tu-dresden.de/web/Existential-rules-acyclicity

Preliminaries

Existential Rules

$$\forall x, y, z \,.\, \Big(\mathsf{HasParent}(x, y) \wedge \mathsf{HasSister}(y, z) \rightarrow \mathsf{HasAunt}(x, z)\Big)$$

$$\forall x \,.\, \Big(\mathsf{Human}(x) \rightarrow \exists y \,.\, \big(\mathsf{HasParent}(x, y) \wedge \mathsf{Human}(y)\big)\Big)$$

$$\forall x, y, w \,.\, \Big(\mathsf{P}(x, \mathsf{a}, y) \wedge \mathsf{R}(y, w) \wedge \mathsf{S}(w, x) \rightarrow \exists v \,.\, \big(\mathsf{R}(w, v) \wedge \mathsf{A}(v)\big)\Big)$$

Existential Rules

$$\mathsf{HasParent}(x, y) \wedge \mathsf{HasSister}(y, z) \rightarrow \mathsf{HasAunt}(x, z)$$

$$\mathsf{Human}(x) \rightarrow \exists y\,.\,\mathsf{HasParent}(x, y) \wedge \mathsf{Human}(y)$$

$$\mathsf{P}(x, \mathsf{a}, y) \wedge \mathsf{R}(y, w) \wedge \mathsf{S}(w, x) \rightarrow \exists v\,.\,\mathsf{R}(w, v) \wedge \mathsf{A}(v)$$

Existential Rules

$$\mathsf{HasParent}(x, y) \wedge \mathsf{HasSister}(y, z) \rightarrow \mathsf{HasAunt}(x, z)$$

$$\mathsf{Human}(x) \rightarrow \exists y \,.\, \mathsf{HasParent}(x, y) \wedge \mathsf{Human}(y)$$

$$\mathsf{P}(x, \mathsf{a}, y) \wedge \mathsf{R}(y, w) \wedge \mathsf{S}(w, x) \rightarrow \exists v \,.\, \mathsf{R}(w, v) \wedge \mathsf{A}(v)$$

Facts

$$\mathsf{HasFriend}(\mathsf{stan}, \mathsf{kyle})$$

$$\mathsf{P}(\mathsf{a}, \mathsf{c}, \mathsf{d})$$

Existential Rules

$$\mathsf{HasParent}(x, y) \land \mathsf{HasSister}(y, z) \rightarrow \mathsf{HasAunt}(x, z)$$

$$\mathsf{Human}(x) \rightarrow \exists y \,.\, \mathsf{HasParent}(x, y) \land \mathsf{Human}(y)$$

$$\mathsf{P}(x, \mathsf{a}, y) \land \mathsf{R}(y, w) \land \mathsf{S}(w, x) \rightarrow \exists v \,.\, \mathsf{R}(w, v) \land \mathsf{A}(v)$$

Facts

$$\mathsf{HasFriend}(\mathsf{stan}, \mathsf{kyle})$$

$$\mathsf{P}(\mathsf{a}, \mathsf{c}, \mathsf{d})$$

BCQs

$$\exists x, y \,.\, \mathsf{HasConflictOfInterest}(x, y)$$

$$\exists x, y, z, w \,.\, \mathsf{P}(x, y, z) \land \mathsf{R}(x, w) \land \mathsf{A}(w)$$

The Chase Algorithm

$\mathrm{Features}(x, y) \longrightarrow \mathrm{Actor}(y)$

$\mathrm{ActsIn}(x, y) \longrightarrow \mathrm{Features}(y, x)$

$\mathrm{DirectedBy}(x, y) \longrightarrow \mathrm{Directs}(y, x)$

$\mathrm{Directs}(x, y) \wedge \mathrm{Features}(y, z) \longrightarrow \mathrm{DirectsActor}(x, z)$

$\mathrm{Director}(\mathrm{spielberg})$

$\mathrm{ActsIn}(\mathrm{judeLaw}, \mathrm{ai})$

$\mathrm{DirectedBy}(\mathrm{ai}, \mathrm{spielberg})$

The Chase Algorithm

Features(x, y) ⟶ Actor(y)

DirectedBy(x, y) ⟶ Directs(y, x)

ActsIn(x, y) ⟶ Features(y, x)

Directs(x, y) ∧ Features(y, z) ⟶ DirectsActor(x, z)

judeLaw

spielberg

Director(spielberg)

ActsIn(judeLaw, ai)

ai

DirectedBy(ai, spielberg)

The Chase Algorithm

$\text{Features}(x, y) \longrightarrow \text{Actor}(y)$

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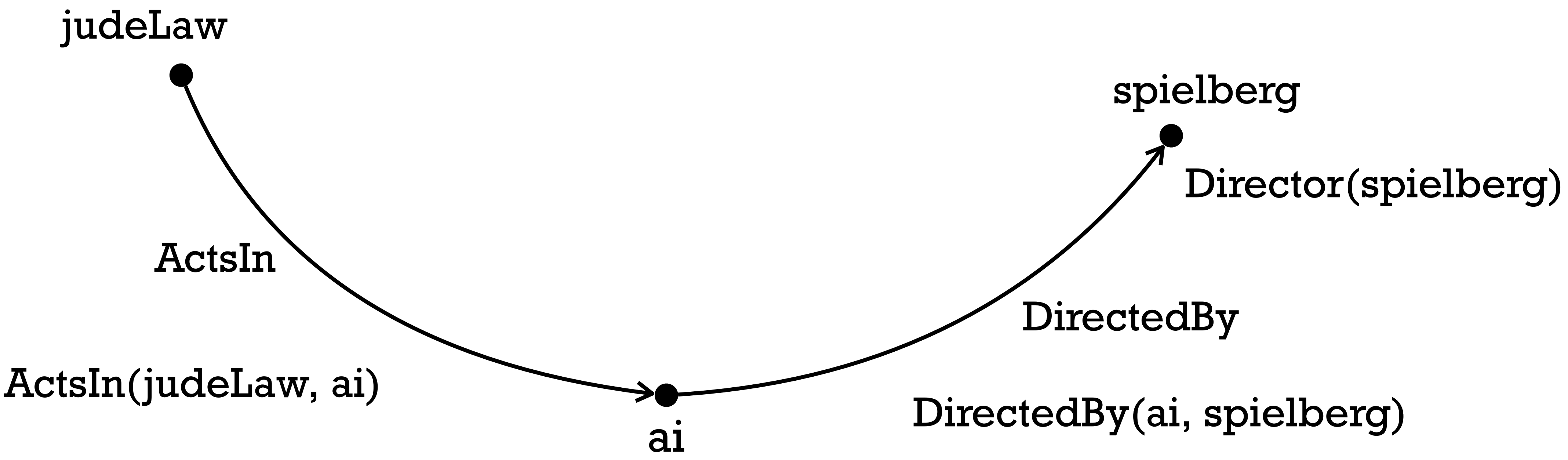

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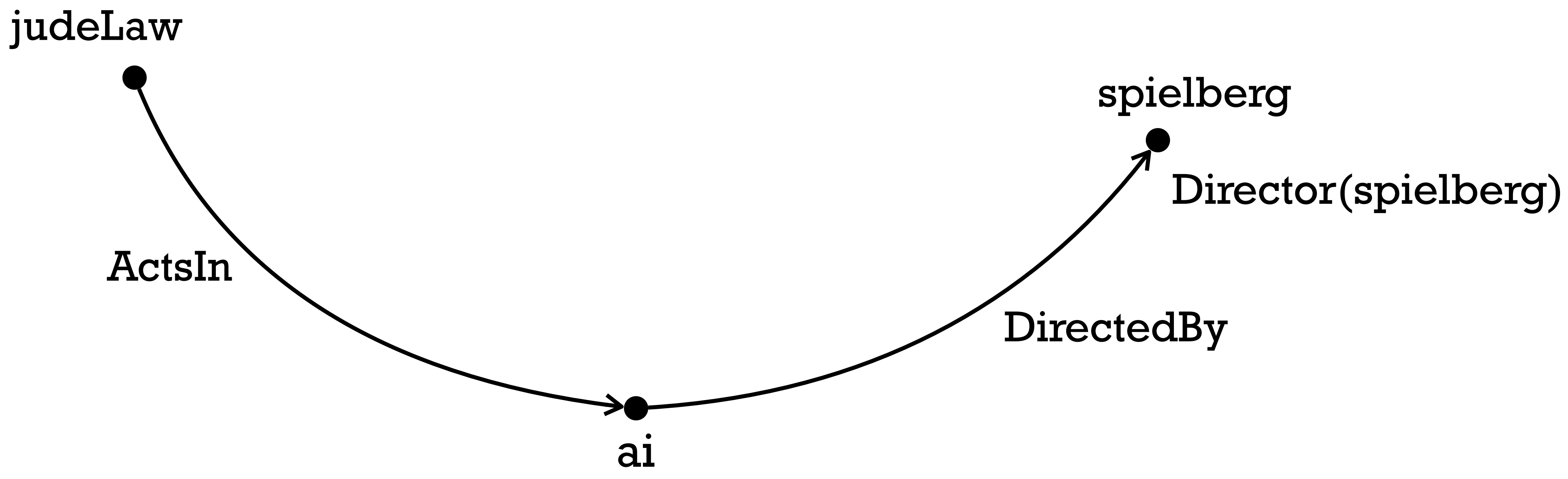

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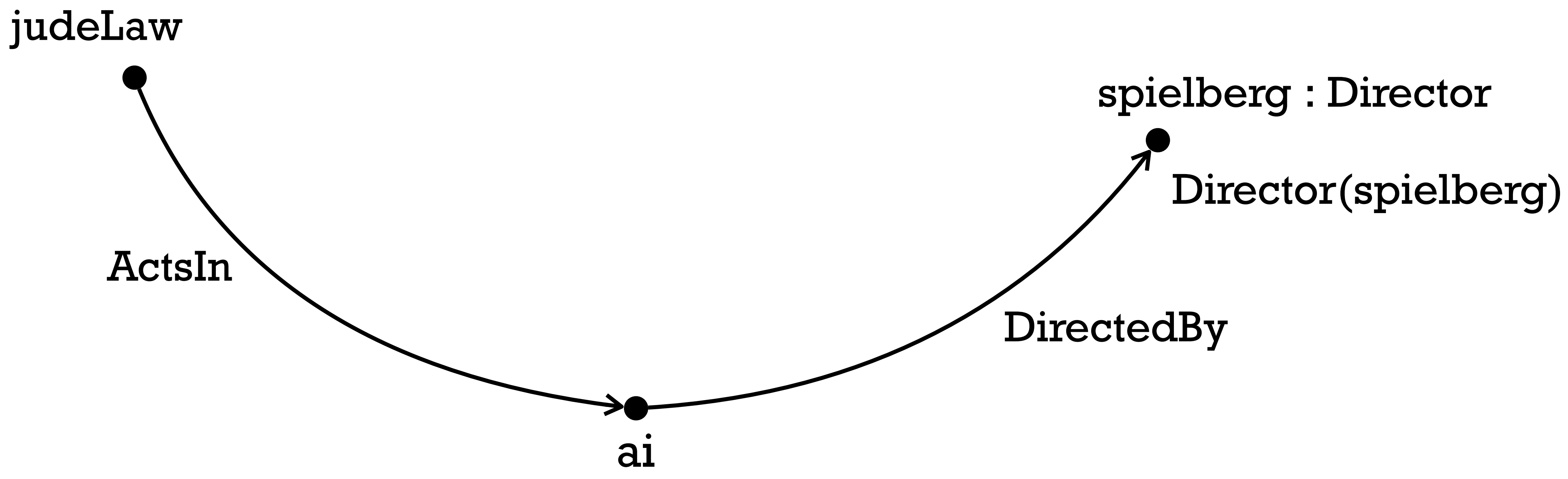

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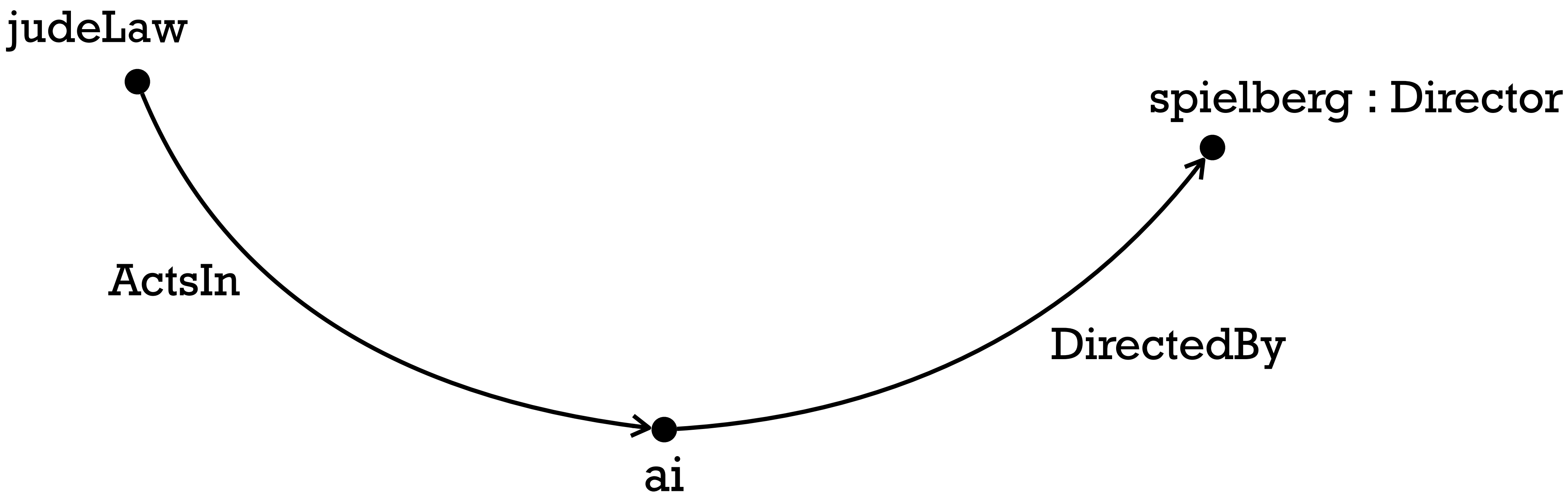

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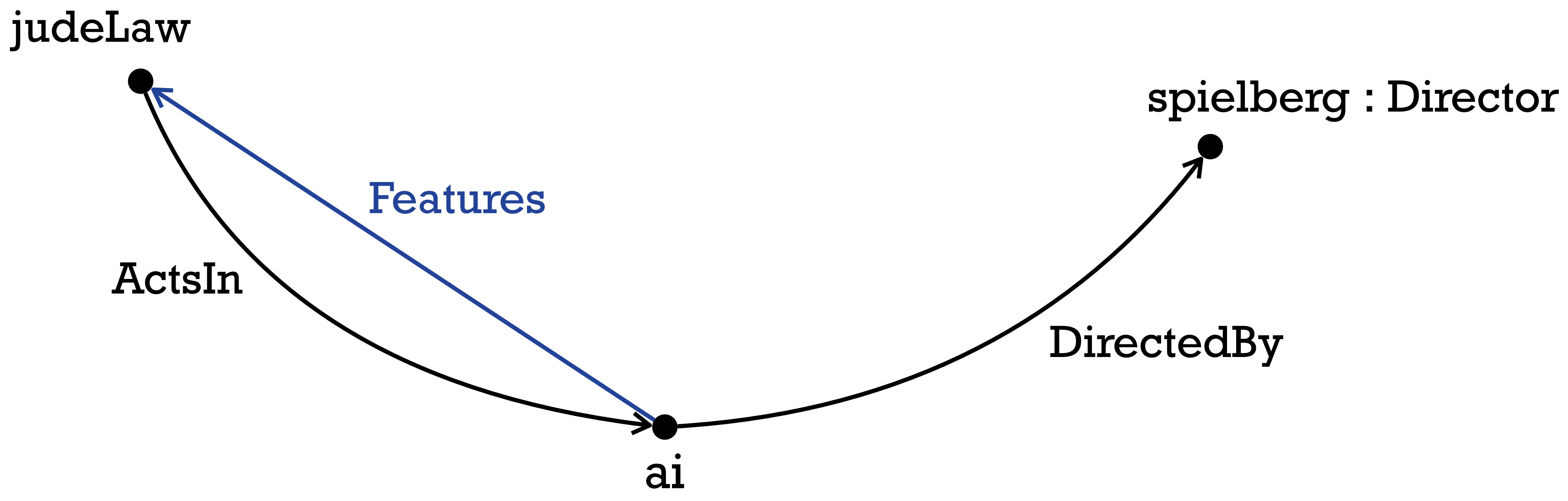

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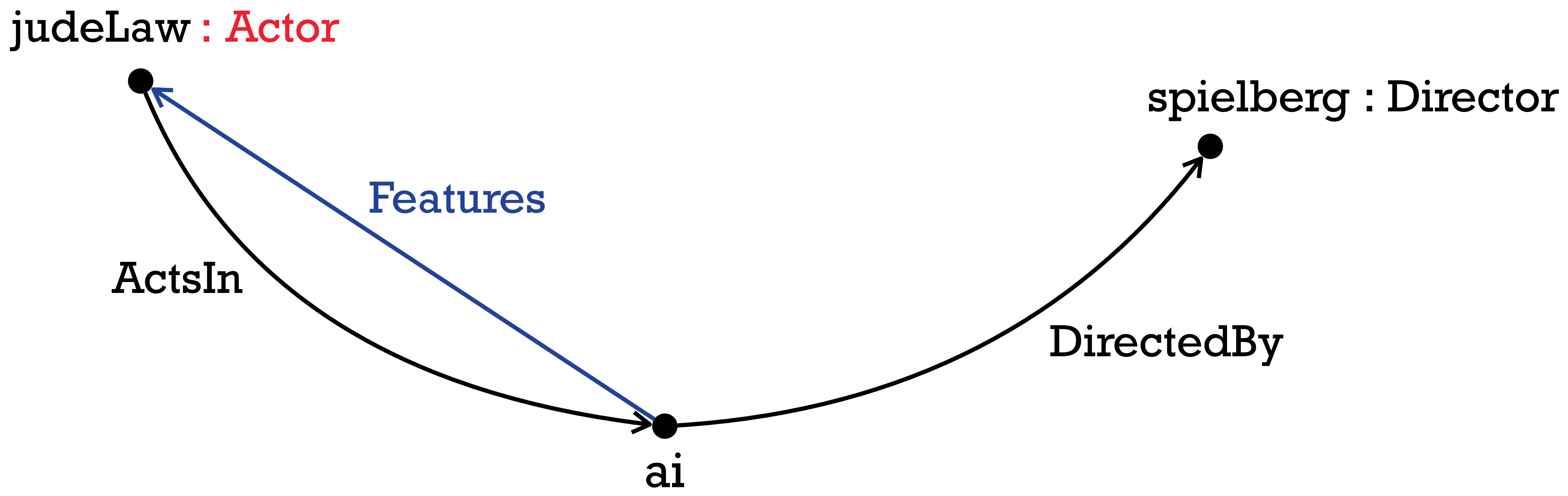

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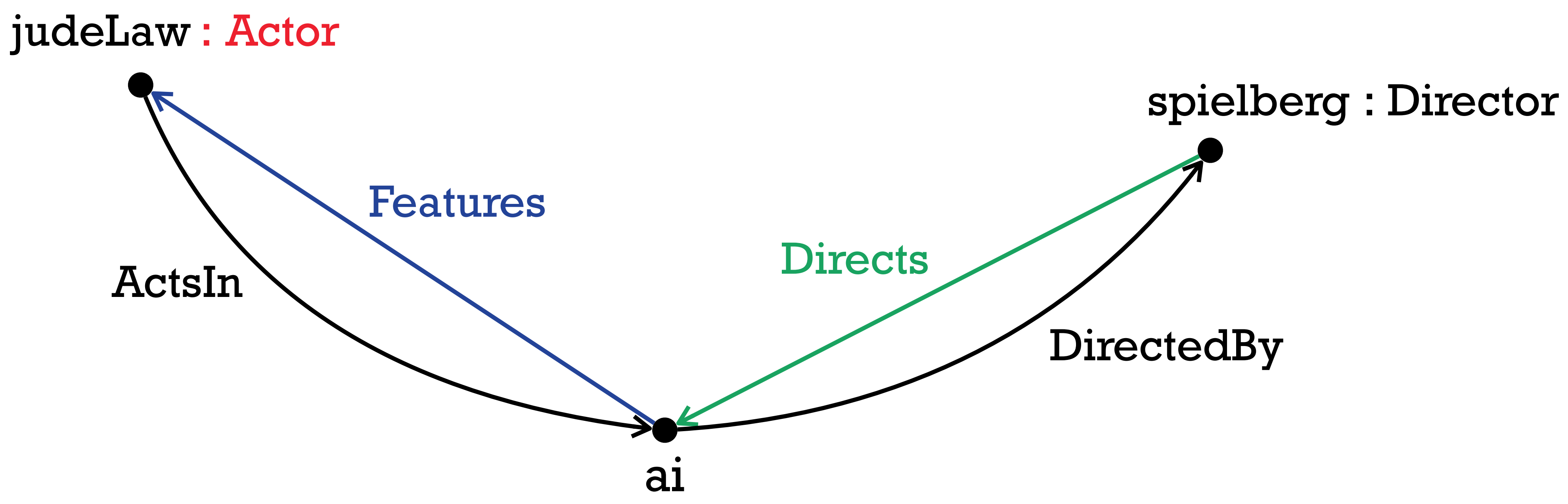

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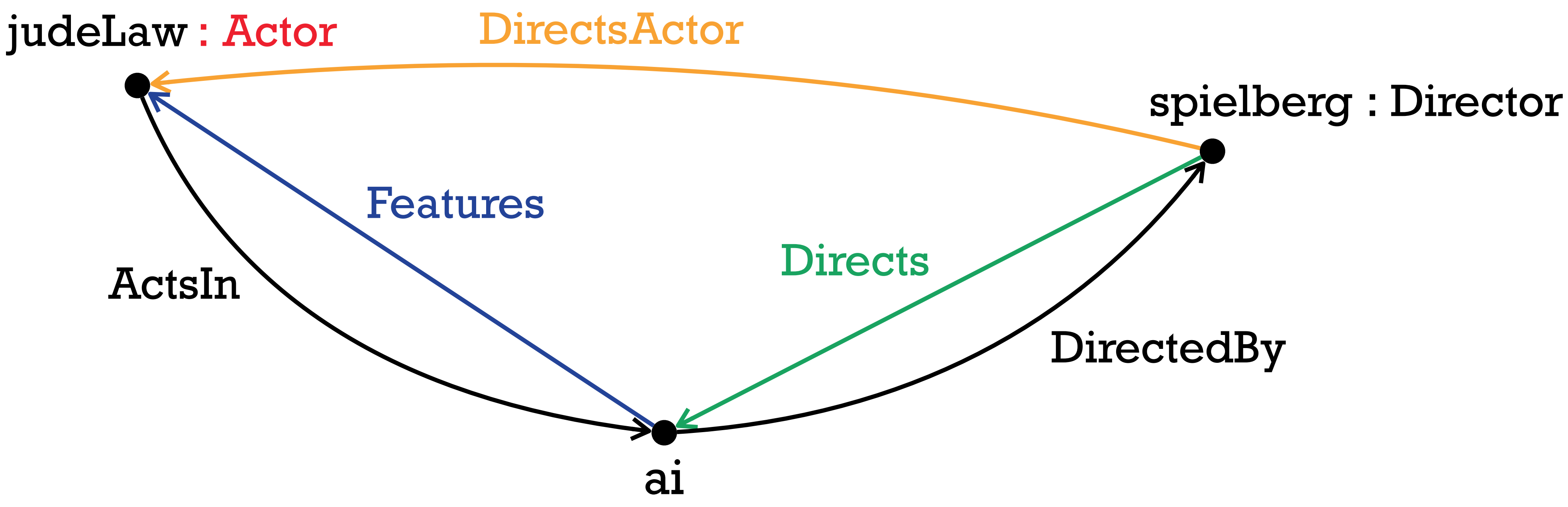

The Skolem Chase

$\text{Bicycle}(x) \longrightarrow \exists v\ .\ \text{HasPart}(x, v) \wedge \text{Wheel}(v)$

$\text{Wheel}(x) \longrightarrow \exists w\ .\ \text{IsPartOf}(x, w) \wedge \text{Bicycle}(w)$

$\text{HasPart}(x, y) \longrightarrow \text{IsPartOf}(y, x)$

$\text{IsPartOf}(x, y) \longrightarrow \text{HasPart}(y, x)$

b : Bicycle

The Skolem Chase

$$\mathrm{Bicycle}(x) \longrightarrow \mathrm{HasPart}(x, f_v(x)) \wedge \mathrm{Wheel}(f_v(x)) \qquad \mathrm{HasPart}(x, y) \longrightarrow \mathrm{IsPartOf}(y, x)$$

$$\mathrm{Wheel}(x) \longrightarrow \mathrm{IsPartOf}(x, f_w(x)) \wedge \mathrm{Bicycle}(f_w(x)) \qquad \mathrm{IsPartOf}(x, y) \longrightarrow \mathrm{HasPart}(y, x)$$

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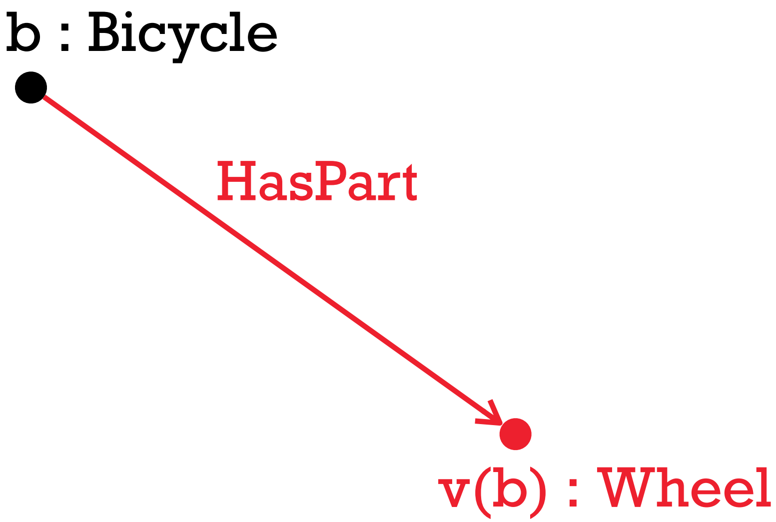

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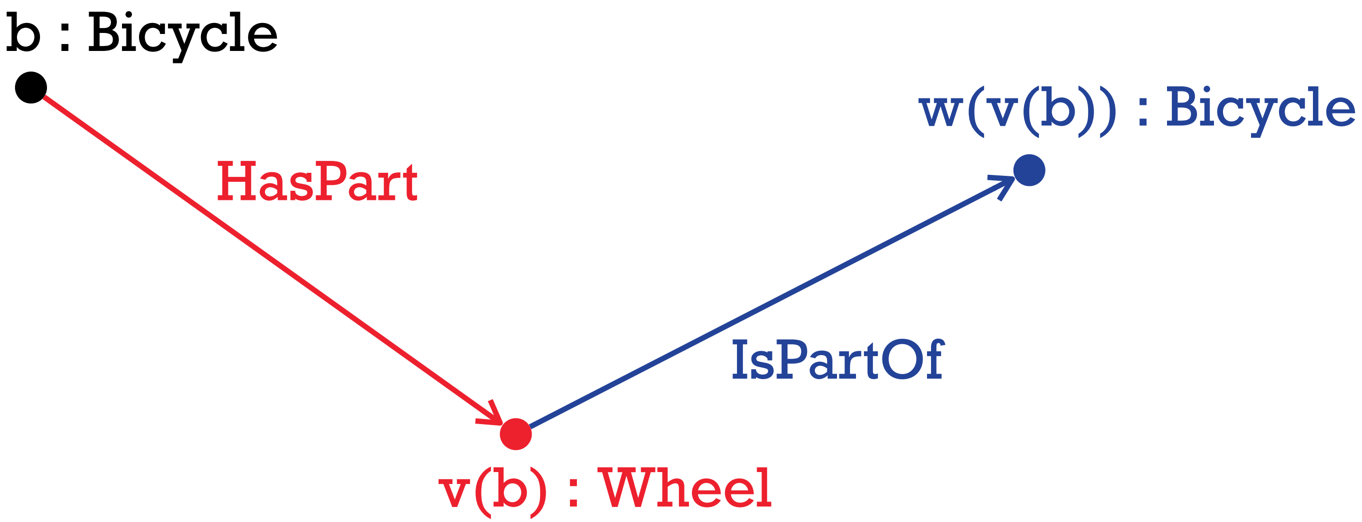

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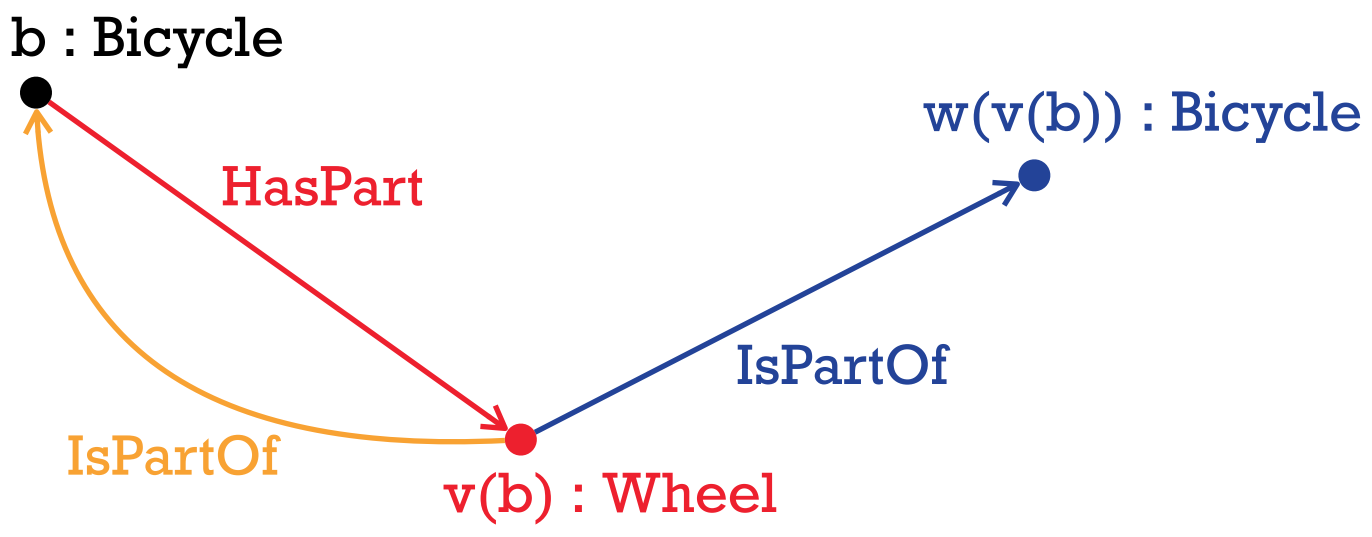

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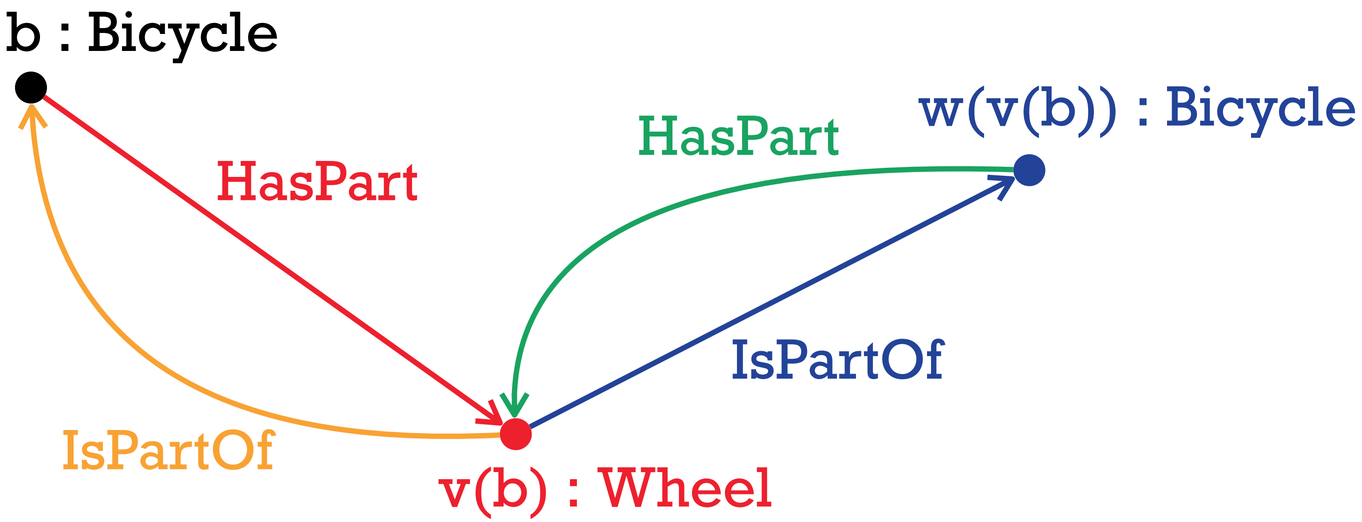

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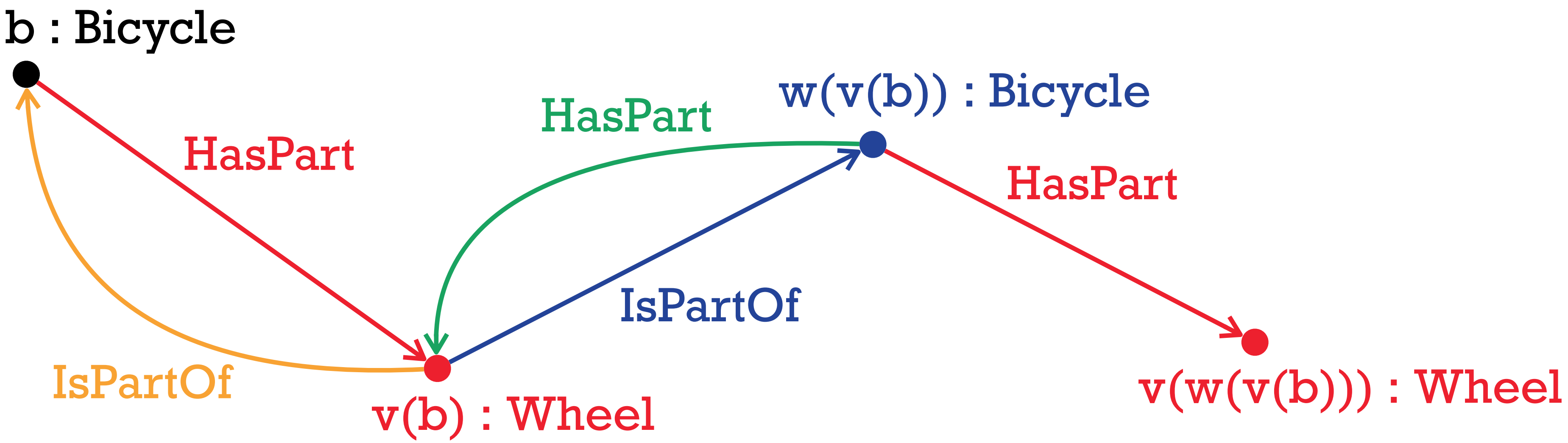

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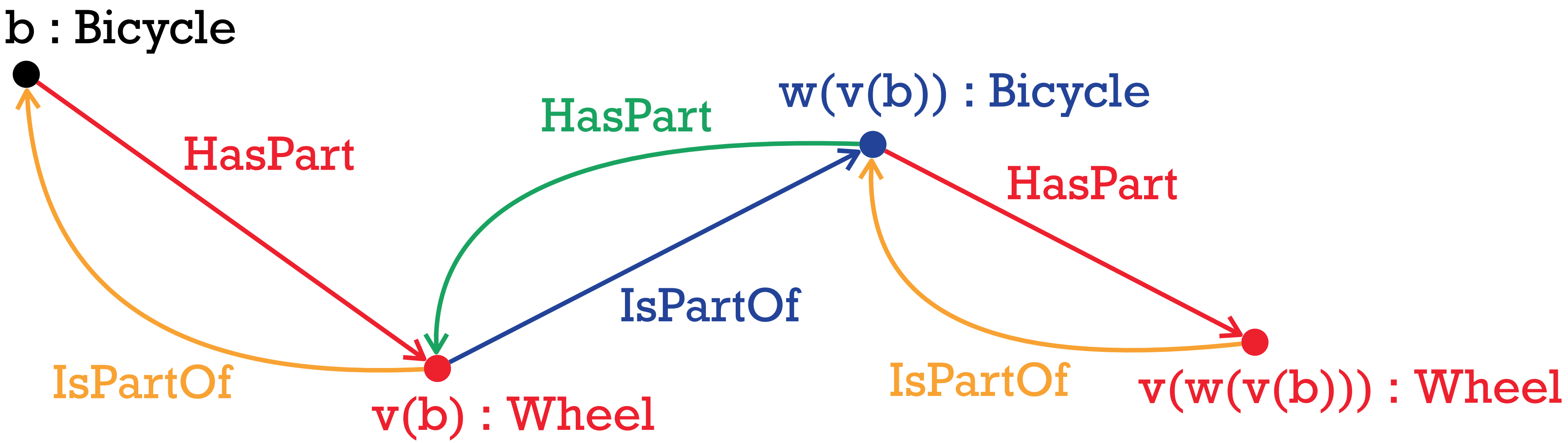

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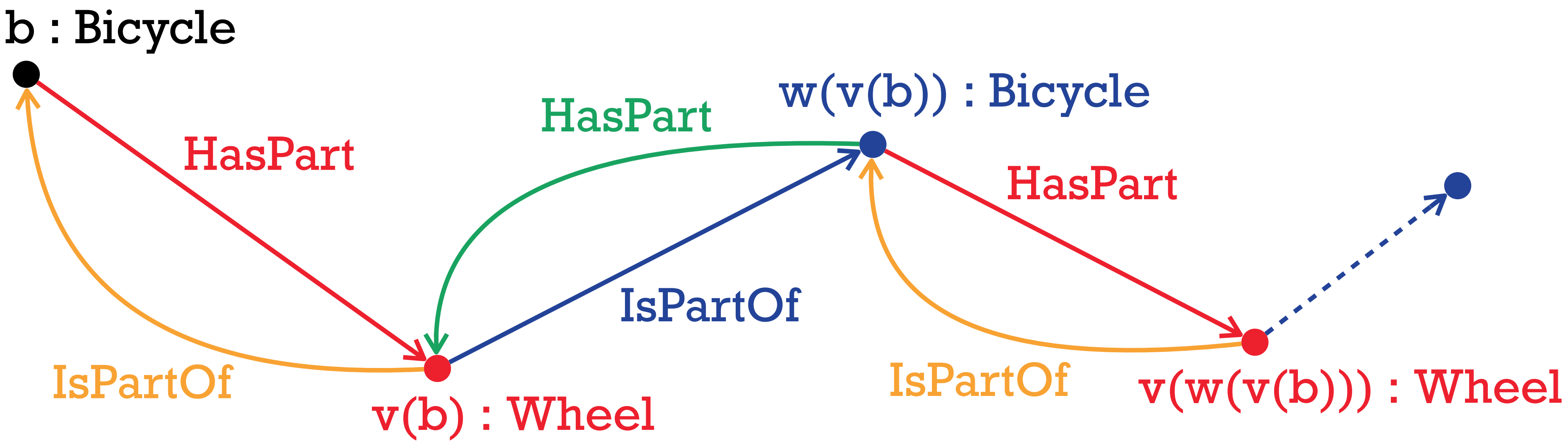

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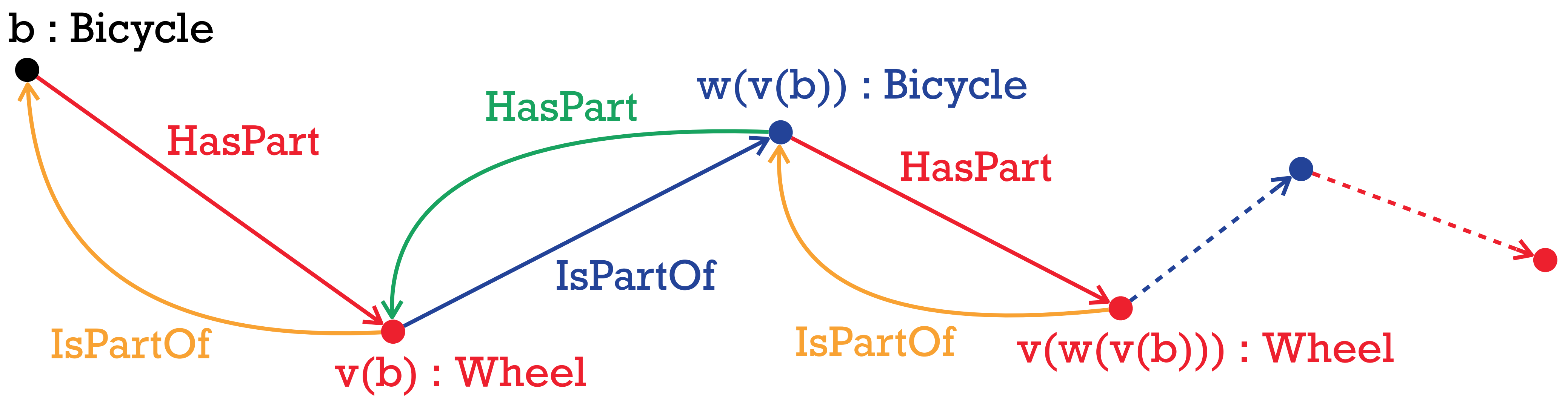

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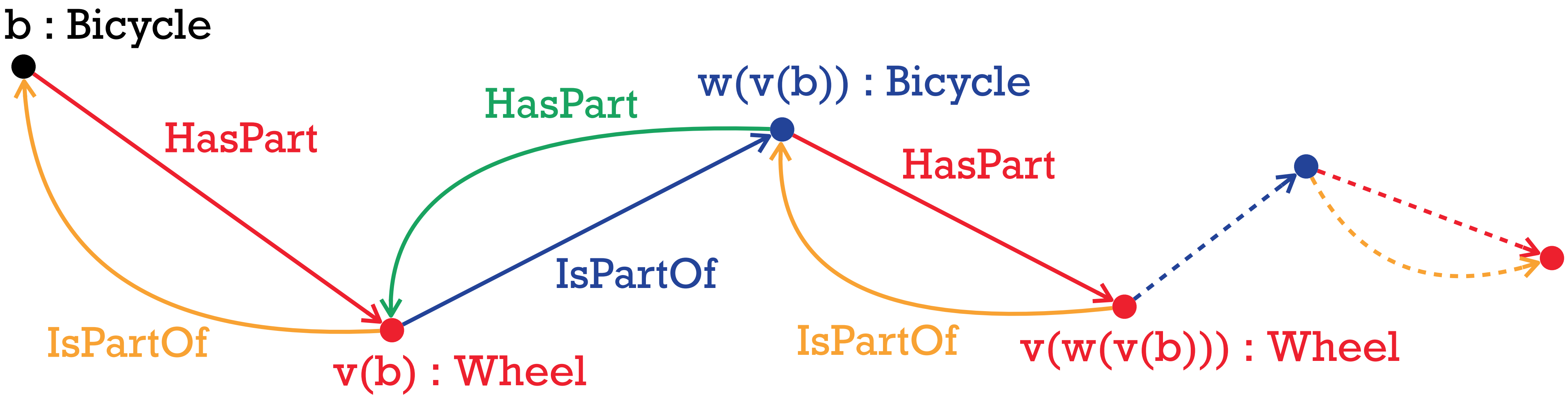

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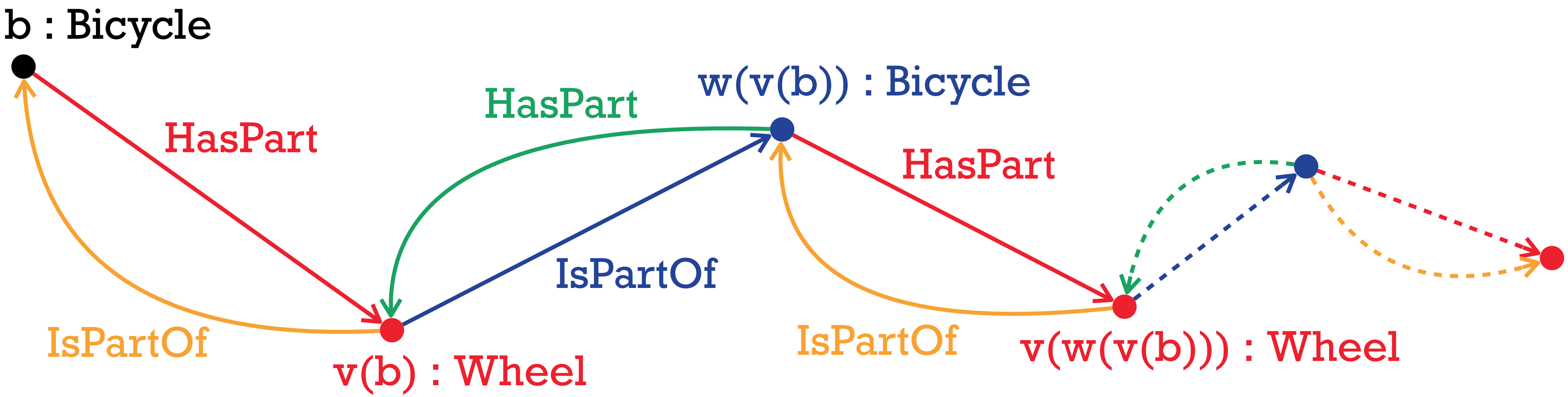

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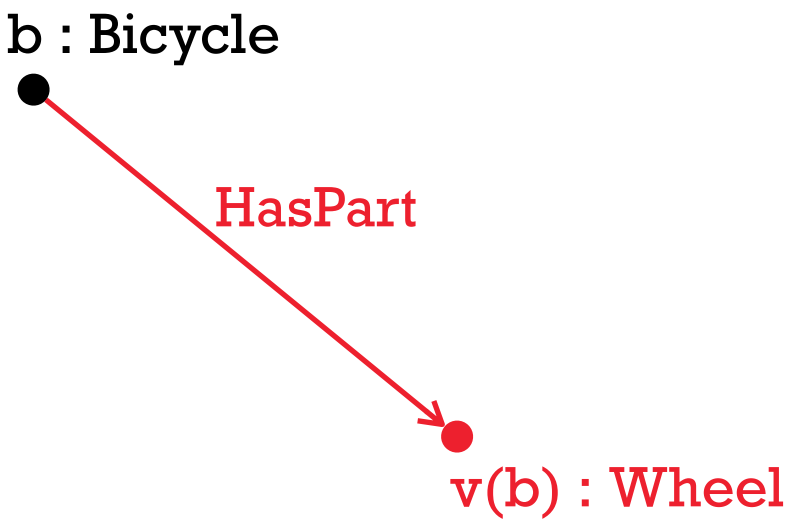

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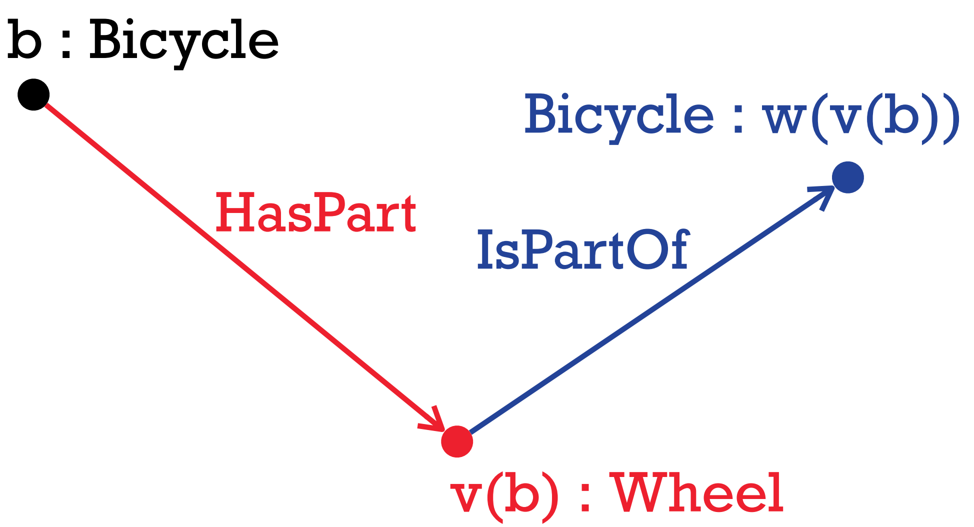

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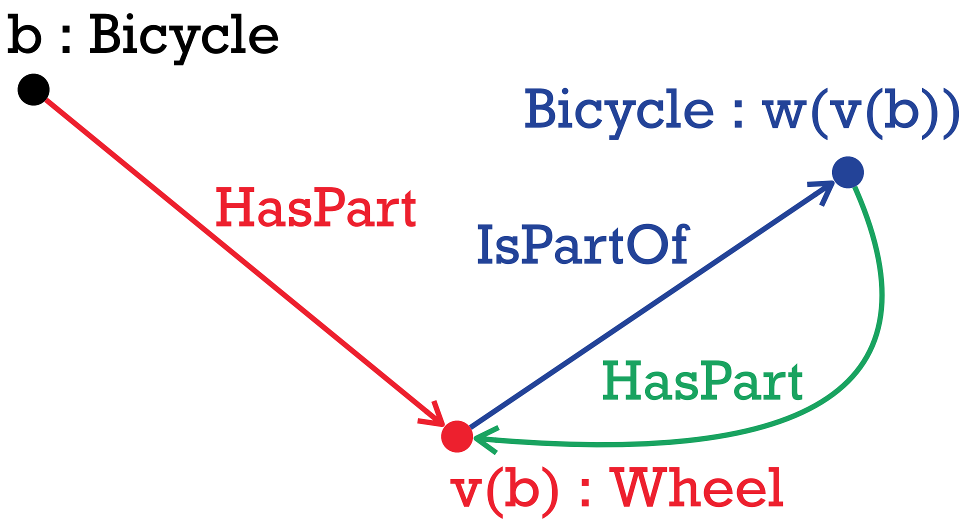

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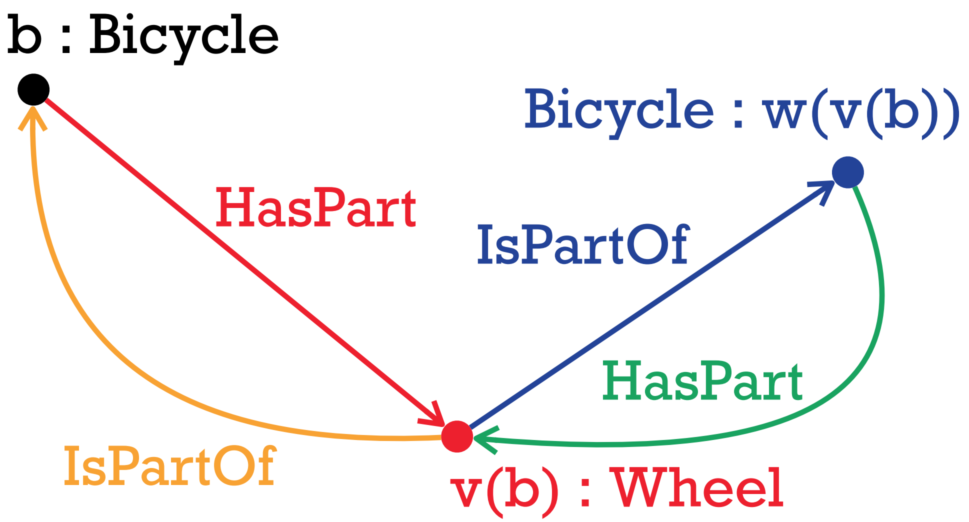

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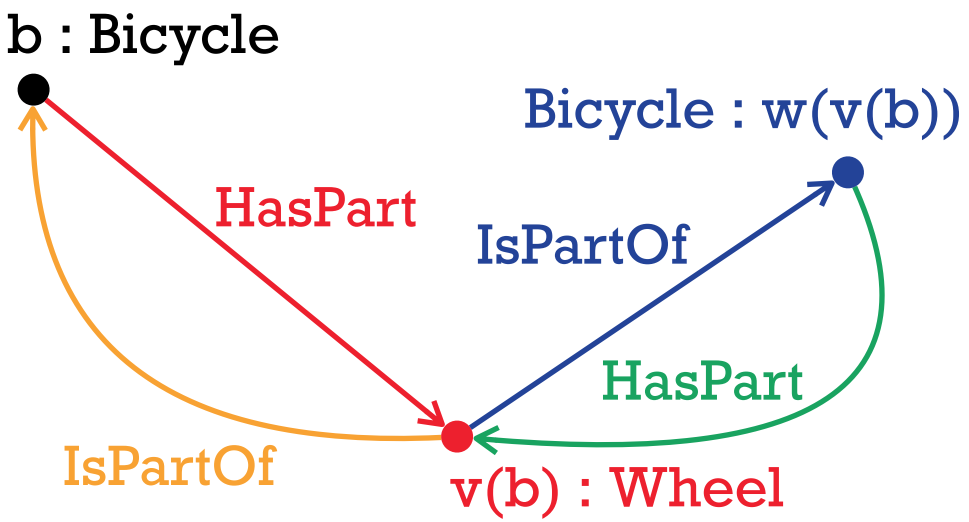

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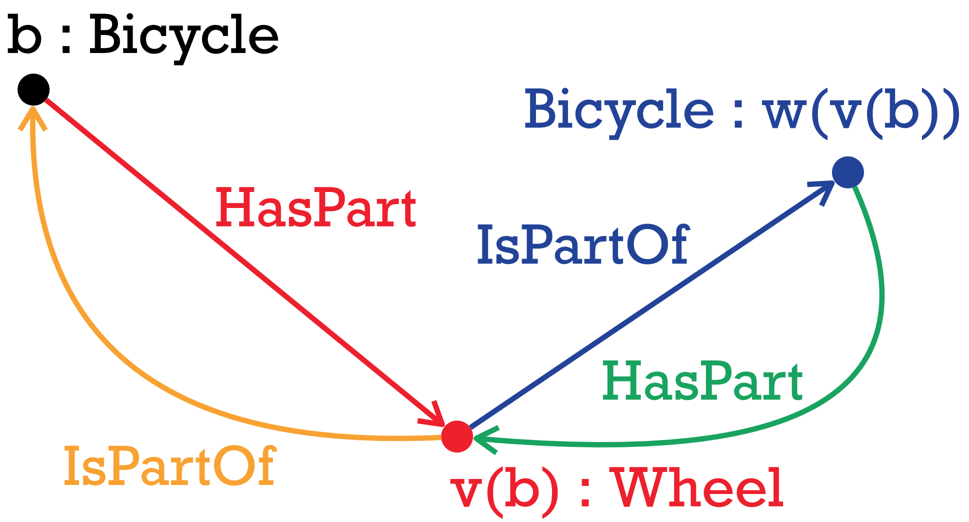

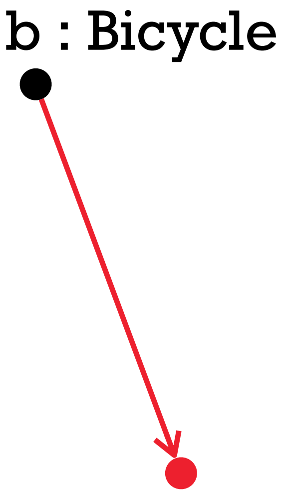

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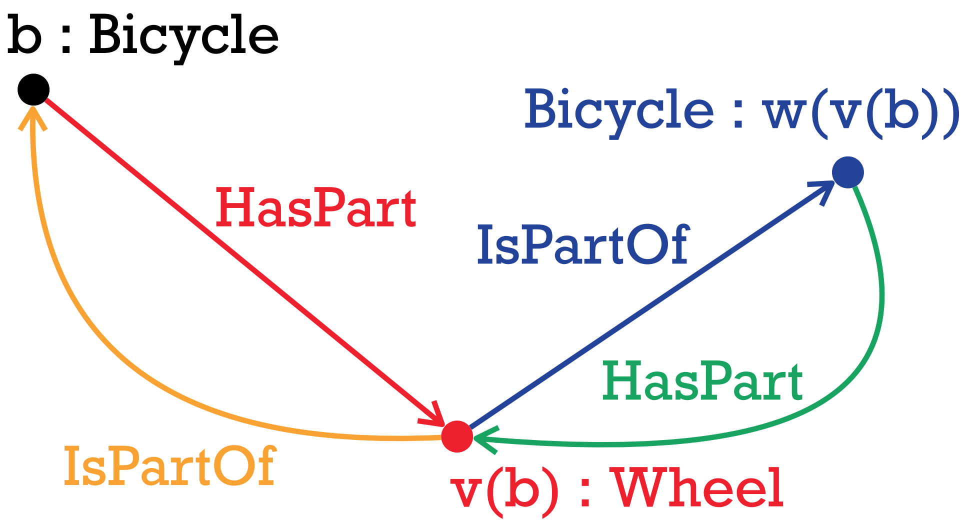

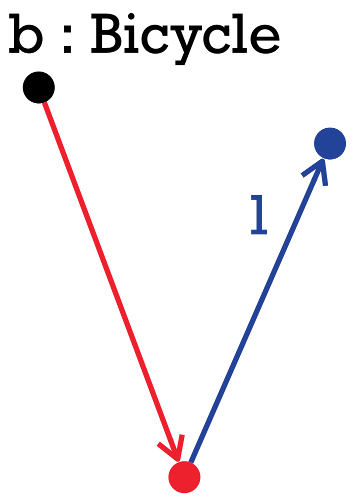

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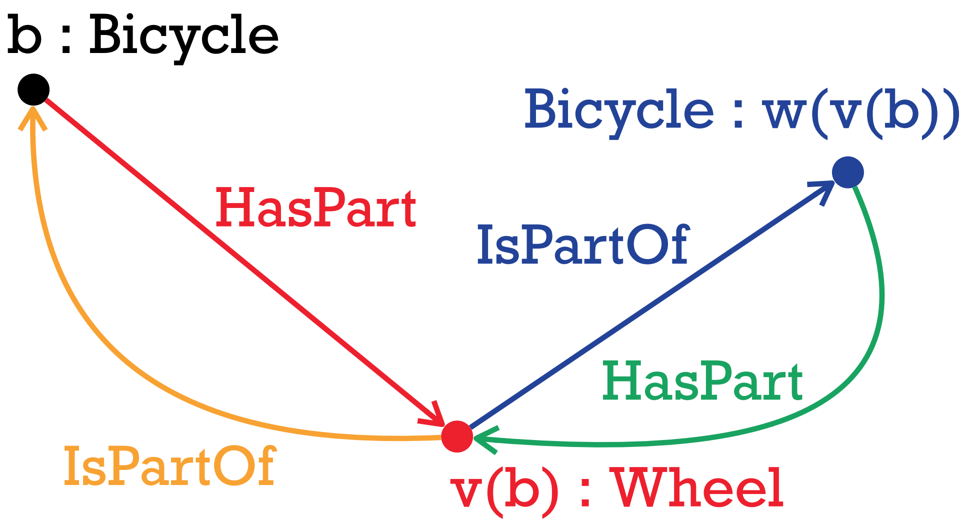

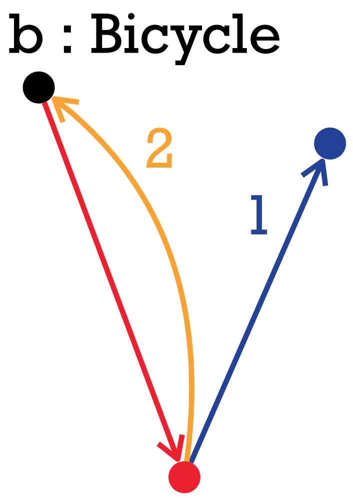

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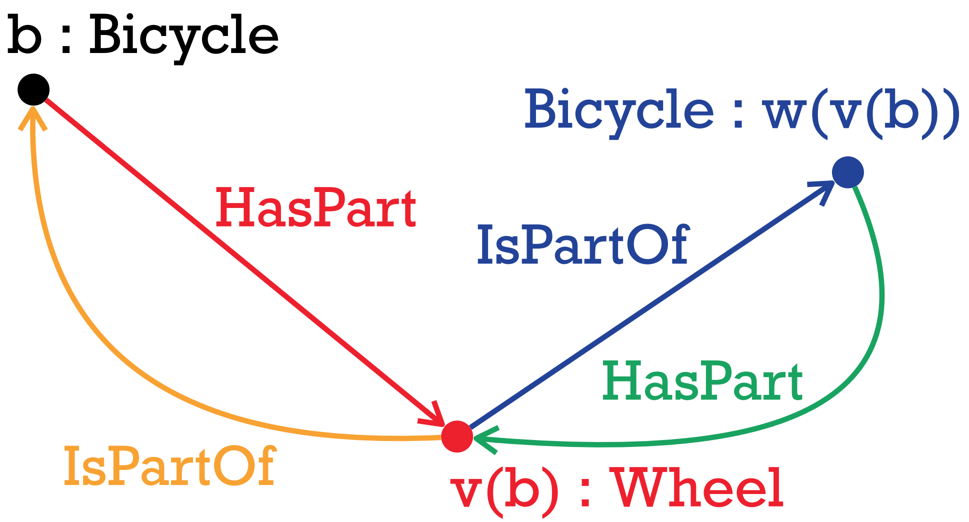

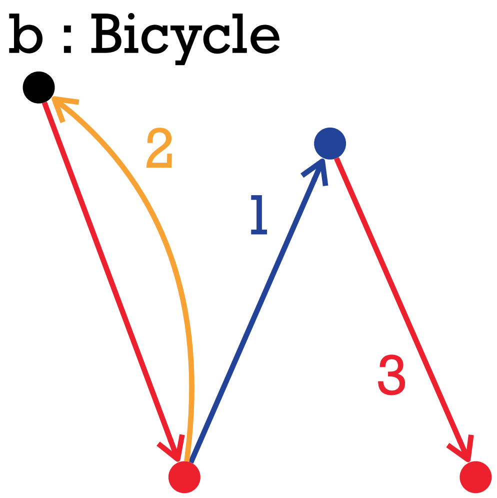

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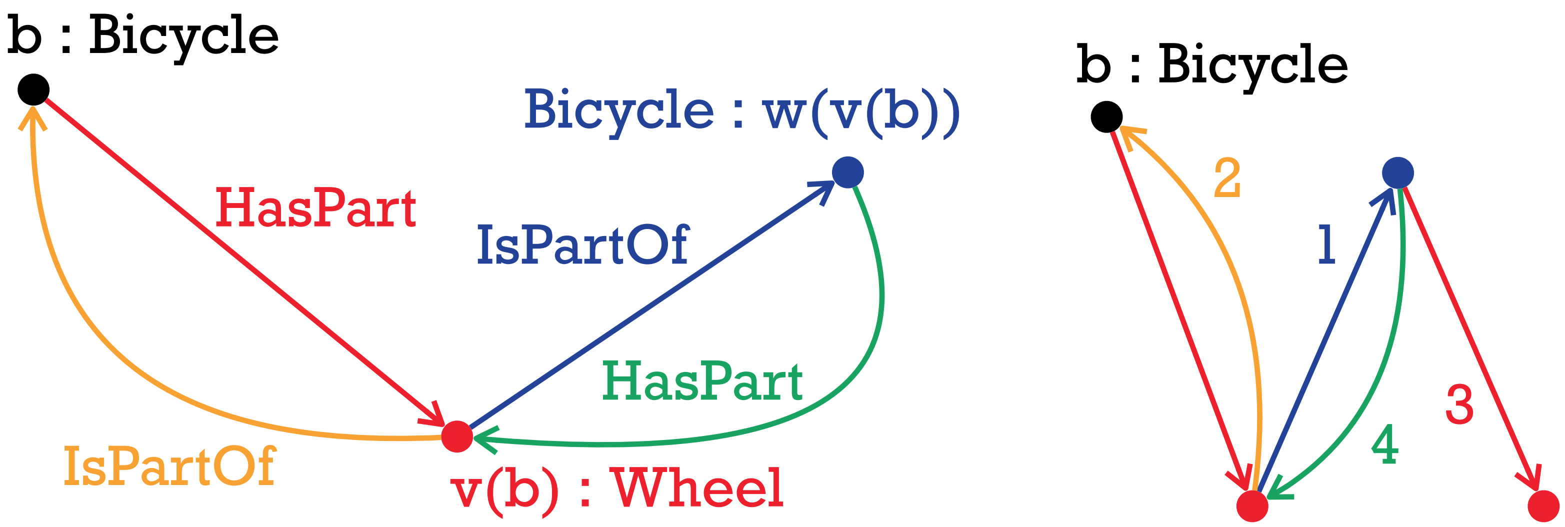

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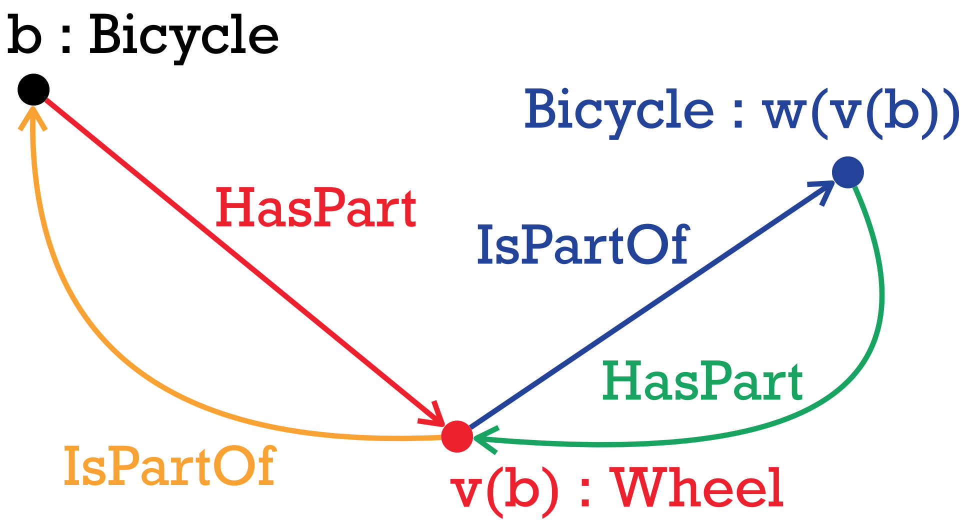

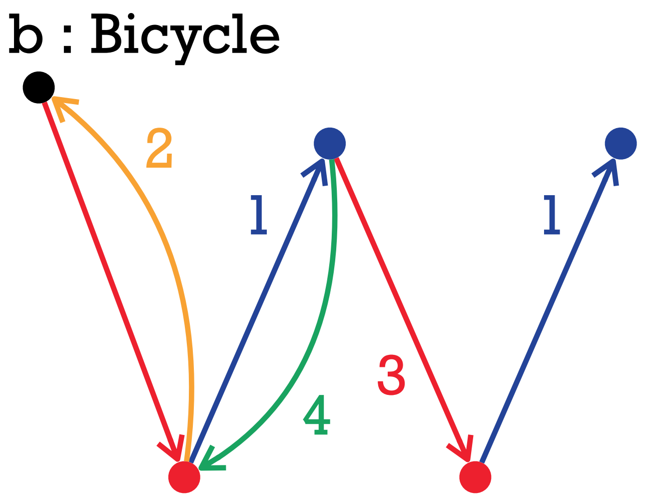

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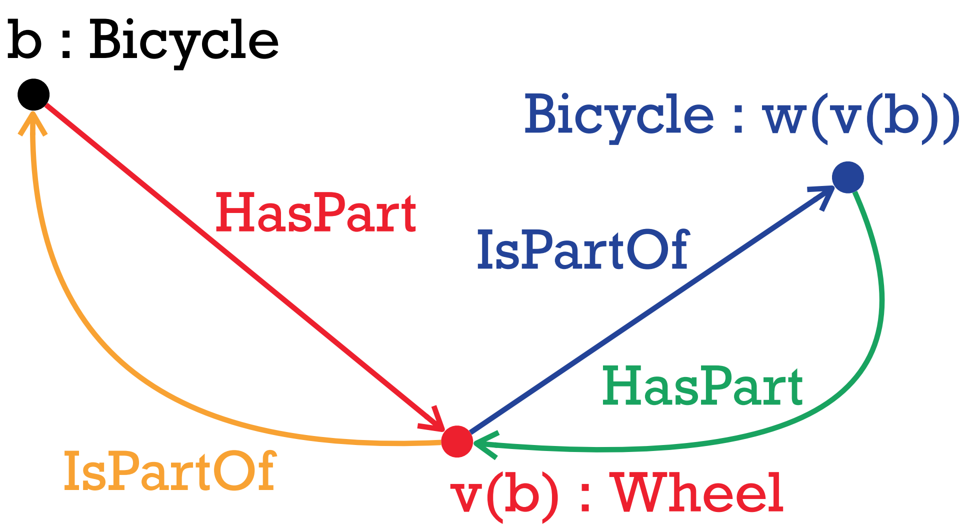

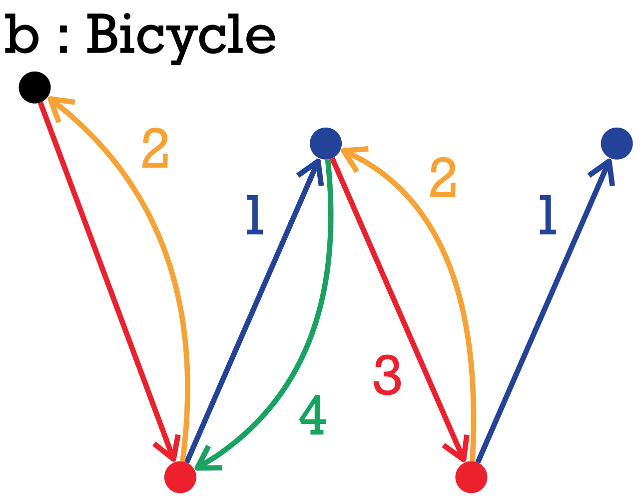

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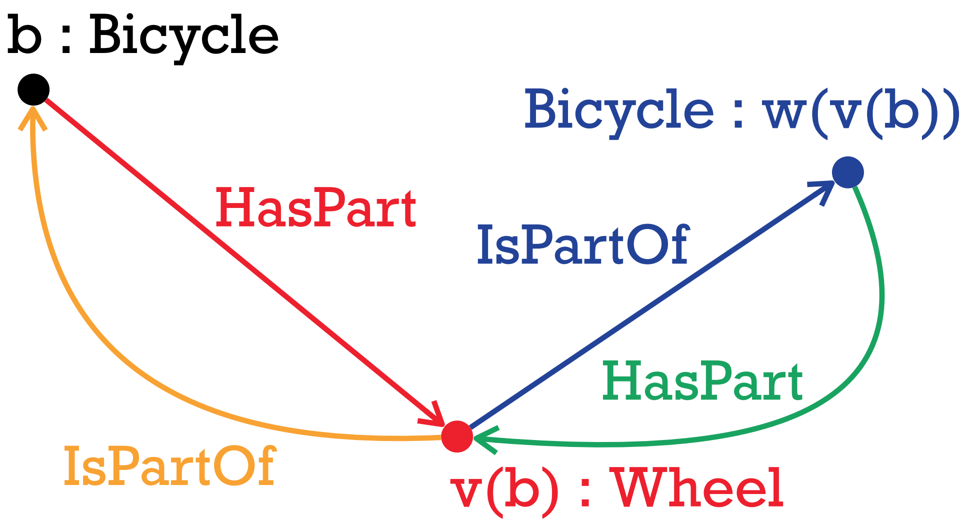

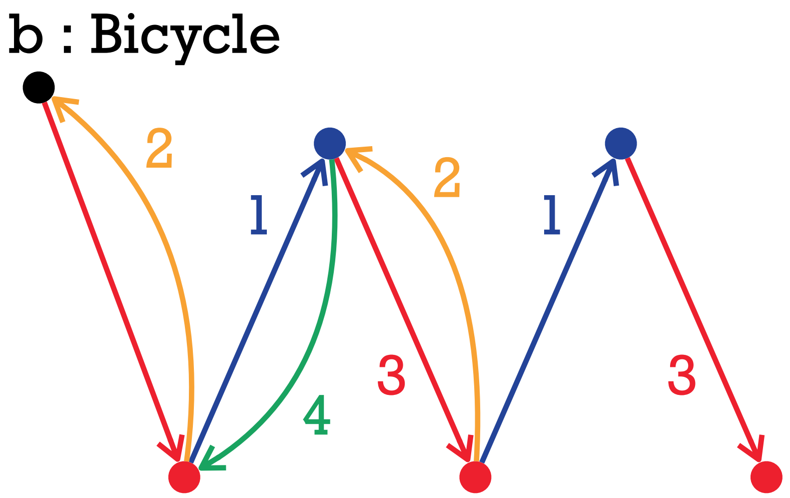

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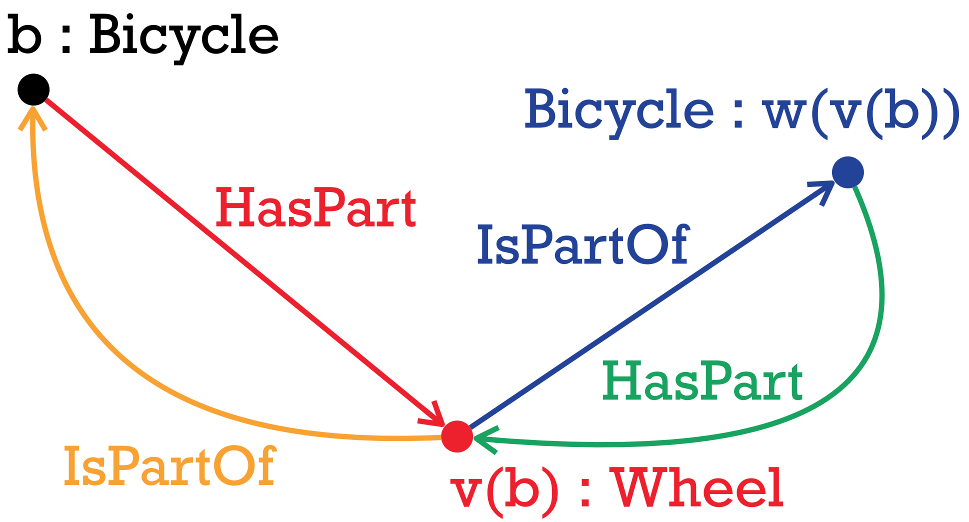

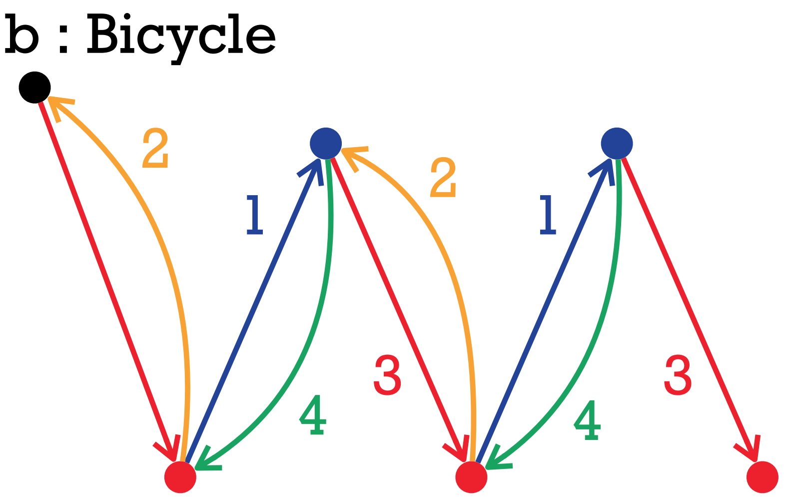

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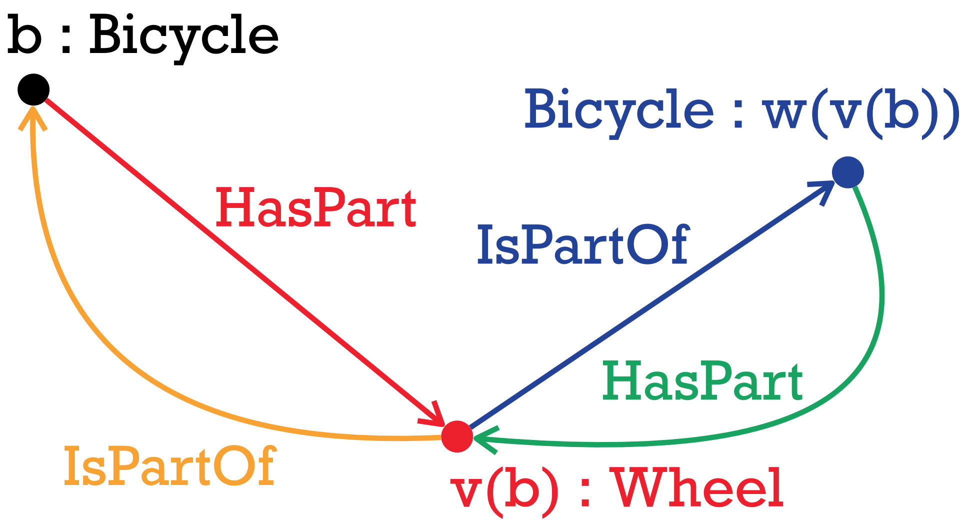

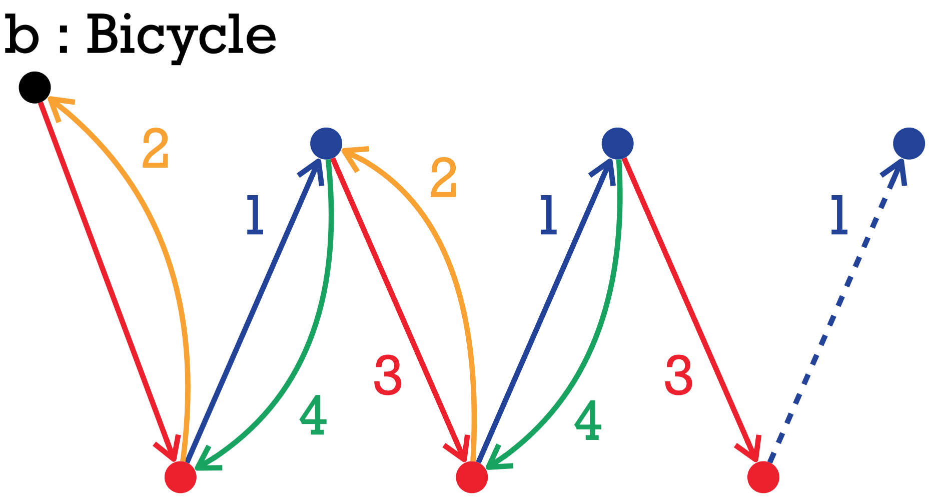

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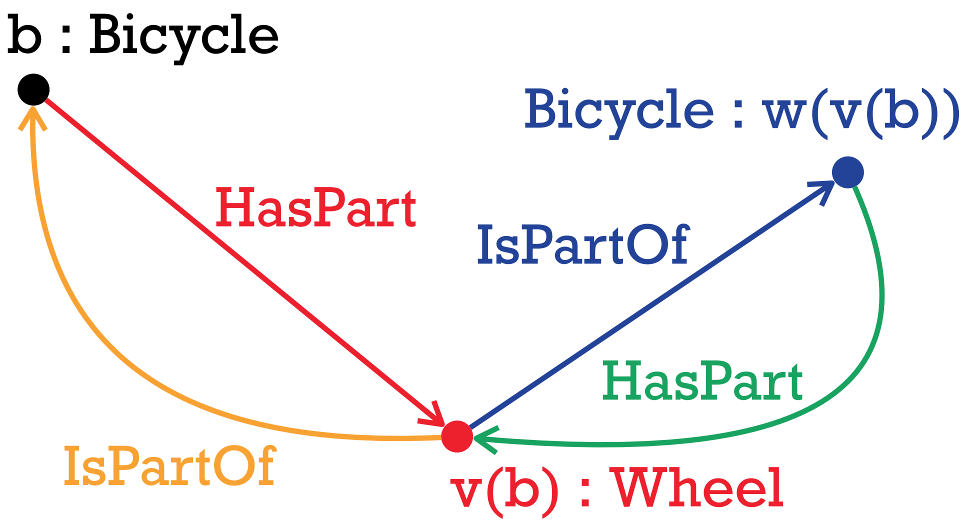

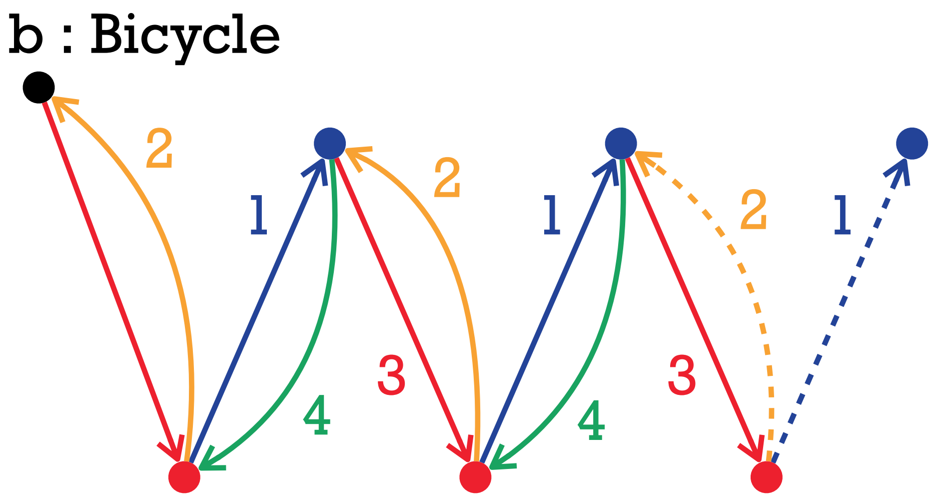

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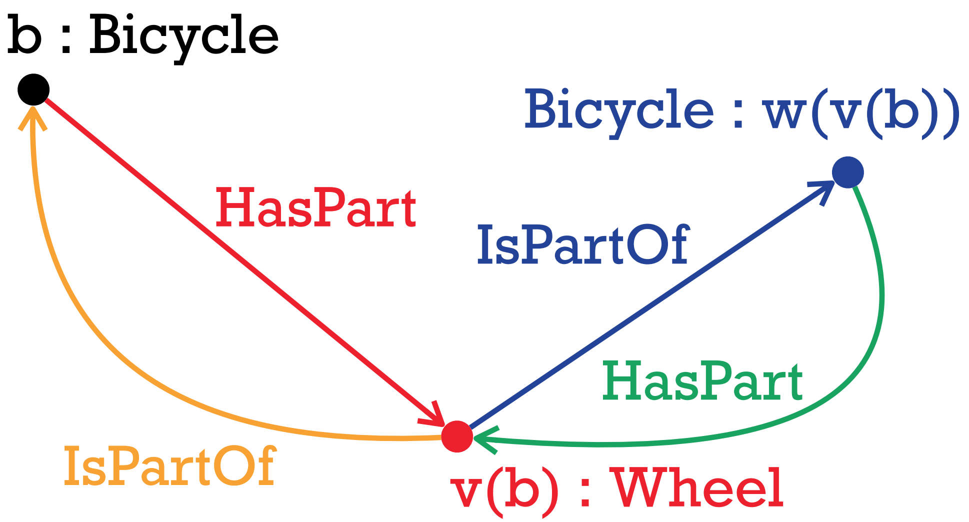

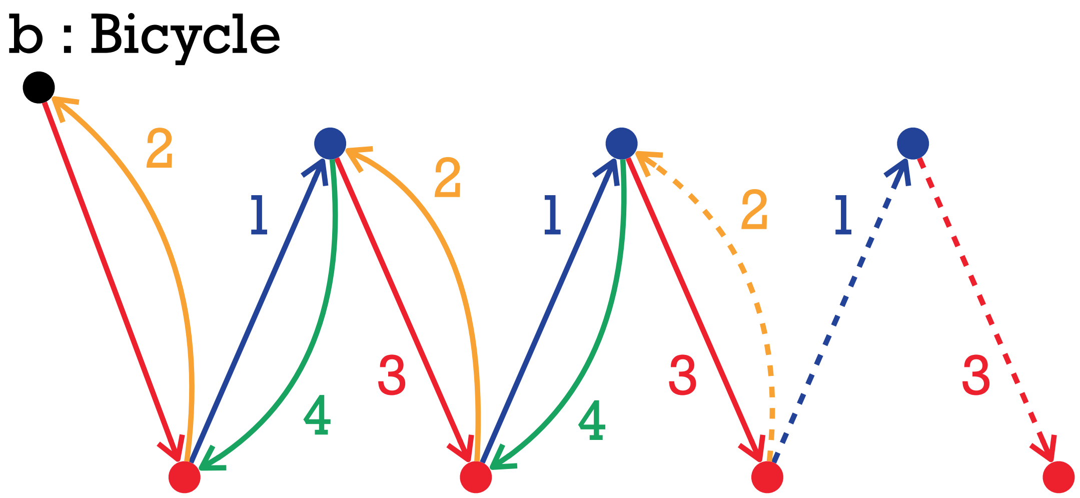

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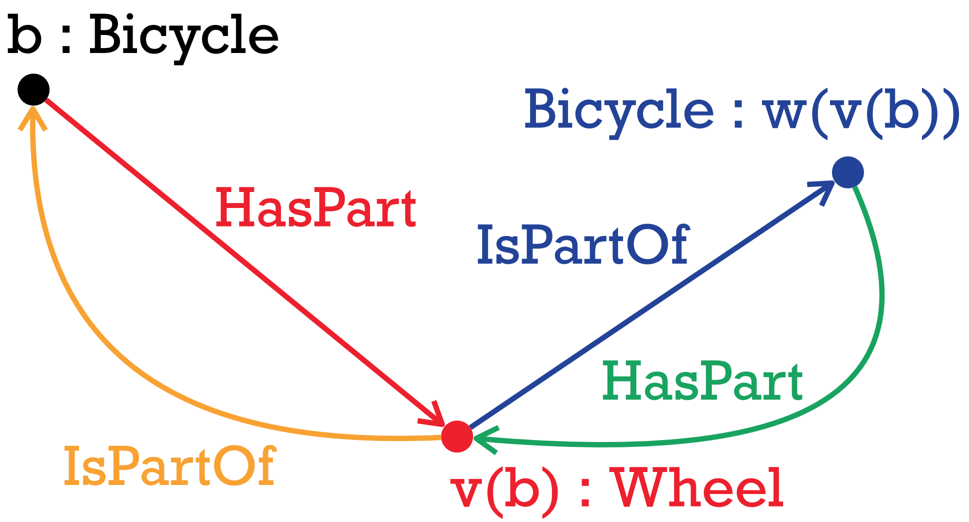

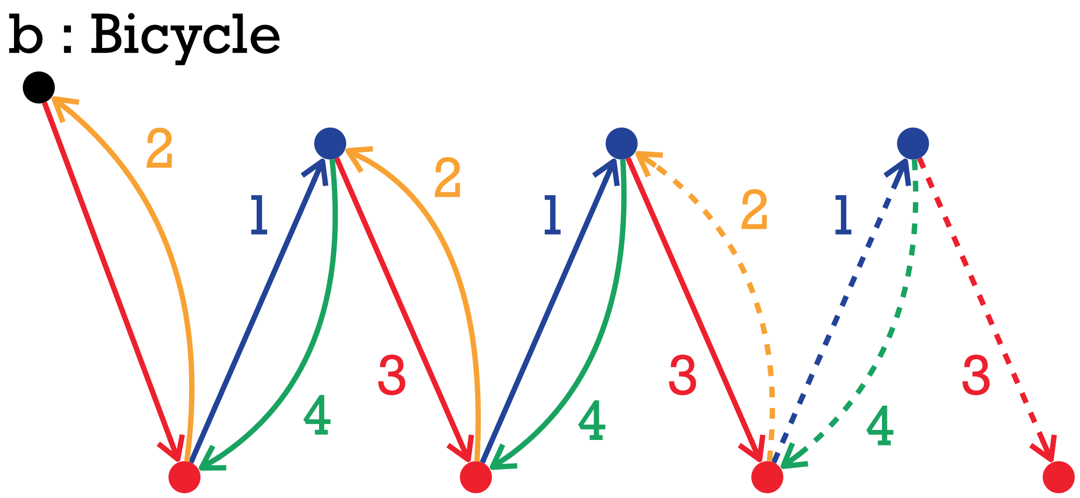

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b : Bicycle

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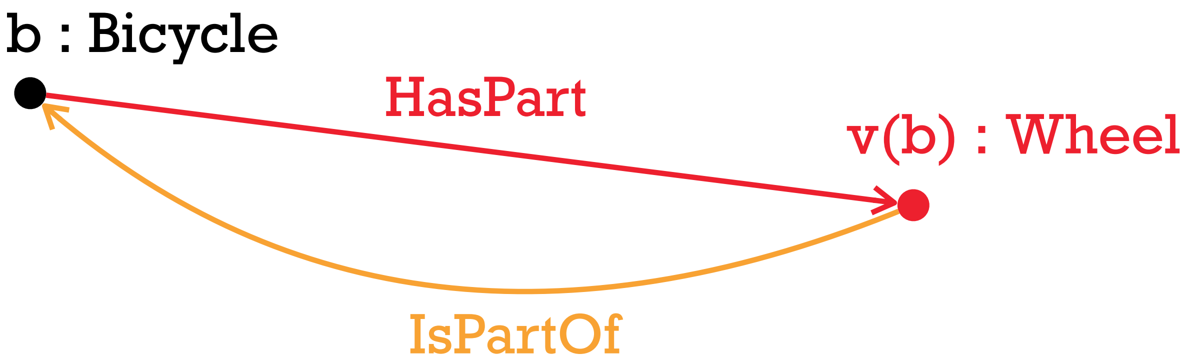

Acyclicity Notions

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Restricted Chase (Non)Termination for Existential Rules with Disjunctions

David Carral, Irina Dragoste, and Markus Krötzsch

[IJCAI 2017]

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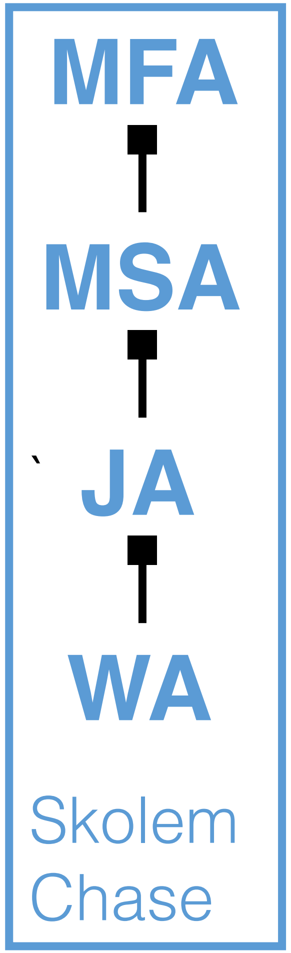

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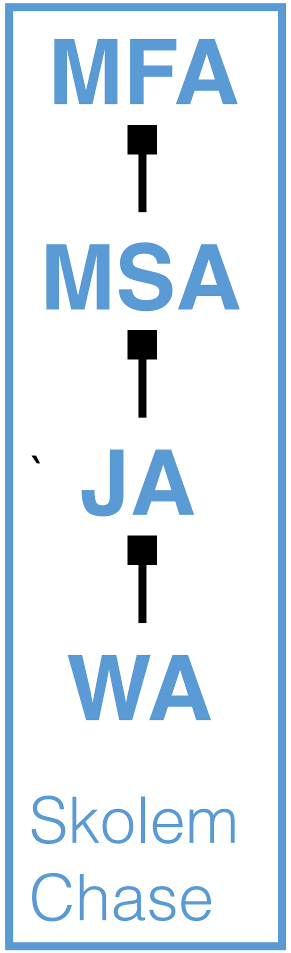

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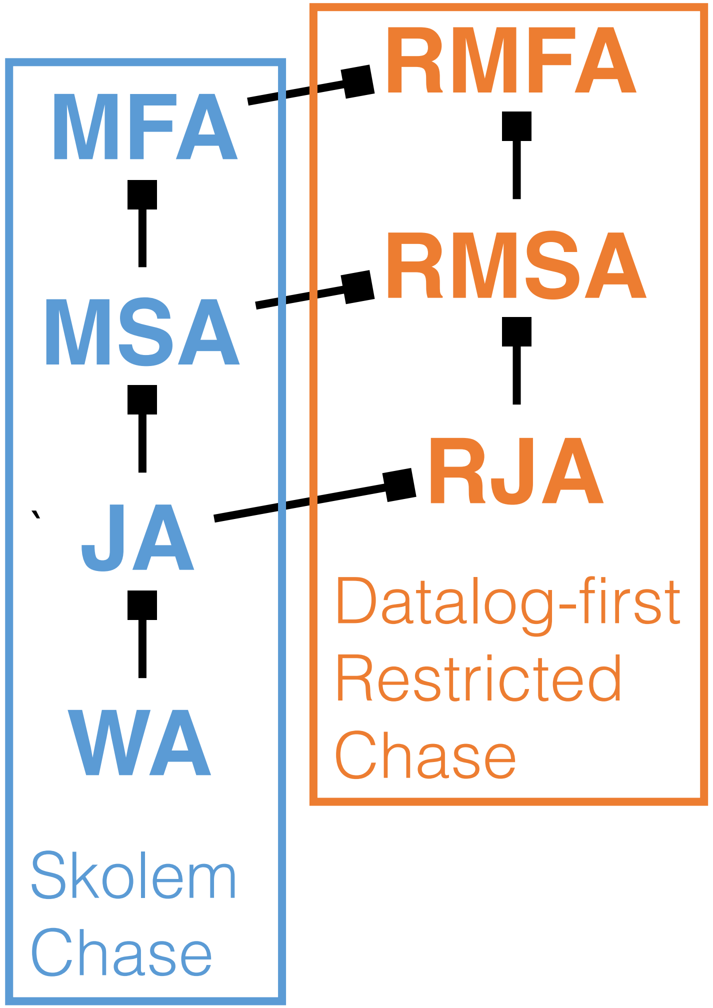

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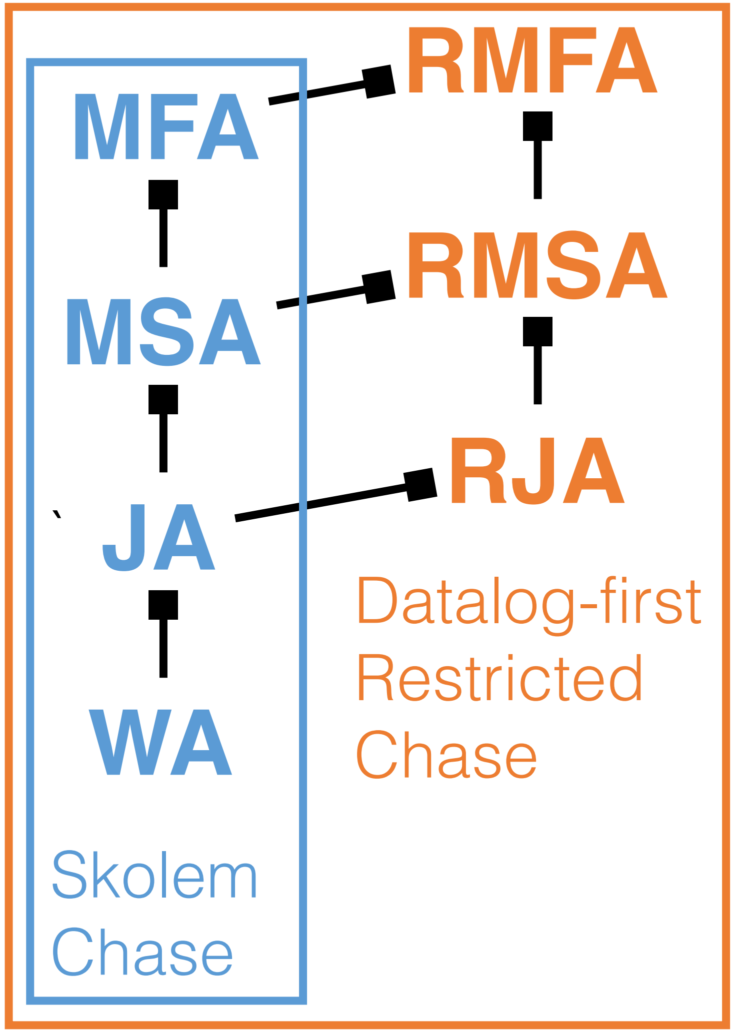

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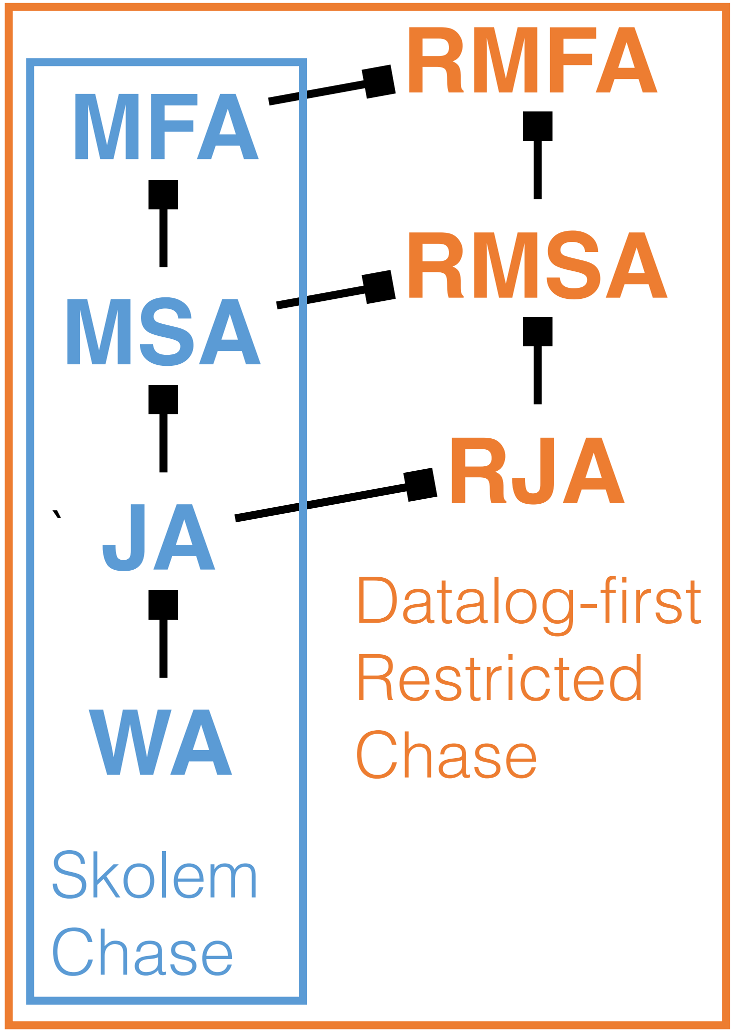

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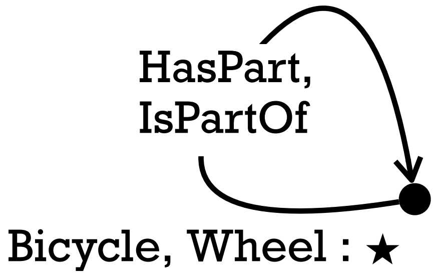

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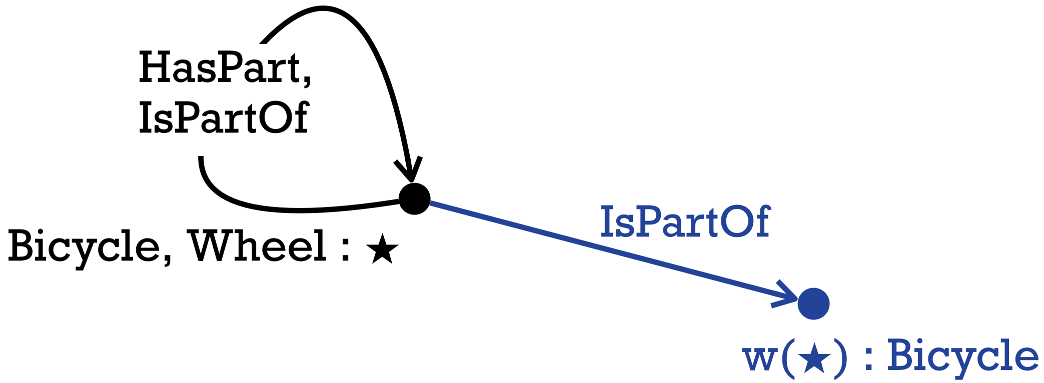

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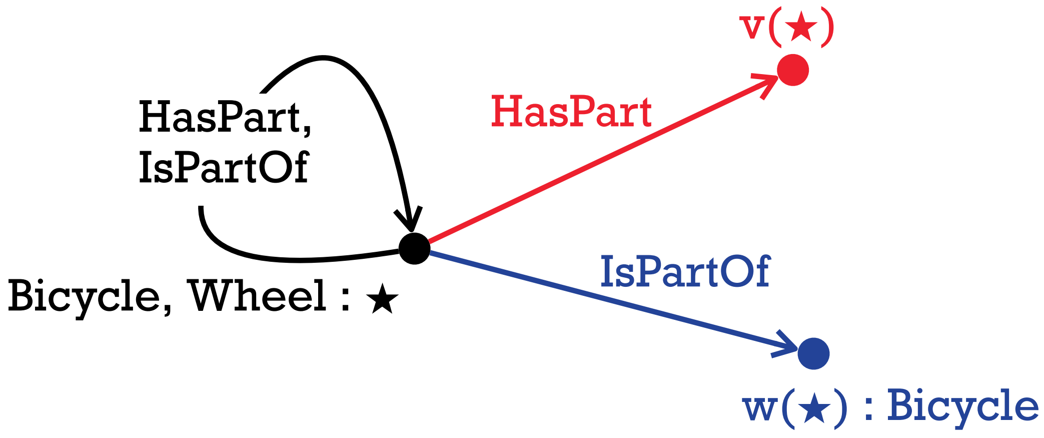

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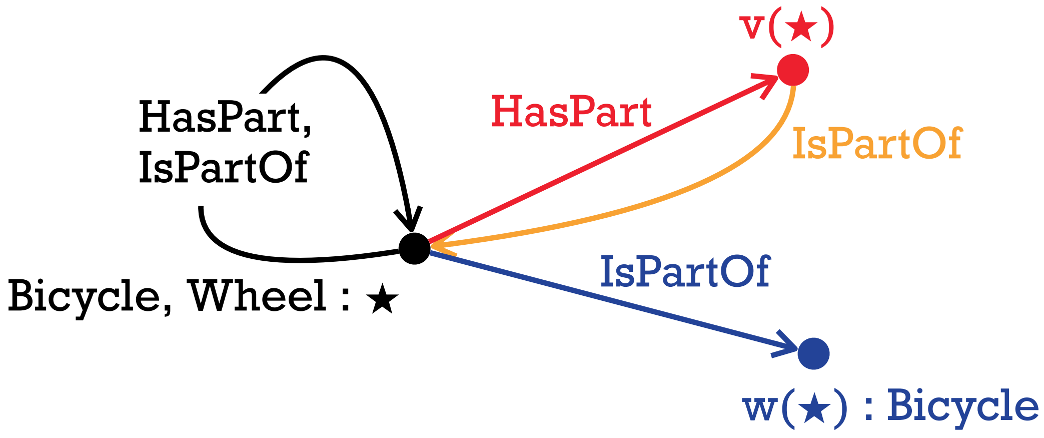

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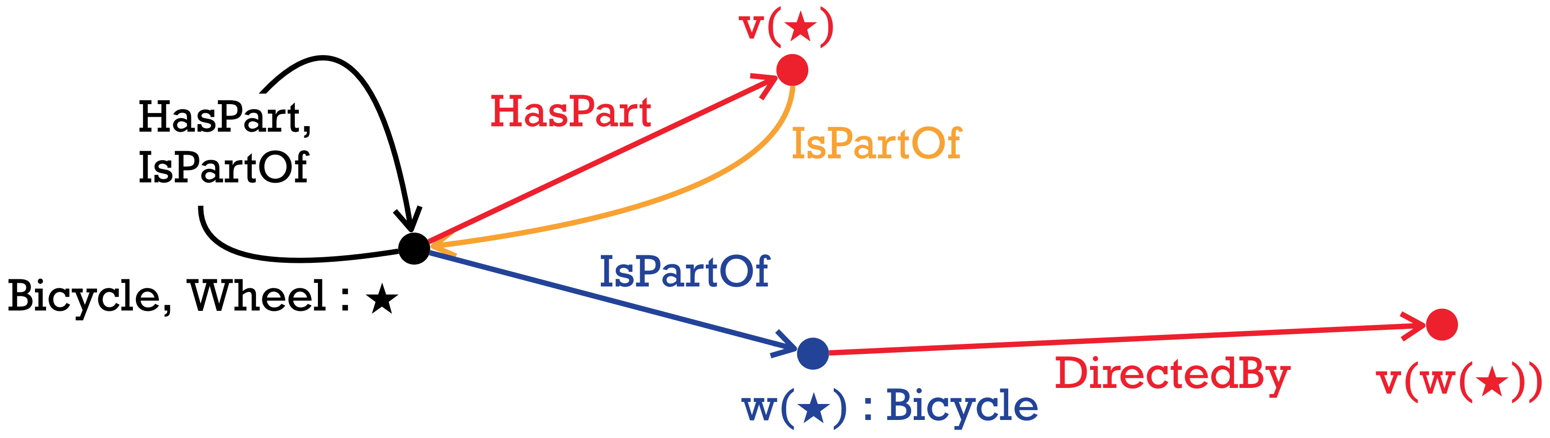

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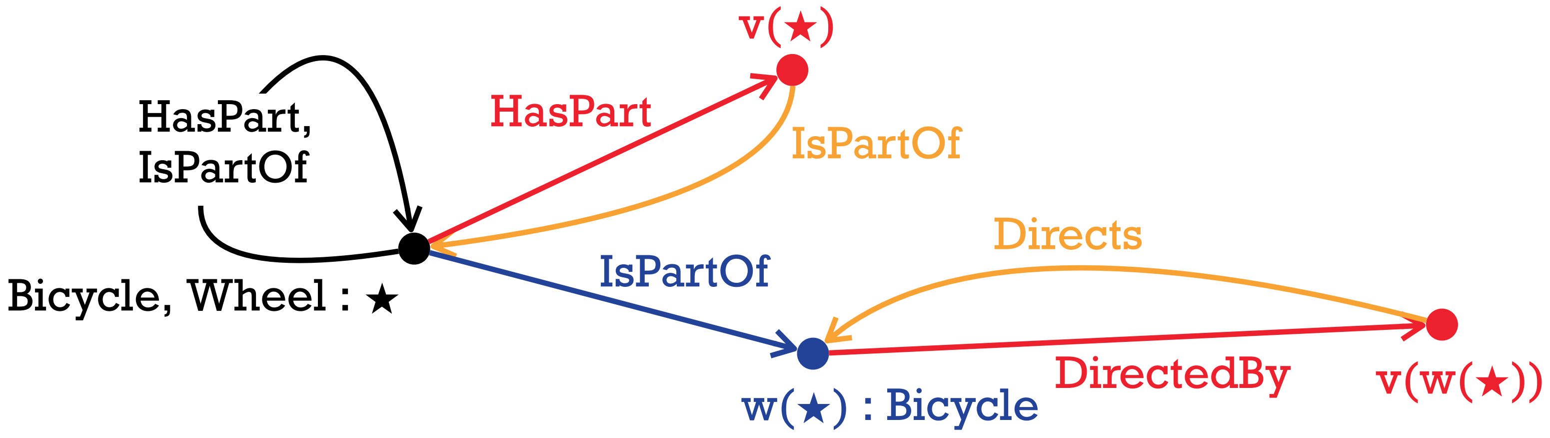

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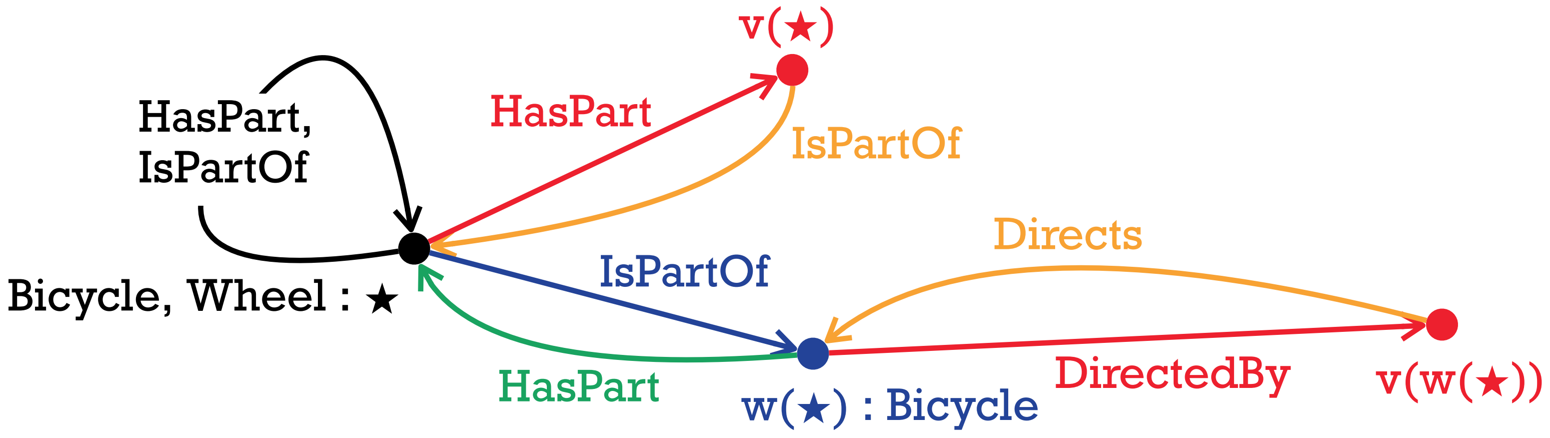

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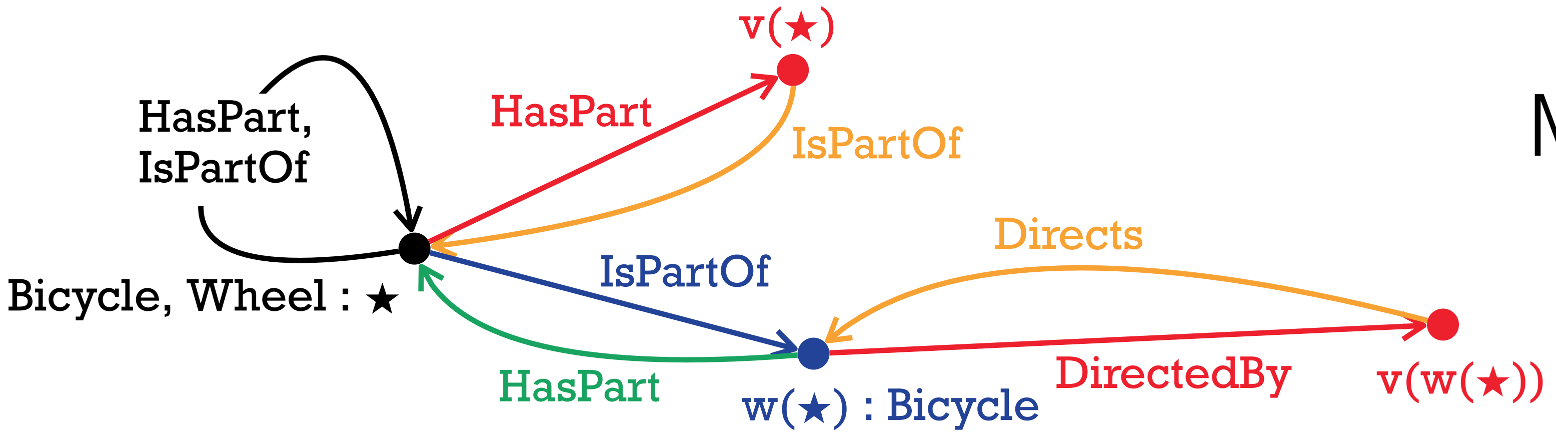

MFA

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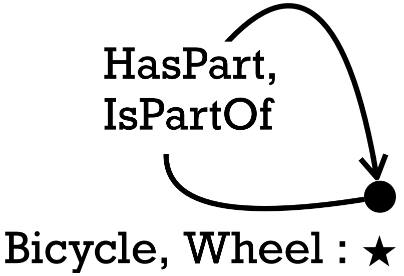

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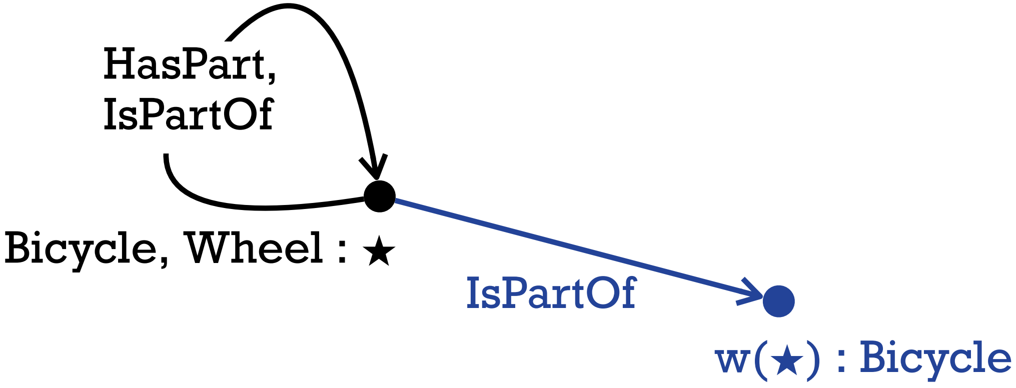

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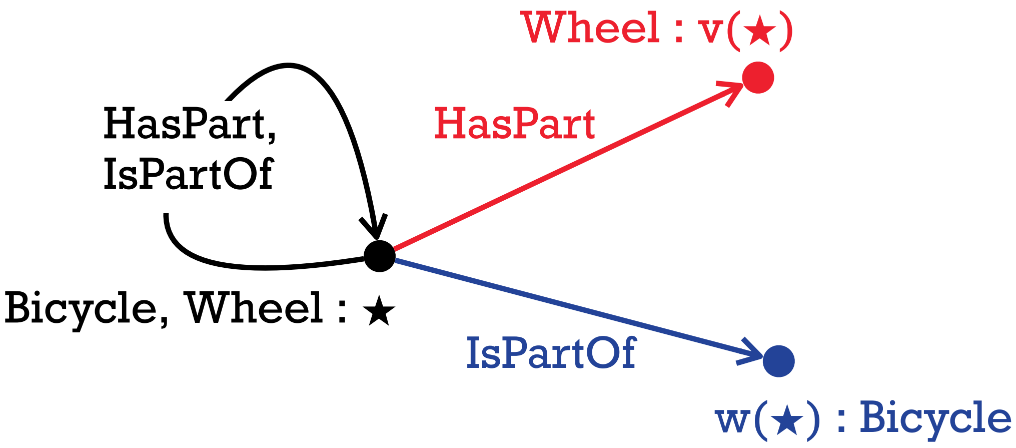

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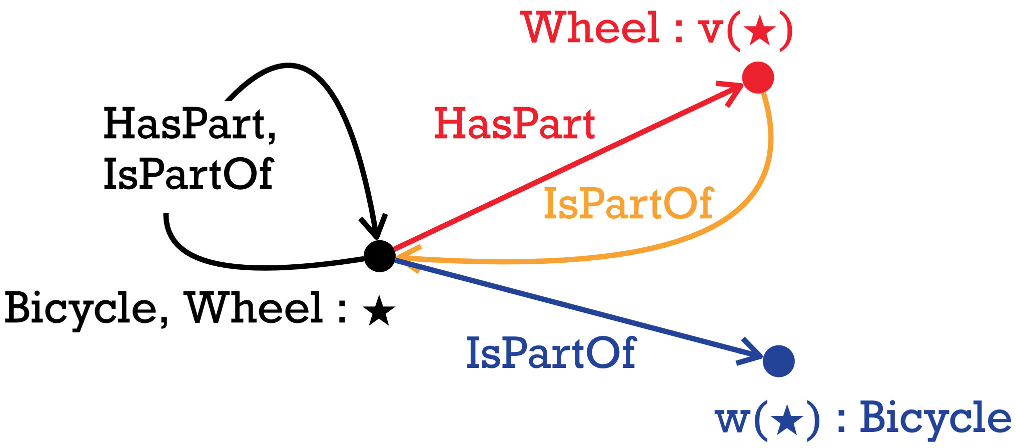

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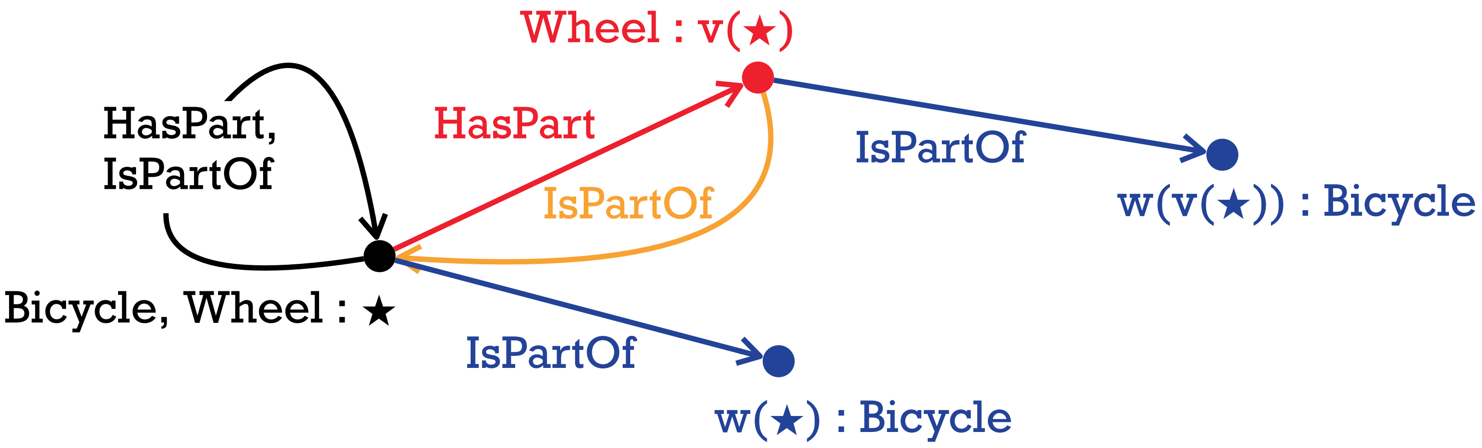

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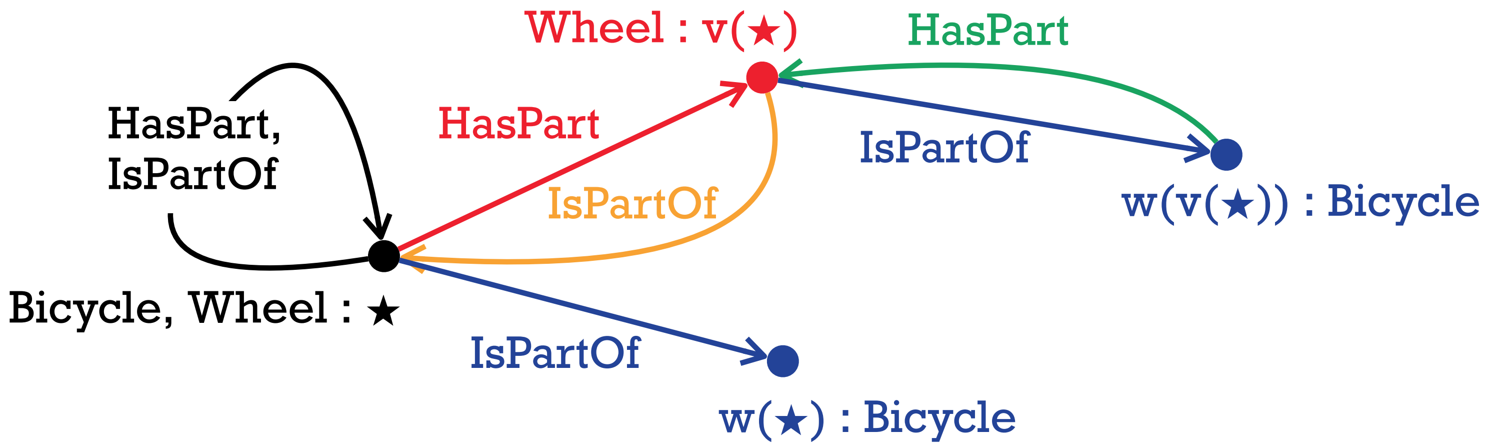

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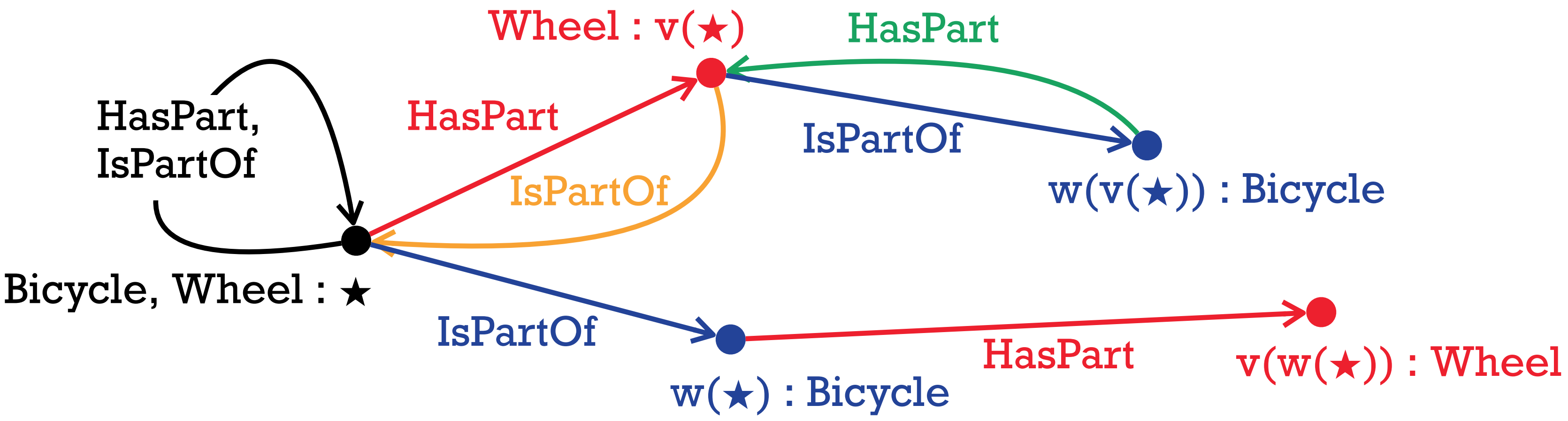

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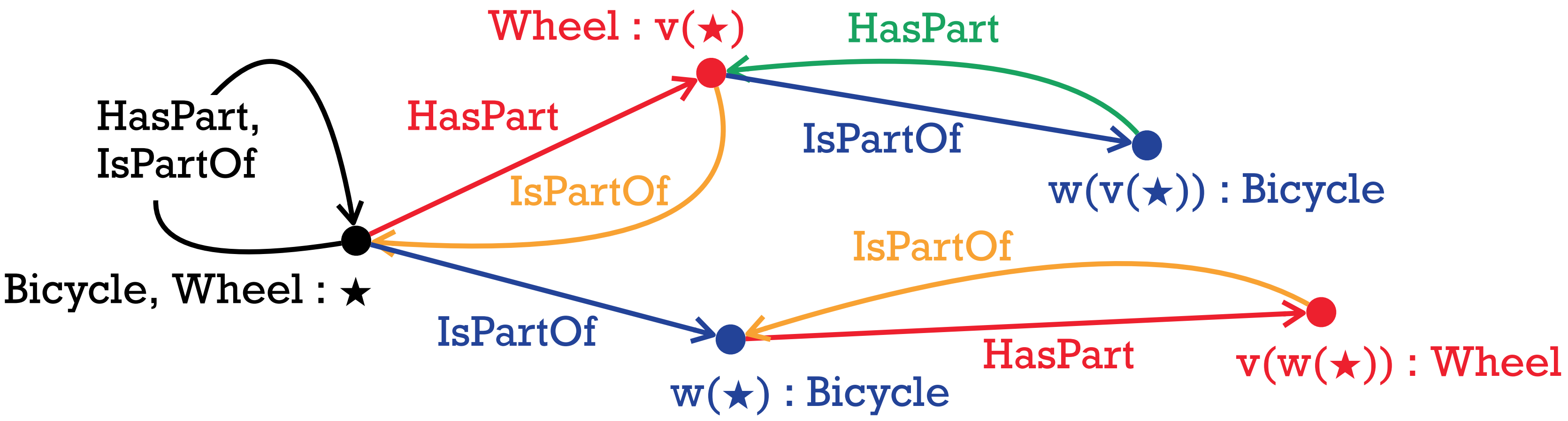

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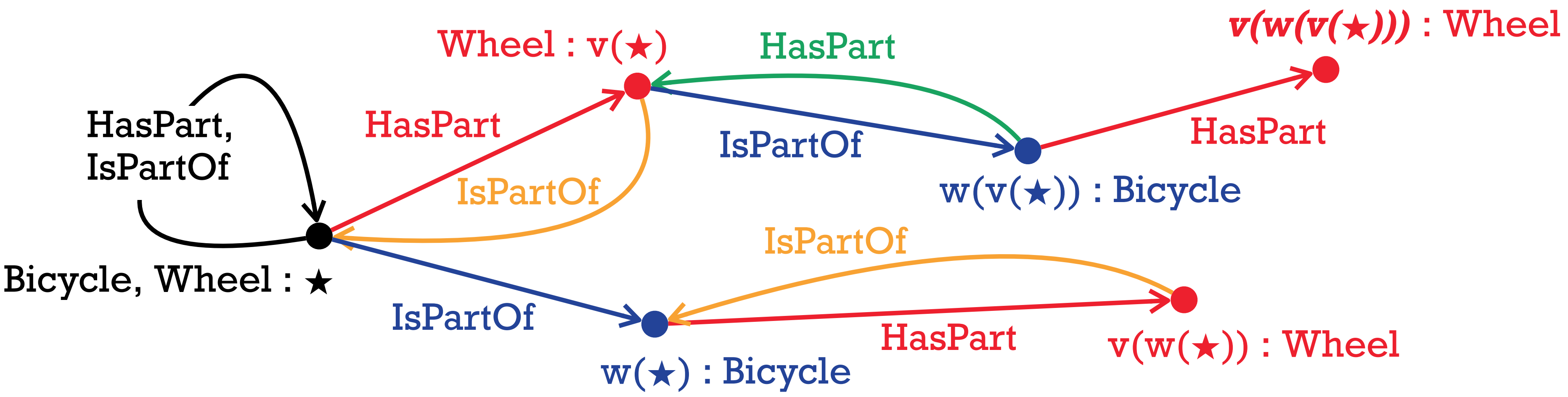

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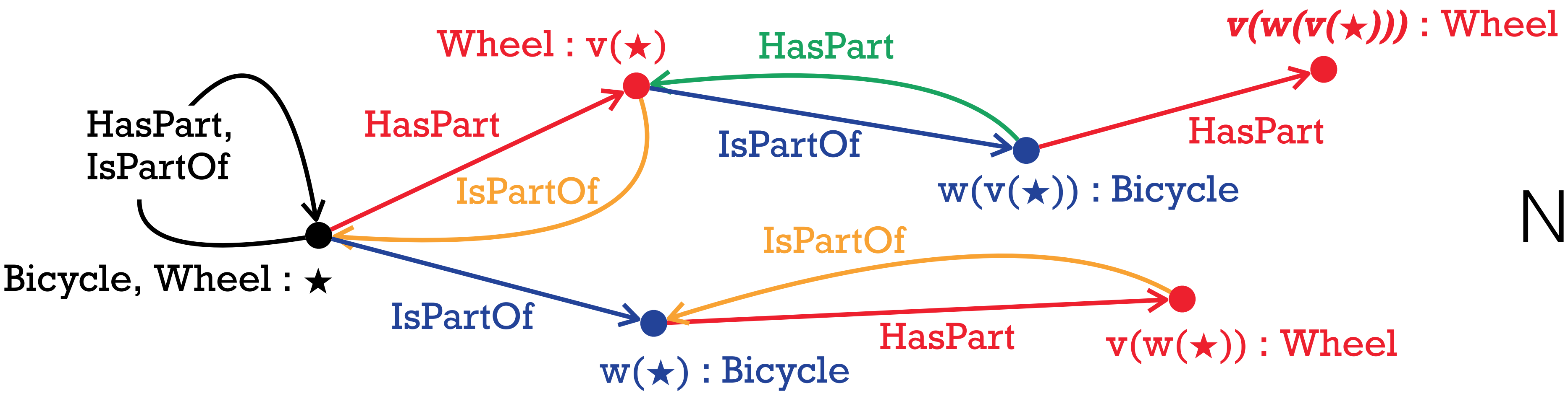

Not MFA!

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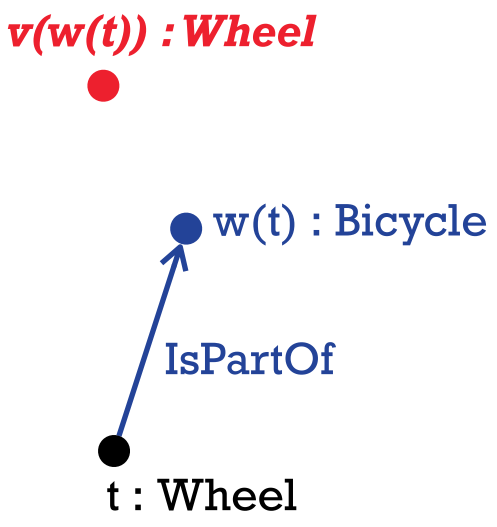

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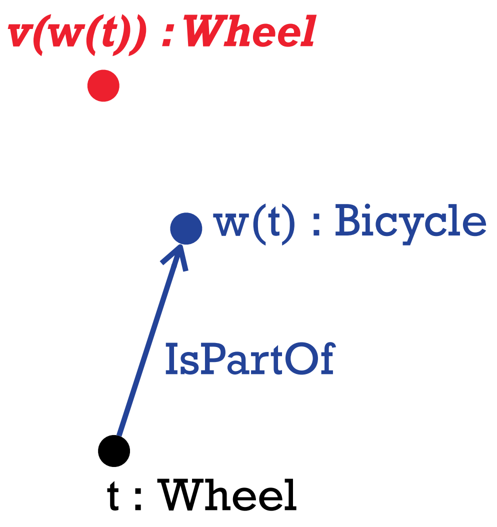

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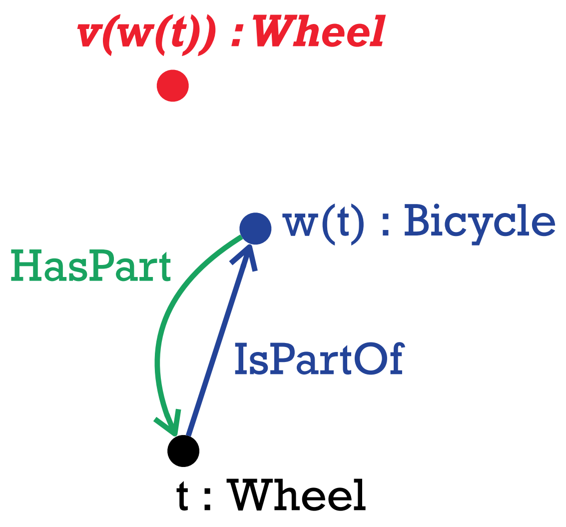

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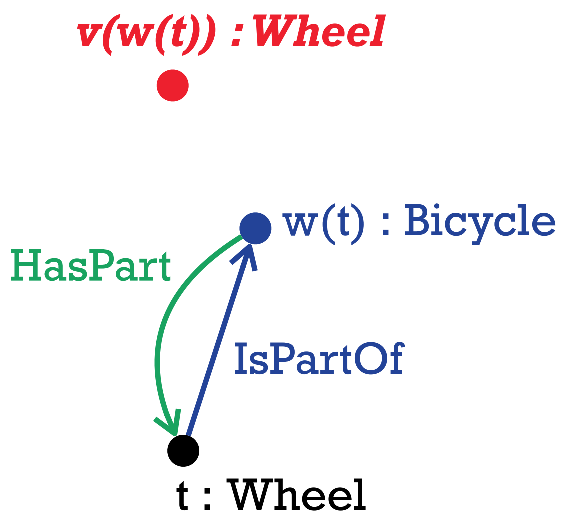
- Because the **green rule** is Datalog, $\mathrm{DirectedBy}(v(t), t)$ is also part of the chase.
- The **red rule** may not be applied to introduce $\mathrm{Director}(v(w(t)))$ since its application with respect to the substitution $\{x\ /\ w(t)\}$ is restricted.



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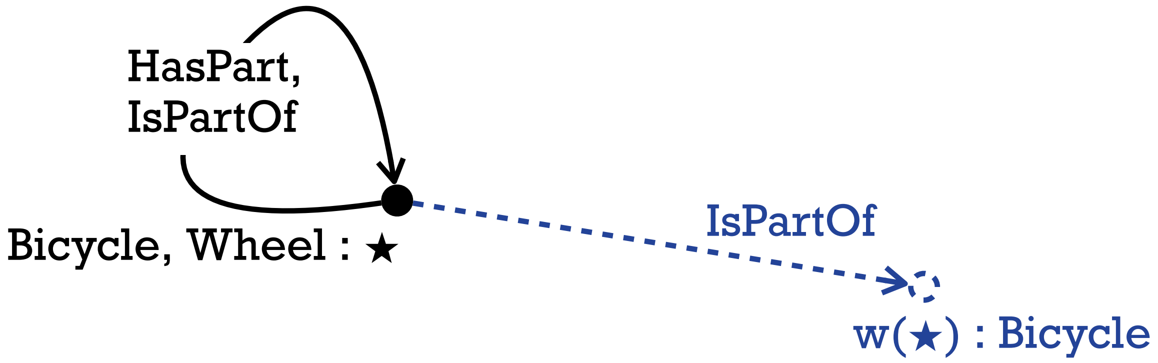

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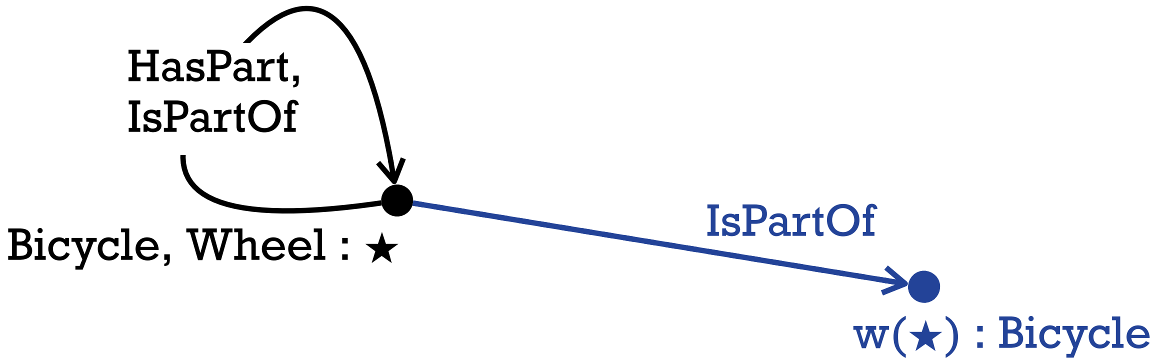

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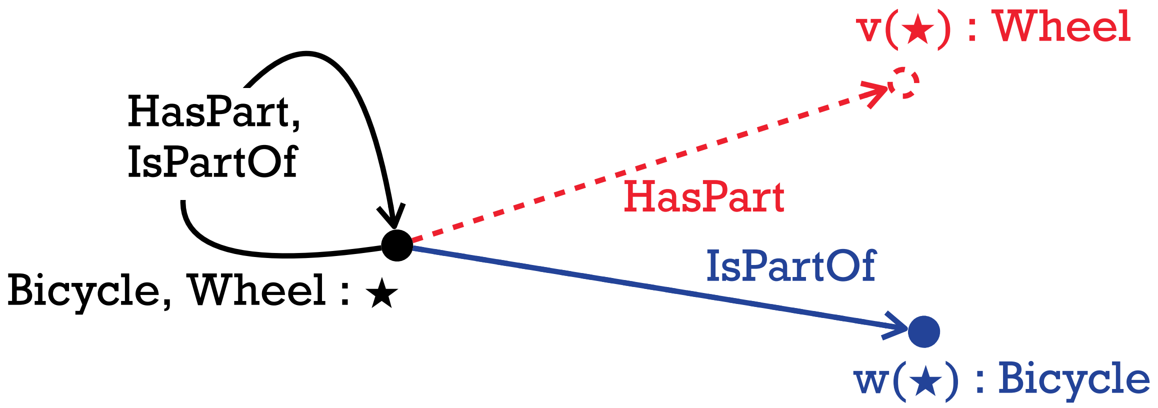

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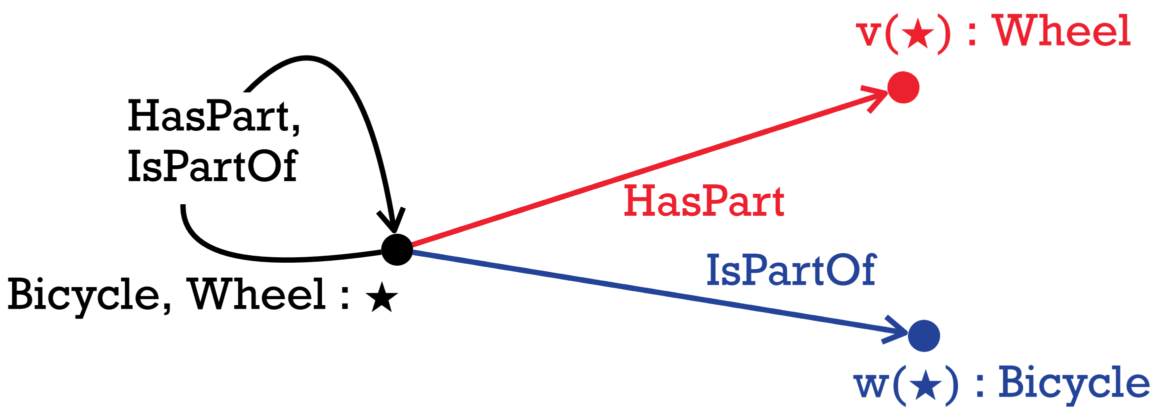

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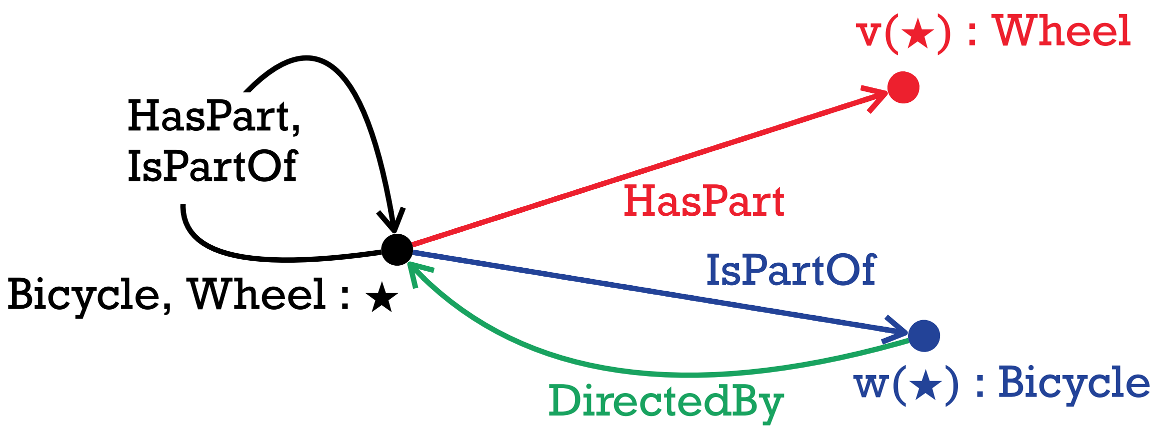

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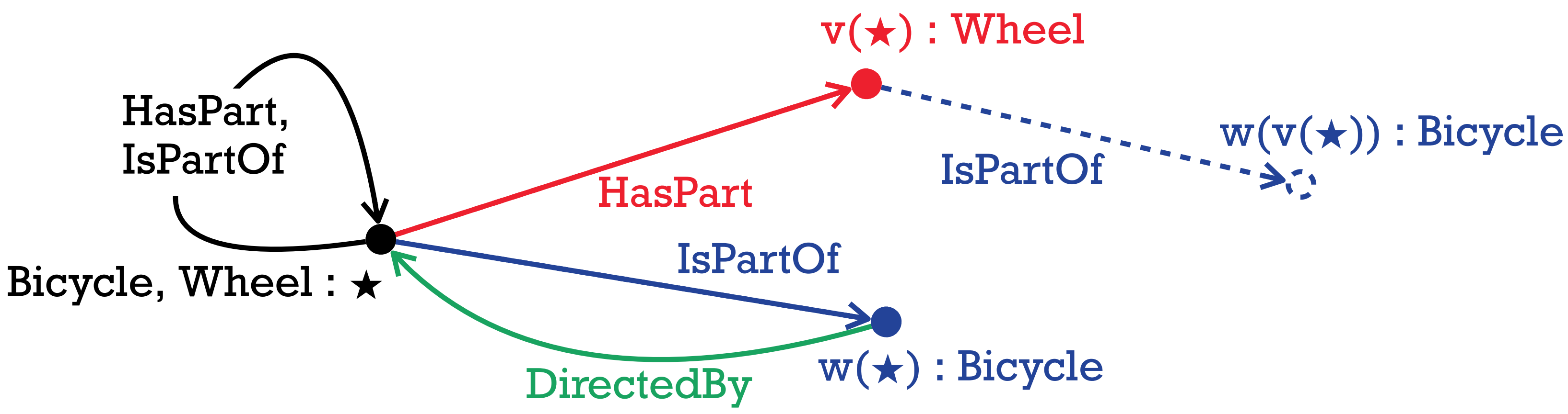

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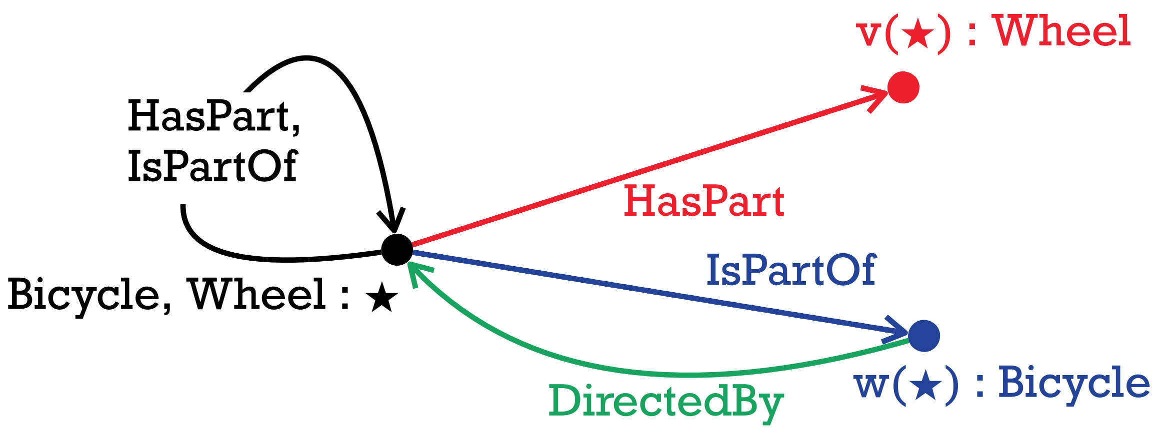

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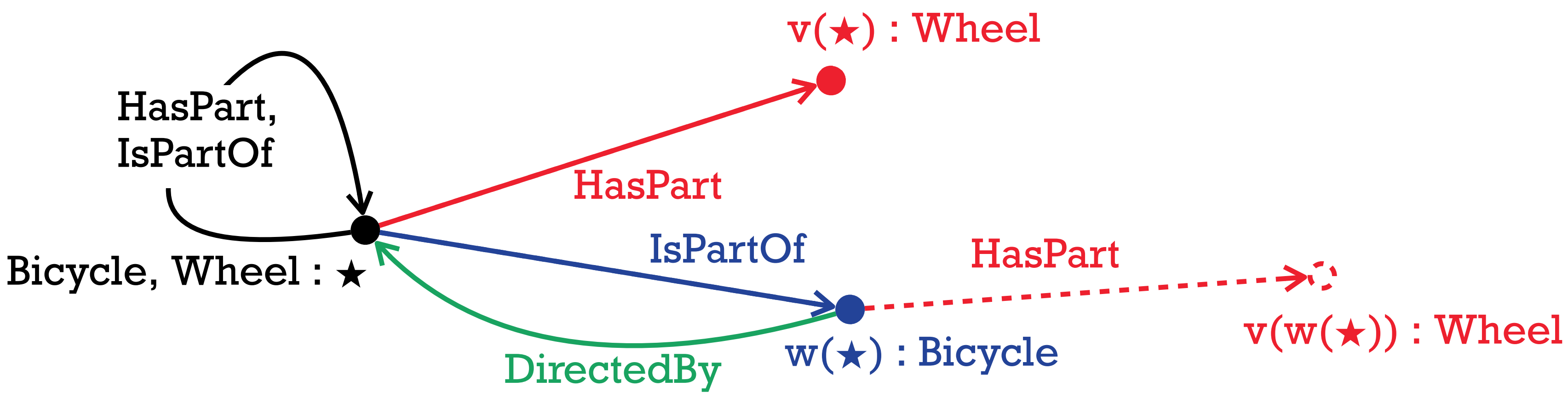

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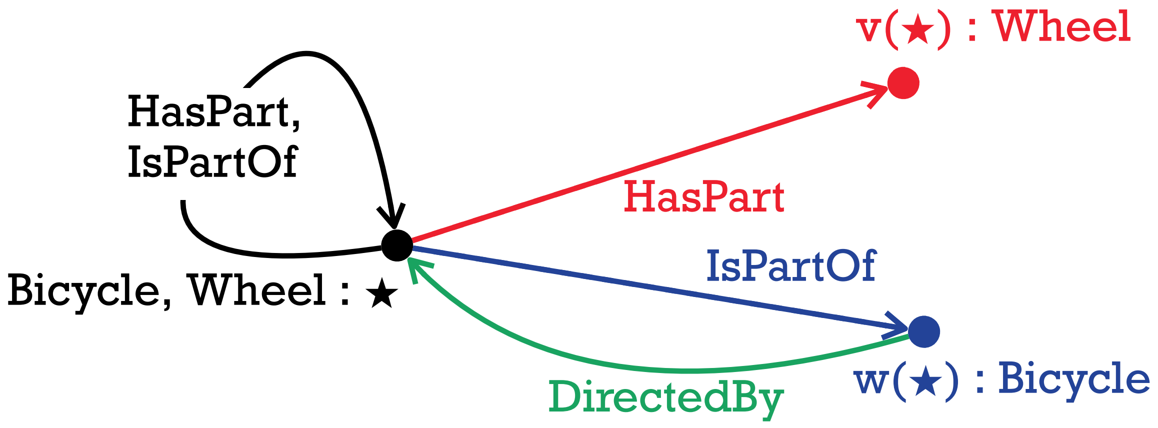

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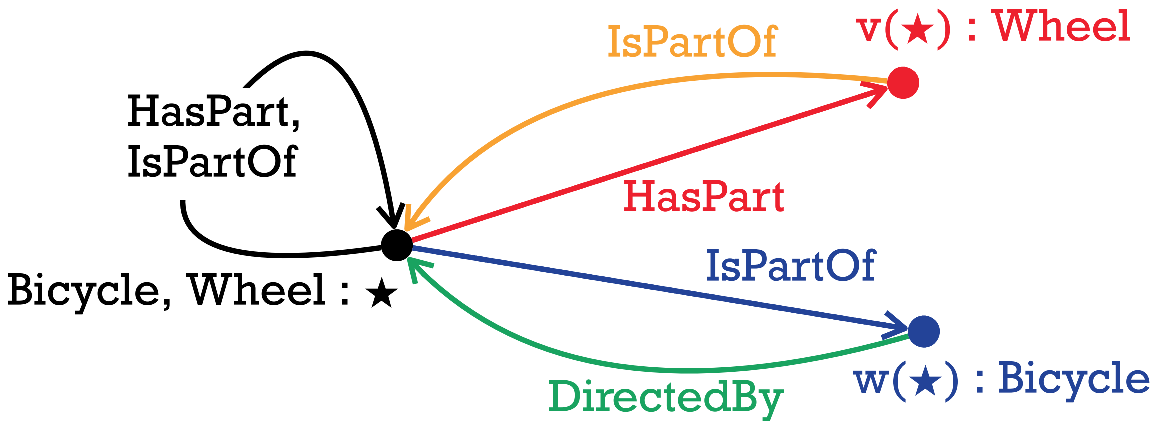

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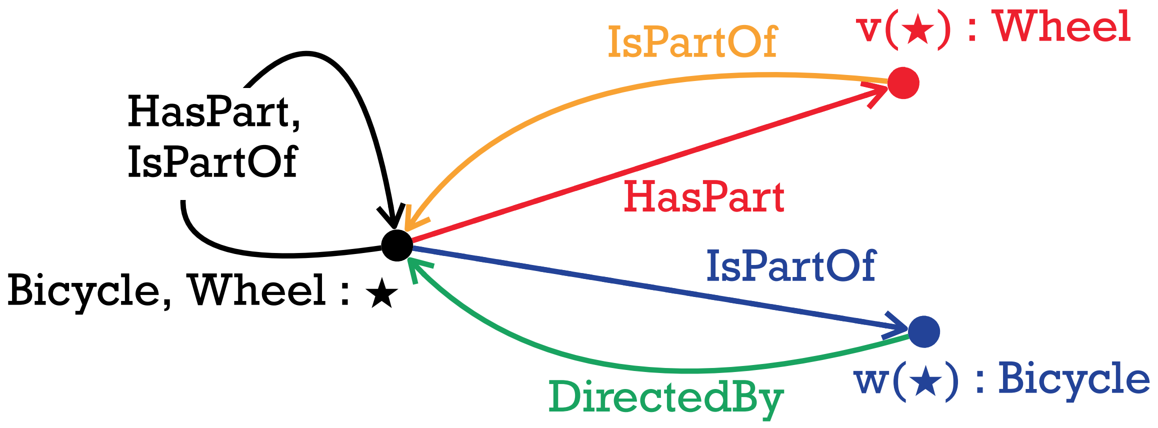

RMFA

Real-world Coverage

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* We selected all rule sets from the MOWLCorp with less than 1000 rules of the form $A(x) \rightarrow \exists y . R(x, y) \wedge B(y)$

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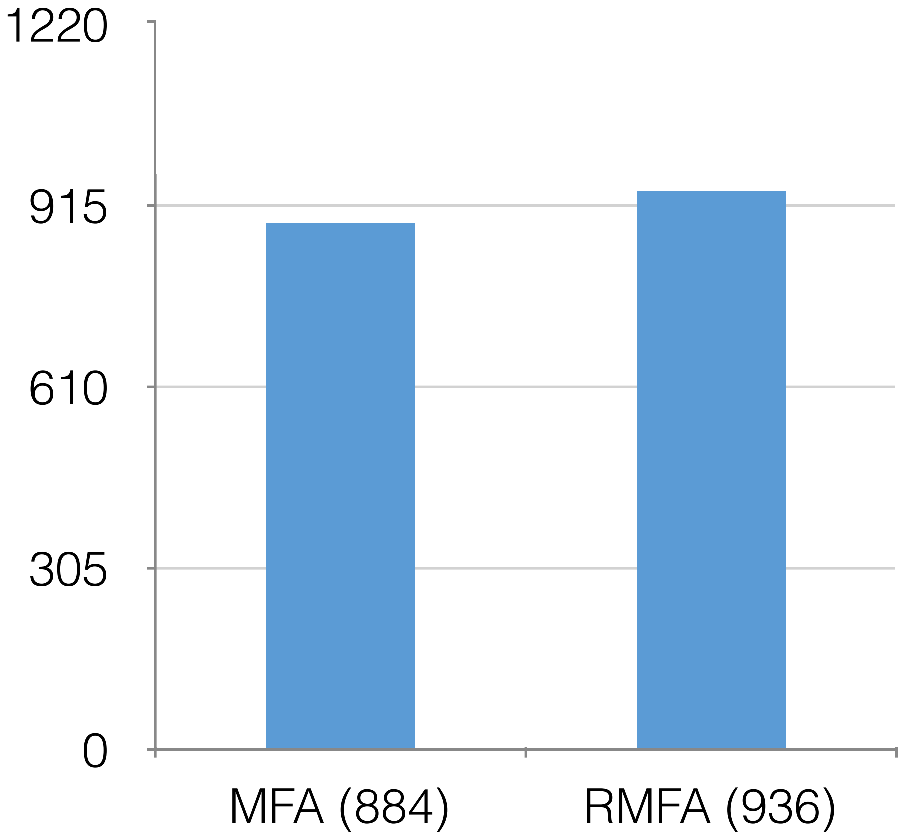

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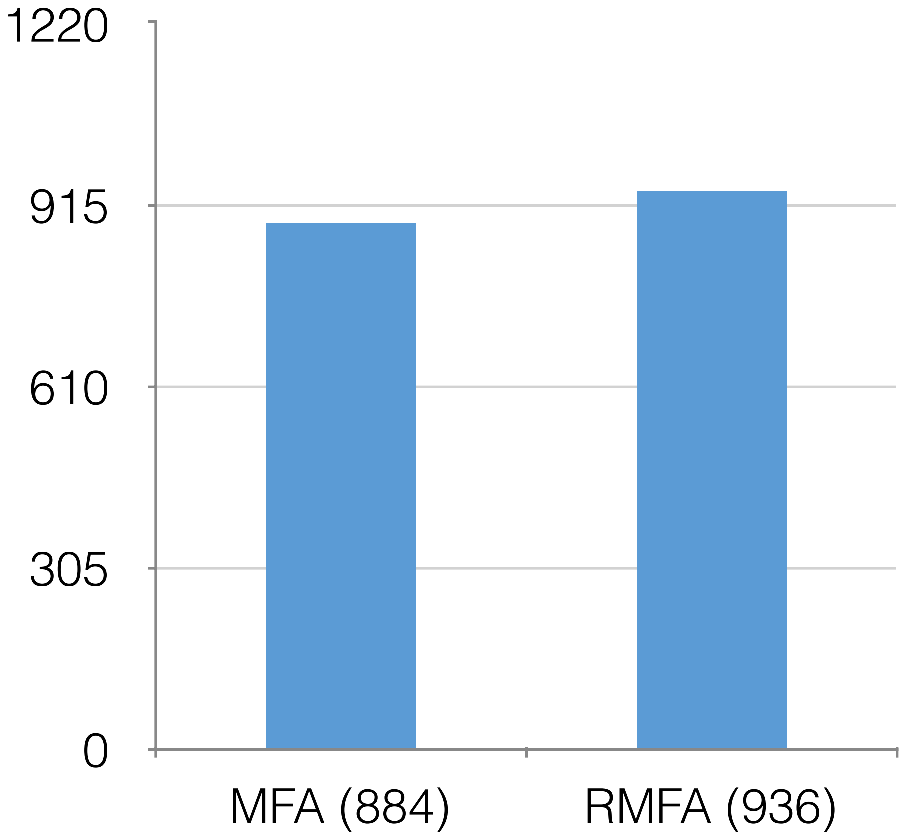

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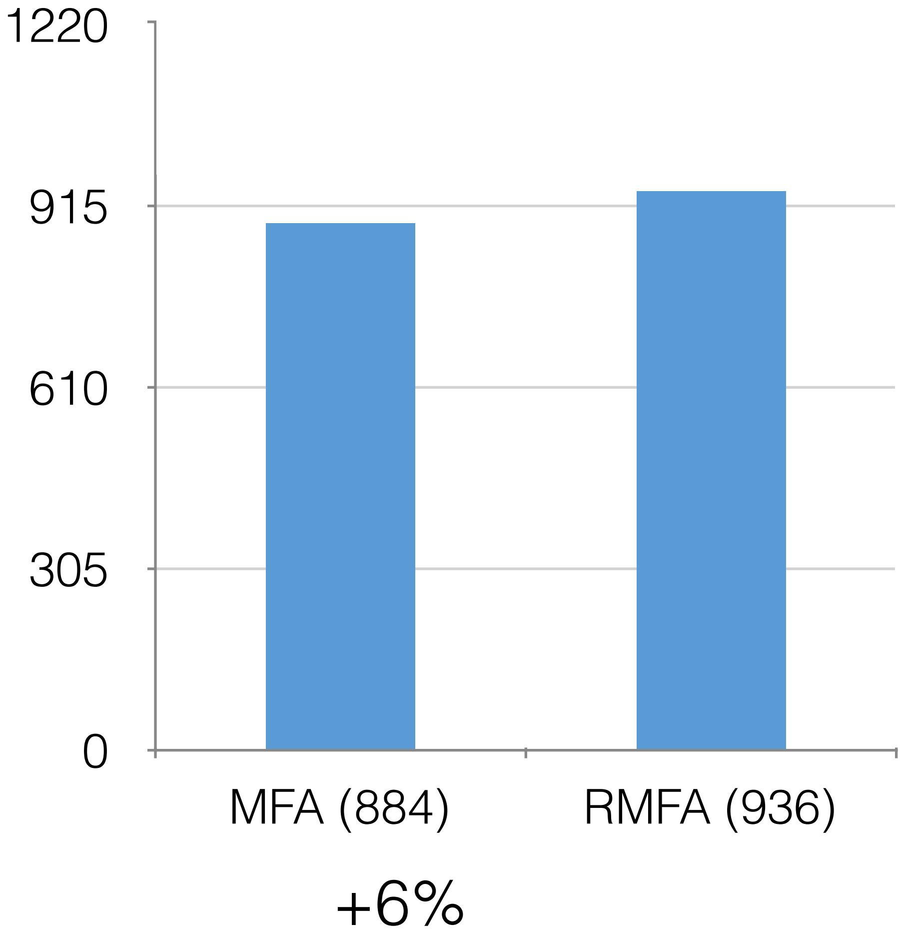

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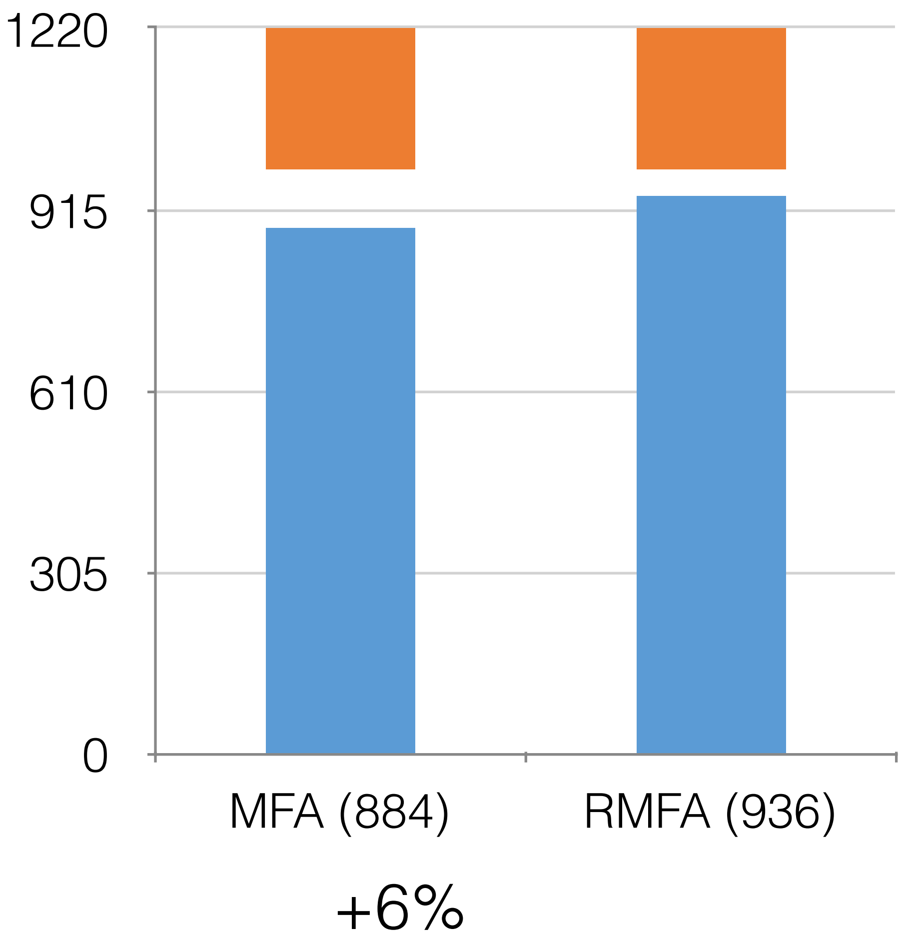

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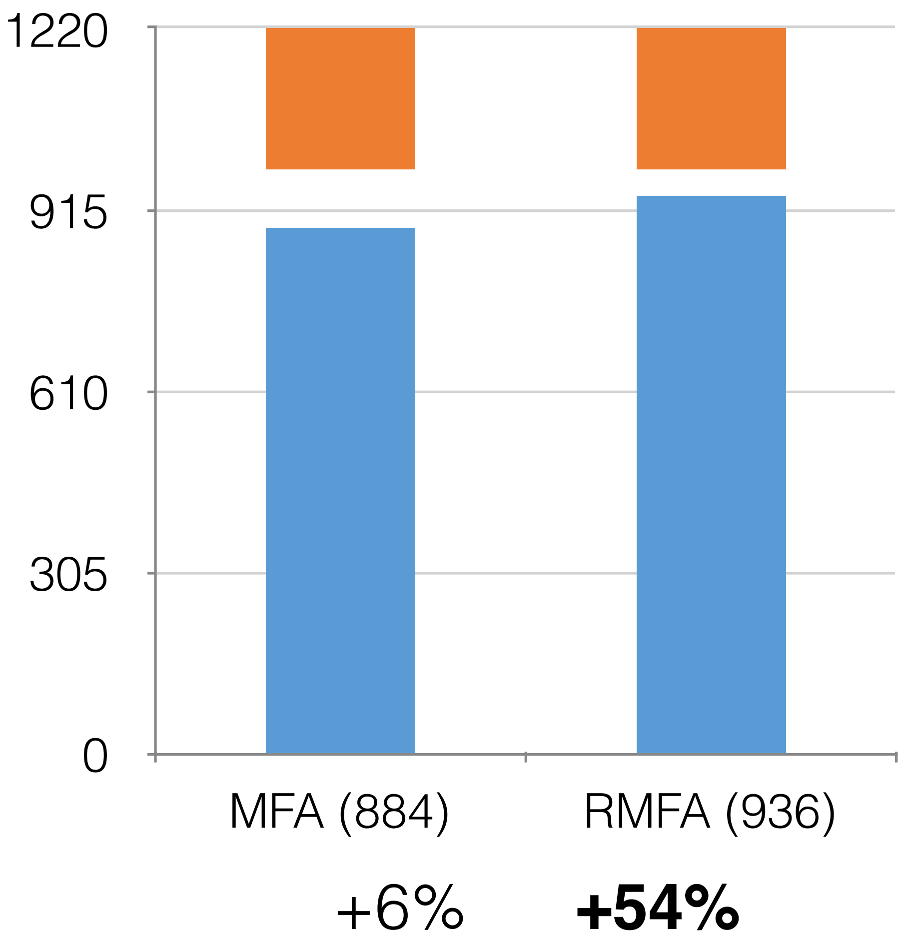

Membership Checks

	RJA	RMSA	RMFA
No restrictions	ExpTime	ExpTime	2-ExpTime
Variables per rule bounded	P	P	2-ExpTime

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BCQ entailment: 2-ExpTime

Ensuring Tractability of the Chase

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Tractable Query Answering for Expressive Ontologies and Existential Rules

David Carral, Irina Dragoste, and Markus Krötzsch

[ISWC 2017]

Existential Dependency Graph

$$\mathsf{A}(x) \to \exists y \,.\, \mathsf{S}(x, y) \wedge \mathsf{B}(y)$$

$$\mathsf{B}(x) \to \exists z \,.\, \mathsf{R}(x, z) \wedge \mathsf{D}(z)$$

$$\mathsf{D}(x) \to \mathsf{E}(x)$$

$$\mathsf{E}(x) \to \exists w \,.\, \mathsf{R}(x, w)$$

$$\mathsf{B}(x) \wedge \mathsf{C}(x) \to \mathsf{E}(x)$$

$$\mathsf{S}(x, y) \to \mathsf{C}(x)$$

Existential Dependency Graph

$$\mathsf{A}(x) \to \mathsf{S}(x, y(x)) \wedge \mathsf{B}(y(x))$$

$$\mathsf{B}(x) \to \mathsf{R}(x, z(x)) \wedge \mathsf{D}(z(x))$$

$$\mathsf{D}(x) \to \mathsf{E}(x)$$

$$\mathsf{E}(x) \to \mathsf{R}(x, w(x))$$

$$\mathsf{B}(x) \wedge \mathsf{C}(x) \to \mathsf{E}(x)$$

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$$\mathsf{A}(x) \to \mathsf{S}(x, y(x)) \land \mathsf{B}(y(x))$$
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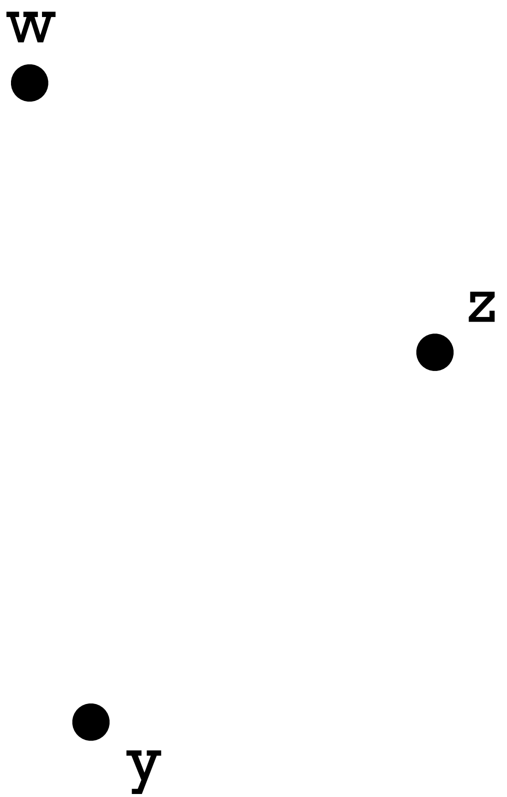


Existential Dependency Graph

$$A(x) \rightarrow S(x, y(x)) \wedge B(y(x))$$

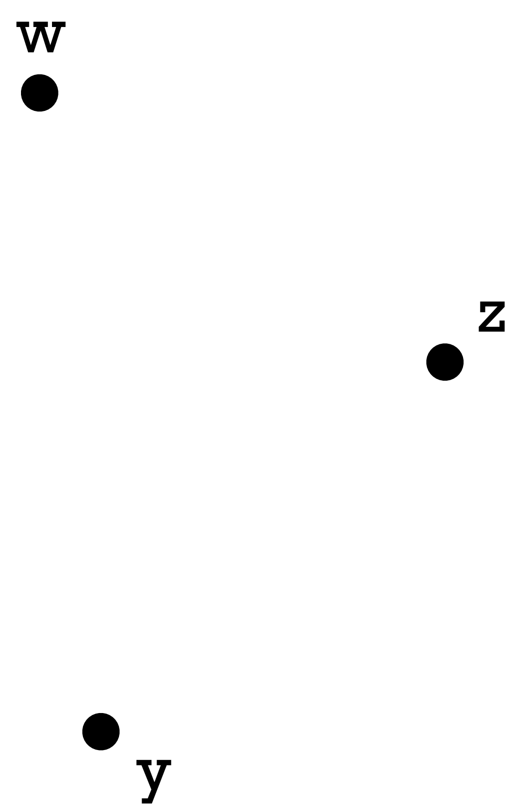
$$B(x) \rightarrow R(x, z(x)) \wedge D(z(x))$$

$$D(x) \rightarrow E(x)$$

$$E(x) \rightarrow R(x, w(x))$$

$$B(x) \wedge C(x) \rightarrow E(x)$$

$$S(x, y) \rightarrow C(x)$$



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$$\mathsf{D}(x) \to \mathsf{E}(x)$$

$$\mathsf{E}(x) \to \mathsf{R}(x, w(x))$$

$$\mathsf{B}(x) \land \mathsf{C}(x) \to \mathsf{E}(x)$$

$$\mathsf{S}(x, y) \to \mathsf{C}(x)$$

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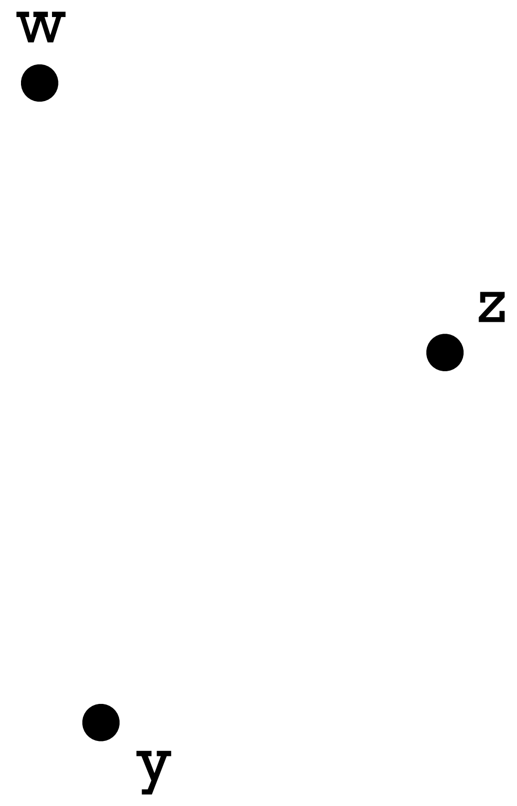
$\mathsf{S}(c, y(c)), B(y(c))$

$\mathsf{R}(y(c), z(y(c))), D(z(y(c)))$



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$$\mathsf{D}(x) \to \mathsf{E}(x)$$

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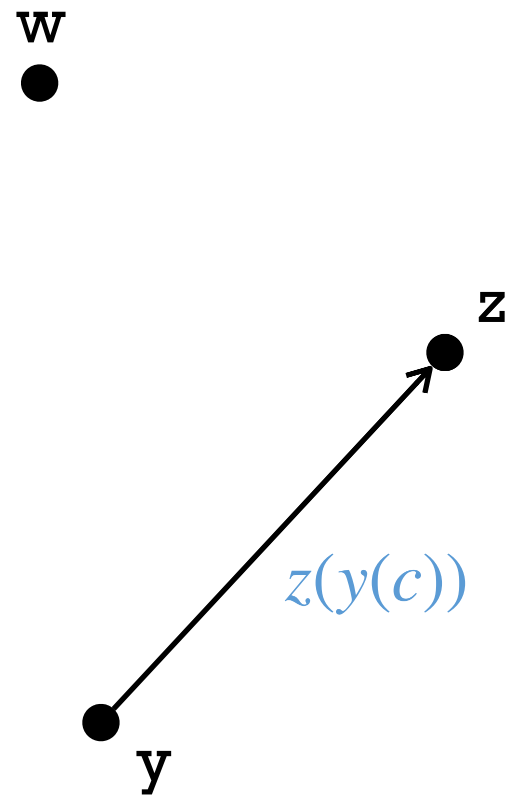

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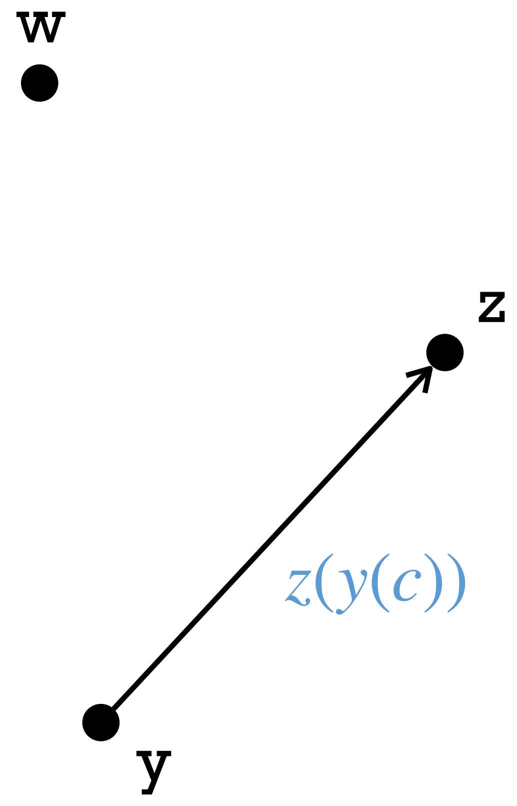


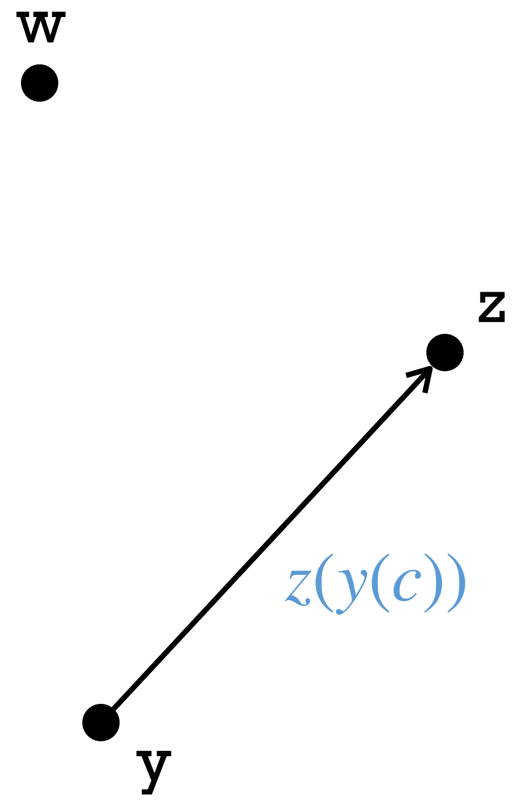
Existential Dependency Graph

$$A(x) \to S(x, y(x)) \wedge B(y(x))$$

$$\boxed{\begin{array}{l} B(x) \to R(x, z(x)) \wedge D(z(x)) \\ D(x) \to E(x) \\ E(x) \to R(x, w(x)) \end{array}}$$

$$B(x) \wedge C(x) \to E(x)$$

$$S(x, y) \to C(x)$$



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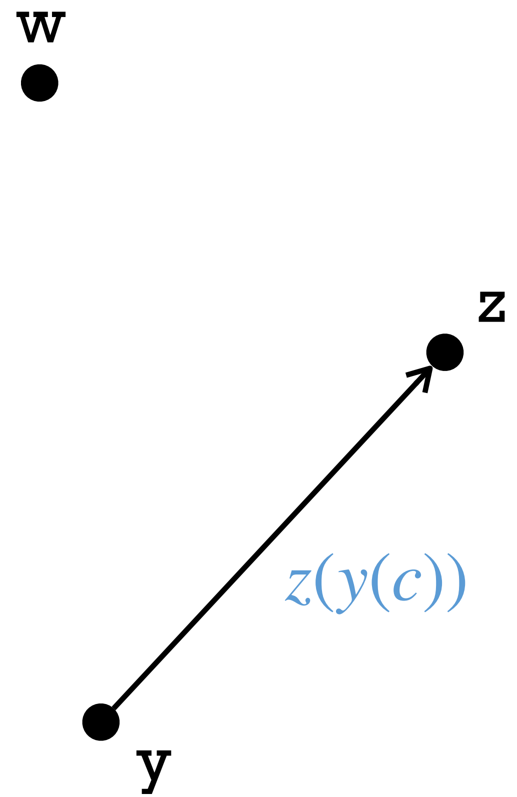
$$S(x, y) \to C(x)$$

$B(c)$

$R(c, z(c)), D(z(c)),$

$E(z(c)),$

$R(z(c), w(z(c)))$



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$$\mathsf{S}(x, y) \to \mathsf{C}(x)$$

$\mathsf{B}(c)$

$\mathsf{R}(c, z(c)), D(z(c)),$

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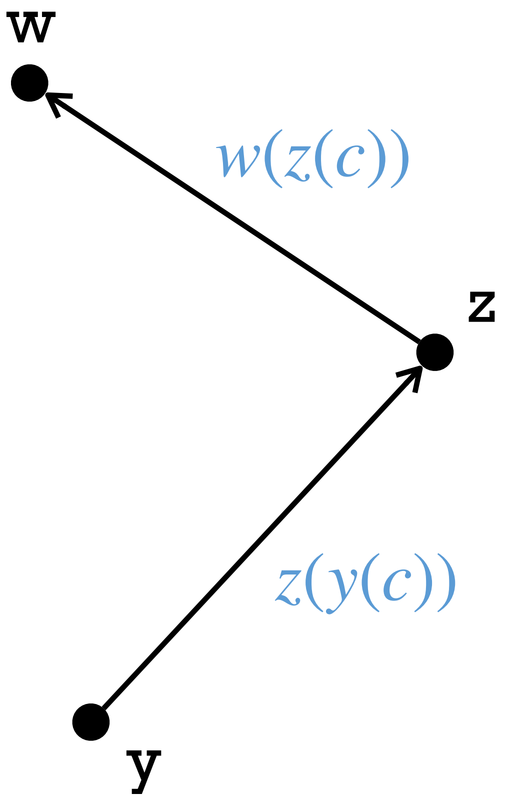

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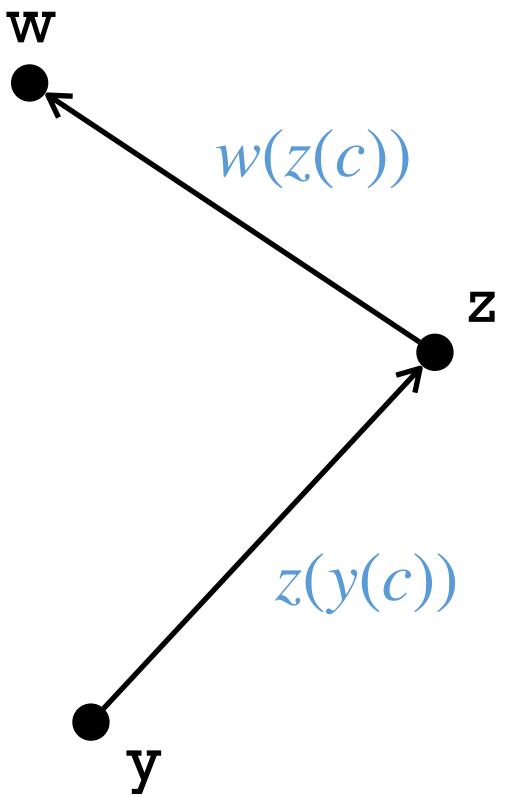

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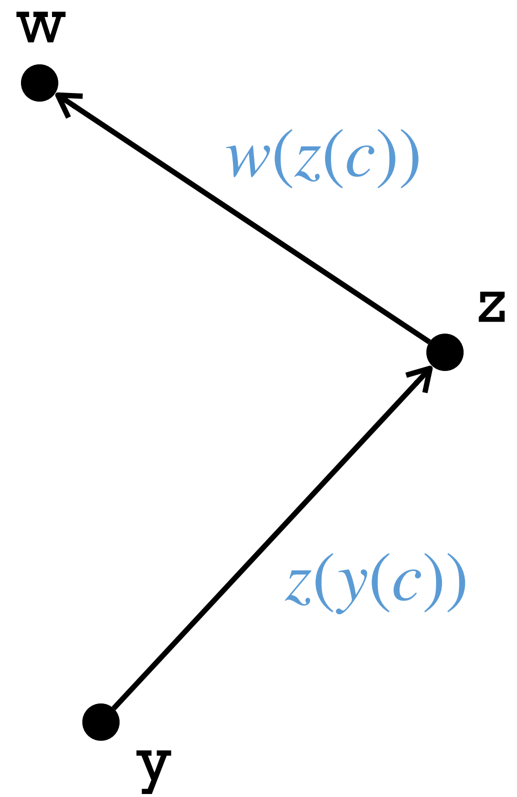
$$\boxed{\begin{array}{rcl} E(x) & \rightarrow & R(x, w(x)) \\ B(x) \wedge C(x) & \rightarrow & E(x) \\ S(x, y) & \rightarrow & C(x) \end{array}}$$



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\mathsf{B}(x) \wedge \mathsf{C}(x) \rightarrow \mathsf{E}(x) \\
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\end{array}}$$

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$\mathsf{C}(y(c)),$

$\mathsf{E}(y(c)),$

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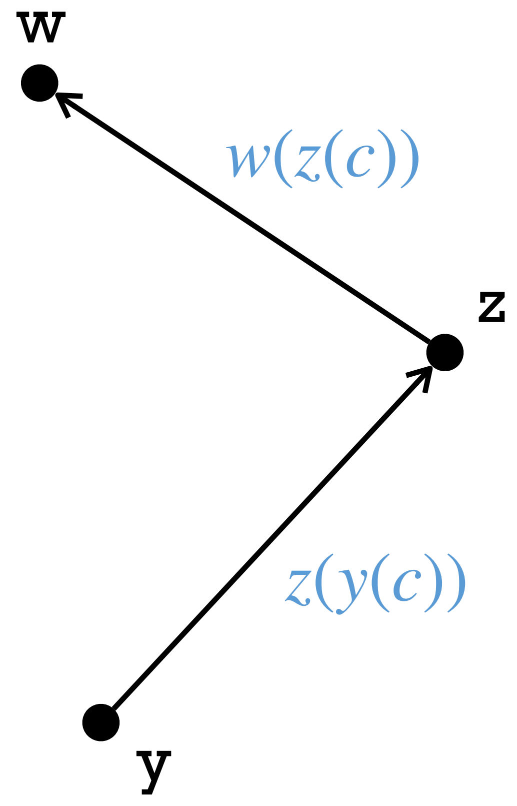

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$\mathsf{E}(y(c)),$

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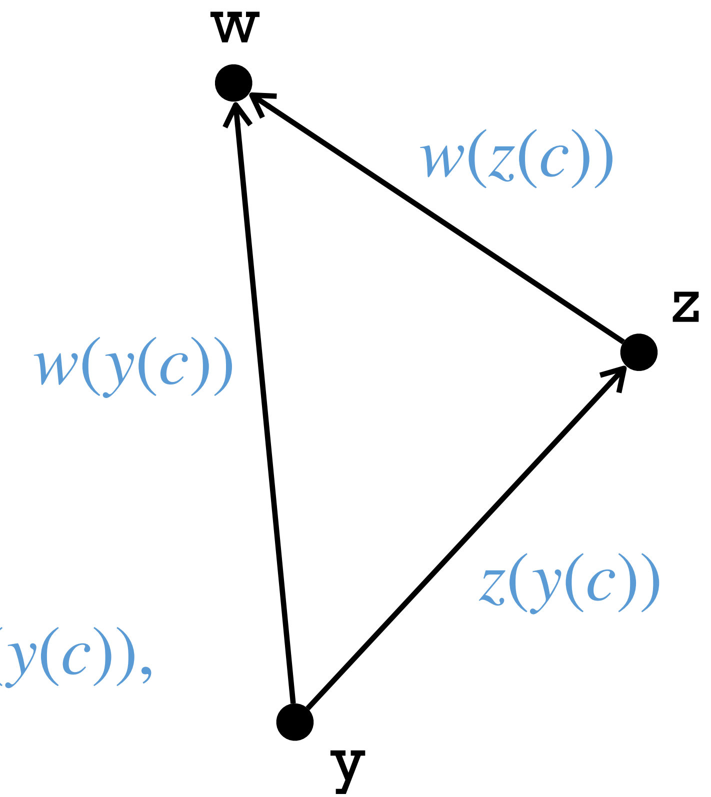

Existential Dependency Graph

$$A(x) \to S(x, y(x)) \wedge B(y(x))$$
$$B(x) \to R(x, z(x)) \wedge D(z(x))$$
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$$E(x) \to R(x, w(x))$$
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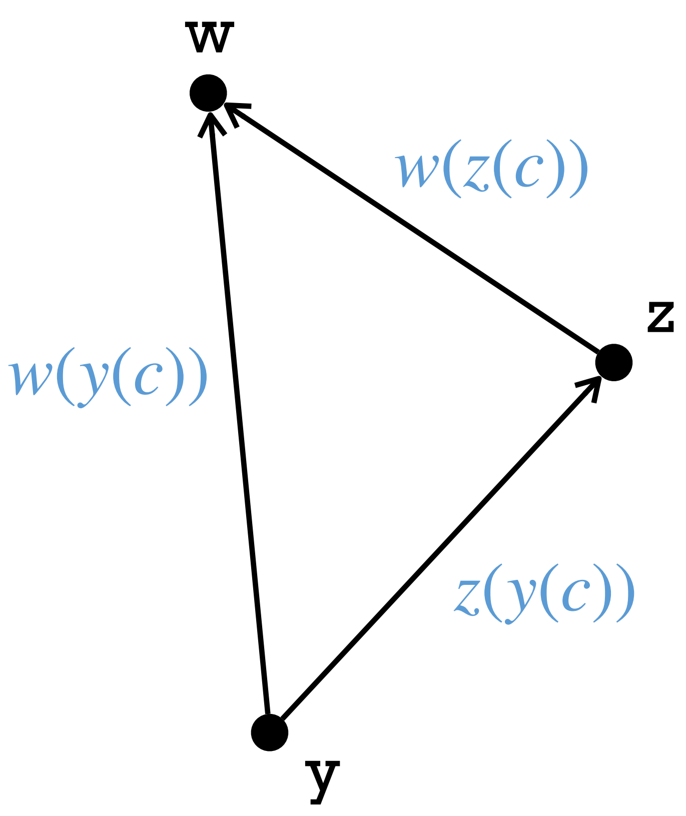

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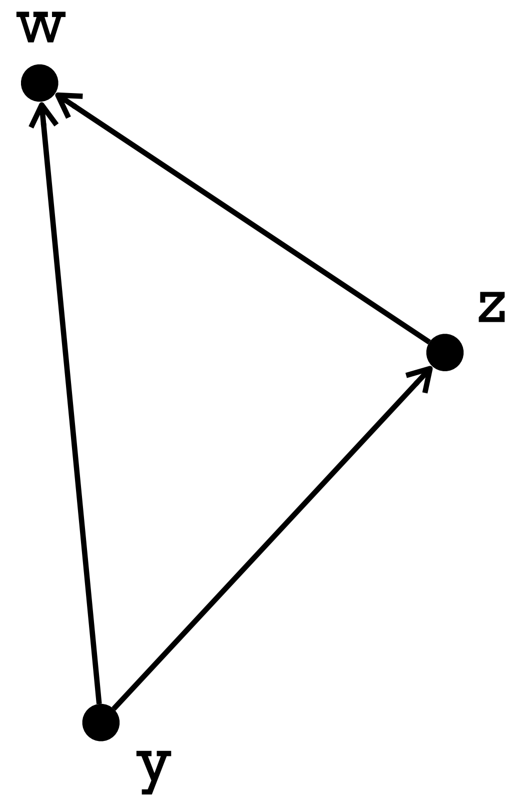
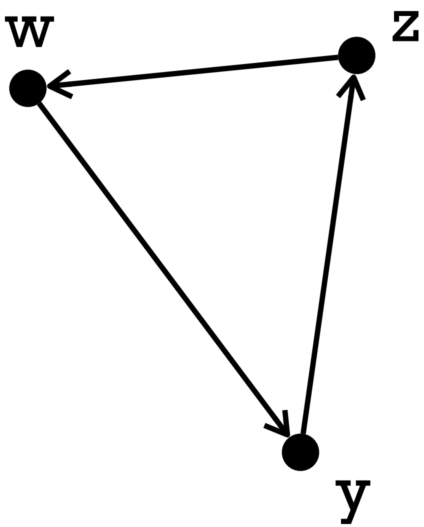


(a) Acyclicity



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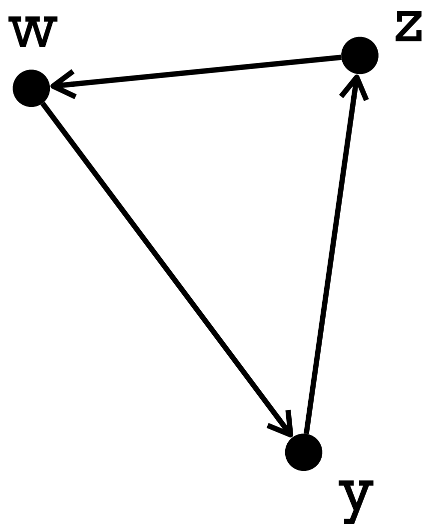

$z(c)$

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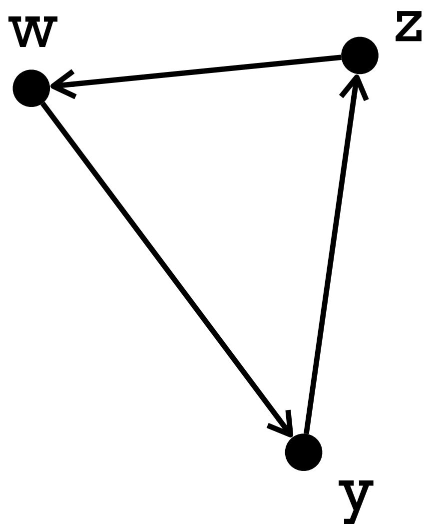
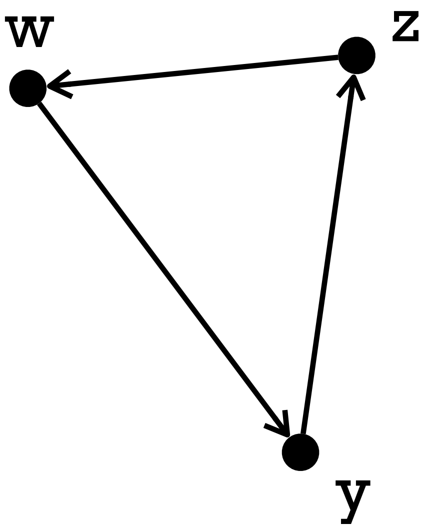


$z(c)$

$w(z(c))$

(a) Acyclicity

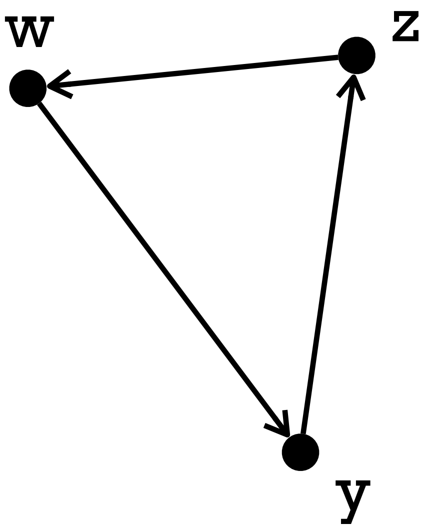


$z(c)$

$y(w(z(c)))$

$w(z(c))$

(a) Acyclicity



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$z(y(w(z(c))))$

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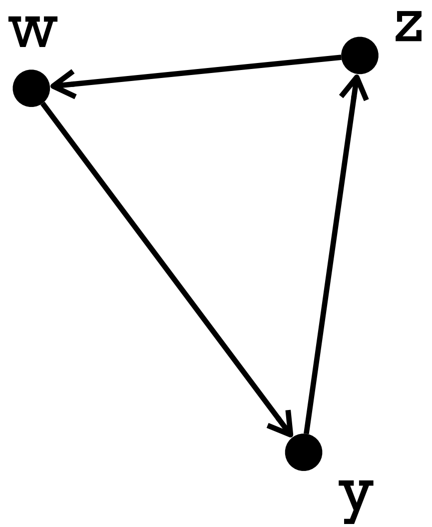

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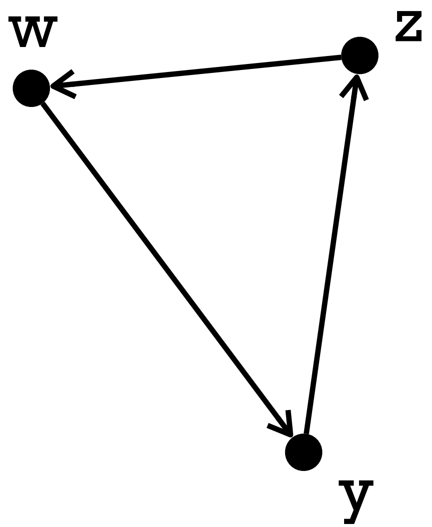

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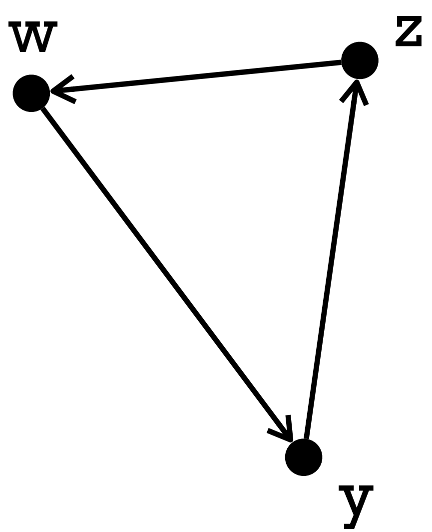

Remark. If the existential dependency graph of a given set of rules is acyclic, then the set of terms introduced during the computation of the chase is finite.

(f) Arity at Most 1

$$\mathsf{Film}(x) \rightarrow \exists y\,.\,\mathsf{IsFilmDirectedBy}(x, y) \wedge \mathsf{Director}(y)$$

$$\mathsf{A}(x) \wedge \mathsf{B}(x, w) \wedge \mathsf{C}(x, z) \rightarrow \exists z\,.\,\mathsf{R}(x, w, z)$$

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Remark. *If the arity of every function symbol in the skolemisation of a program is at most 1, then every term in the chase is of the form* **`x1(...xn(c)...)`** *with* **`c`** *constant.*

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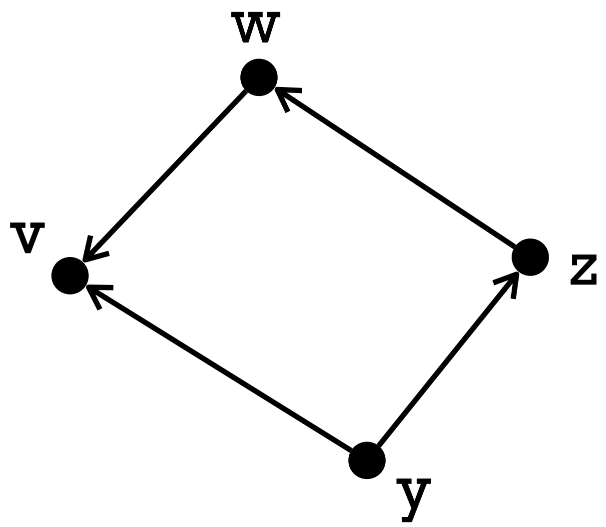

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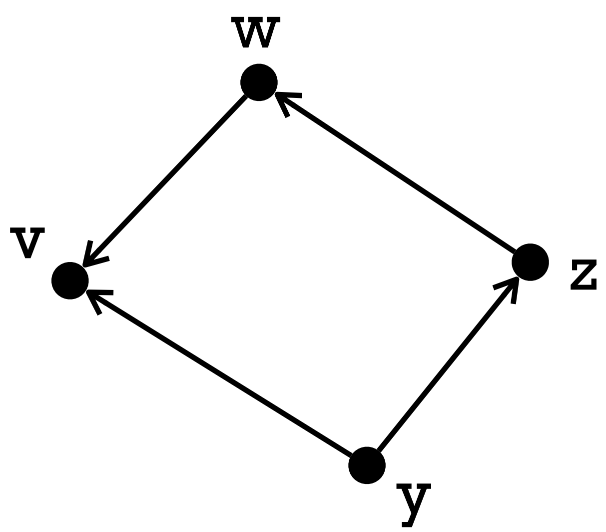

v(w(z(y(c))))
y(v(c))

Ensuring Tractability

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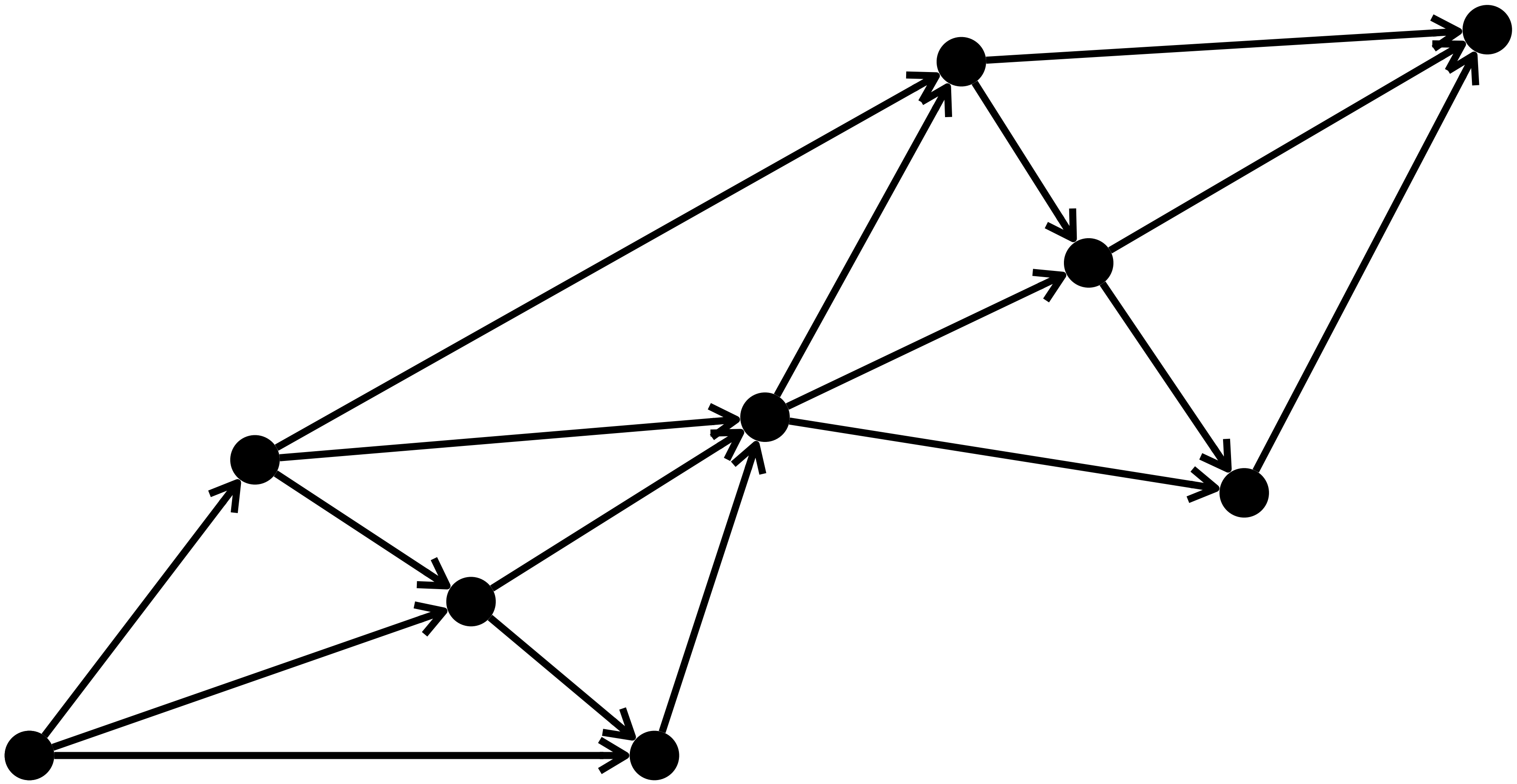
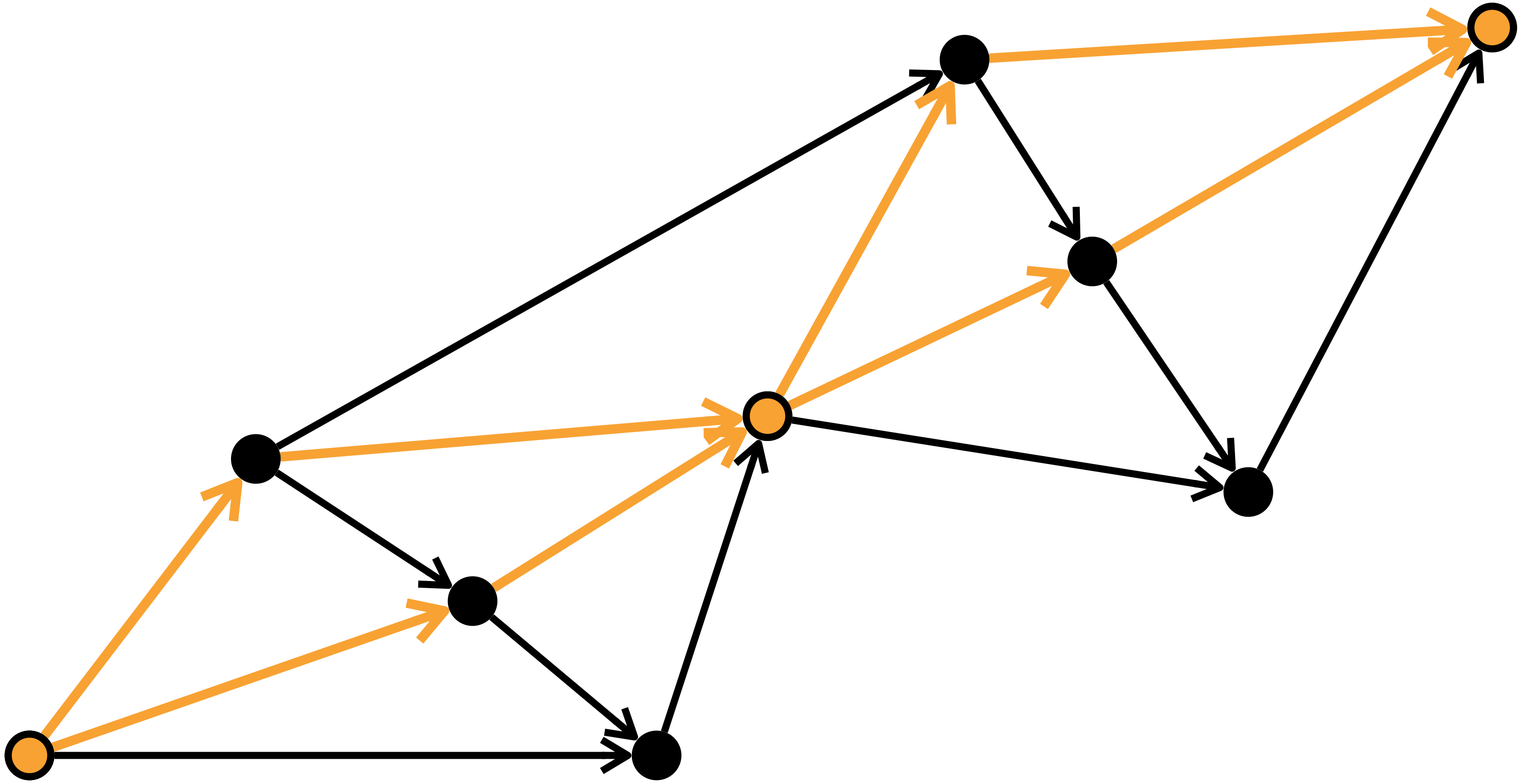
Braids



Braids



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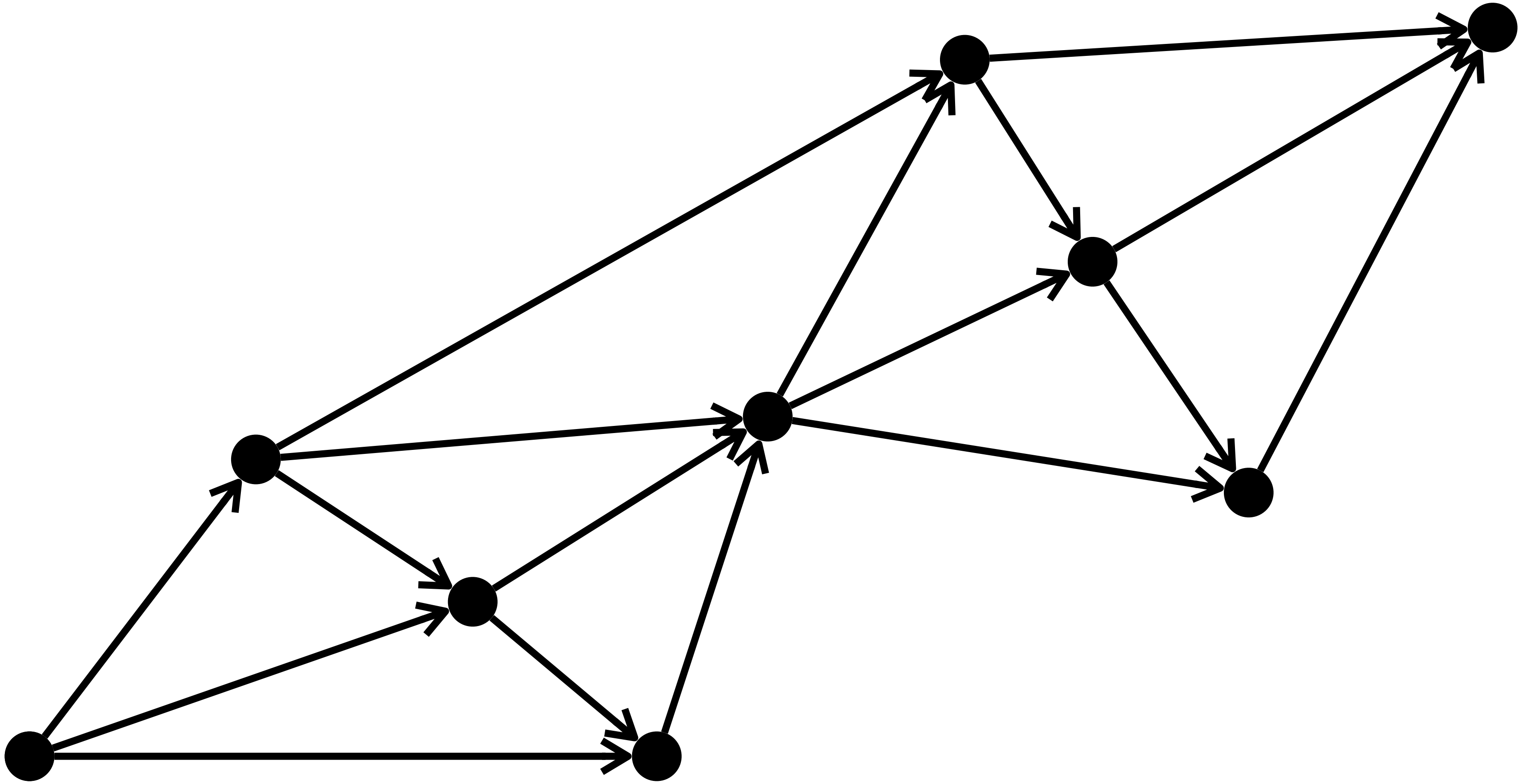

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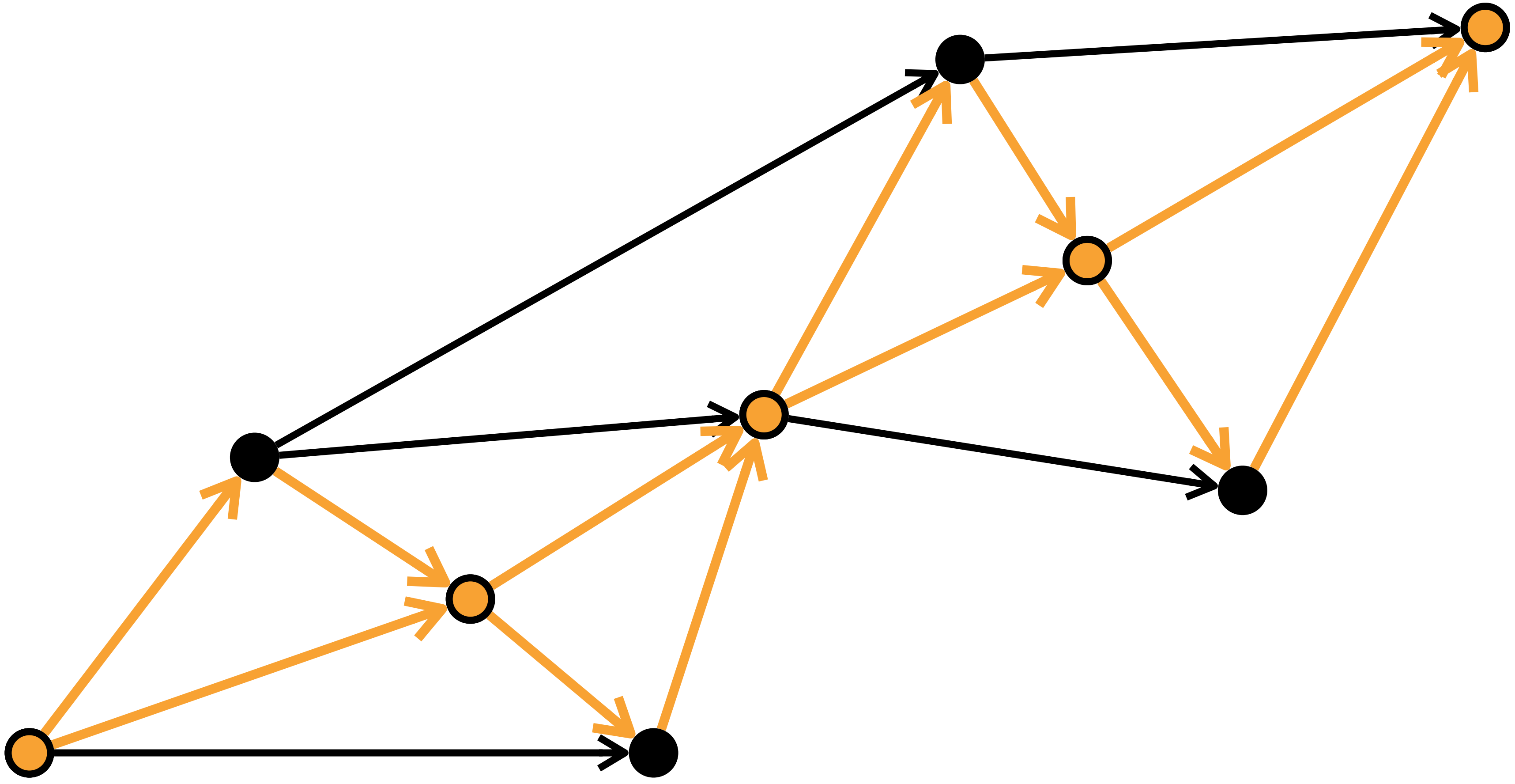

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Ensuring Tractability

Caveats.
1. Fixed query size.
2. Horn rule set.

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Real-world coverage: SRI Ontologies

$$
\begin{aligned}
A_1 \sqcap \ldots \sqcap A_n \sqsubseteq B &\mapsto \bigwedge_{i=1}^{n} A_i(x) \to B(x) \\
A \sqsubseteq B_1 \sqcup \ldots \sqcup B_n &\mapsto A(x) \to \bigvee_{i=1}^{n} B_i(x) \\
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R \sqsubseteq S &\mapsto R(x,y) \to S(x,y) \\
R \circ S \sqsubseteq V &\mapsto R(x,y) \wedge S(y,z) \to S(x,z) \\
R_1 \sqcap \ldots \sqcap R_n \sqsubseteq S &\mapsto \bigwedge_{i=1}^{n} R_i(x,y) \to S(x,y) \\
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SRI Axioms

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Corollary. To guarantee that tractable CQ entailment over a SRI ontology is possible we only need to verify the following:
1. Acyclicity.
2. Braid length in the dependency graph is bounded.

Evaluation Results

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Acyclicity

	MOWL Corpus	Oxford Ontology Repo
Ontologies	1576	225
Acyclic	974 (61.8%)	170 (75.6%)

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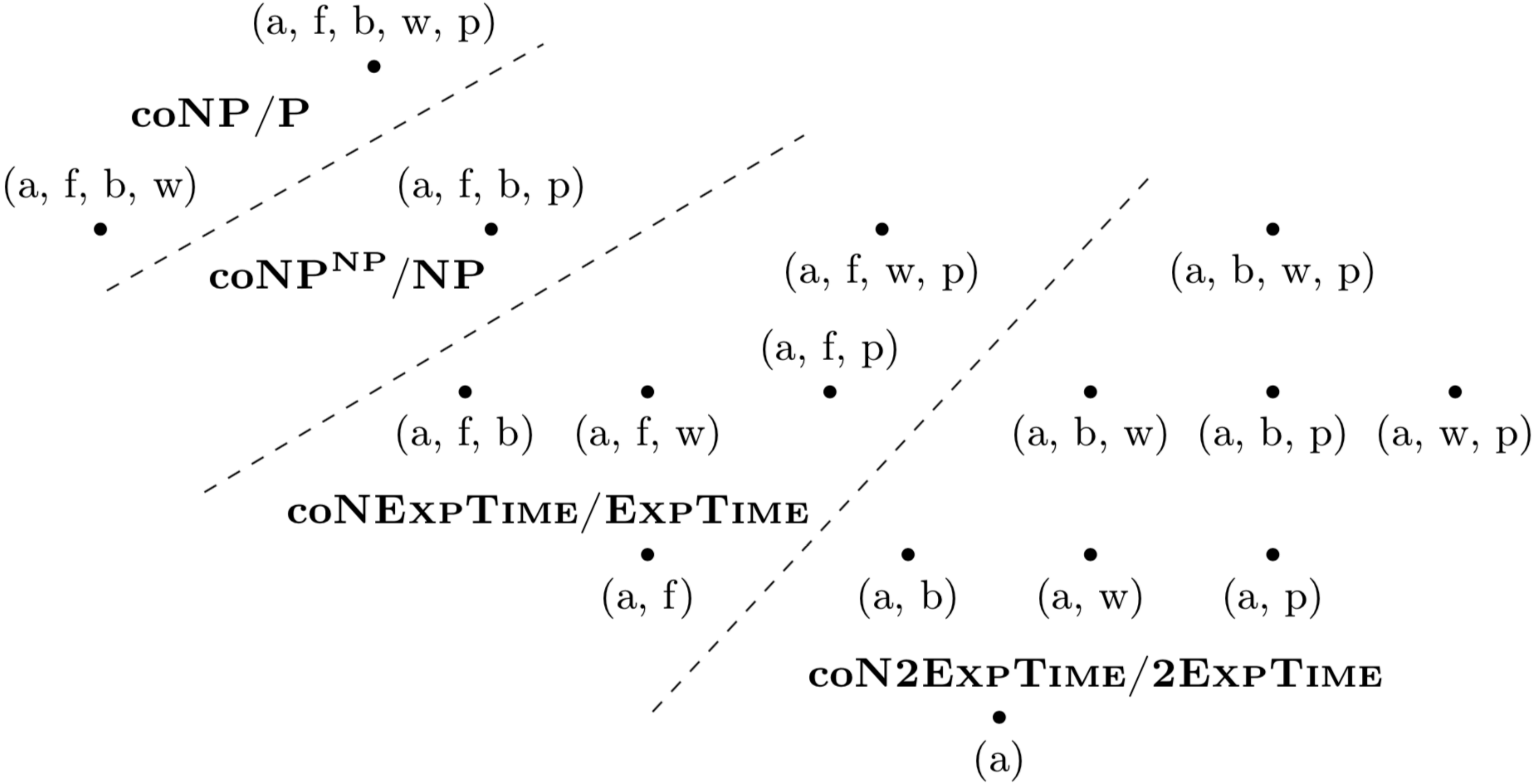
Braid Length

MOWL Corpus + Oxford

(max. length of a braid)	1	2	3	4	5	6	11	22	23	25	Total
(count)	851	153	56	61	11	1	1	2	7	1	1144
	74	88	93	98	99	99	99	99.1	99.3	99.9	100

More Results!



VLog

VLog

Efficient Model Construction for Horn Logic with VLog — System Description

Jacopo Urbani, Markus Krötzsch, Ceriel J. H. Jacobs,
Irina Dragoste, and David Carral

[IJCAR 2018]

An Implementation for the DF Restricted Chase

Consider a rule set R and an instance I.
Let $R_\forall$ and $R_\exists = \{r_1, \ldots, r_n\}$ be the sets of all Datalog and non-Datalog rules in R, respectively.
The Datalog-first restricted chase of R and I, denoted with $\mathsf{Ch}(R, I)$, is computed as follows.

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$I = F_1$

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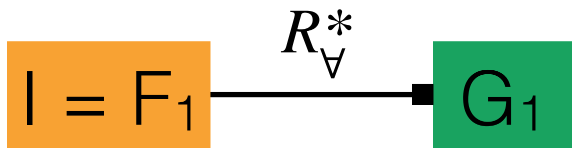

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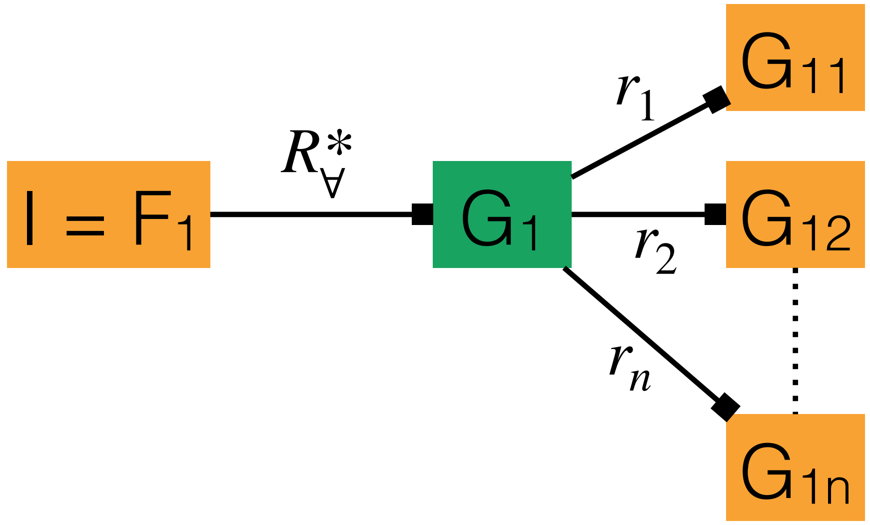

An Implementation for the DF Restricted Chase

Consider a rule set R and an instance I.
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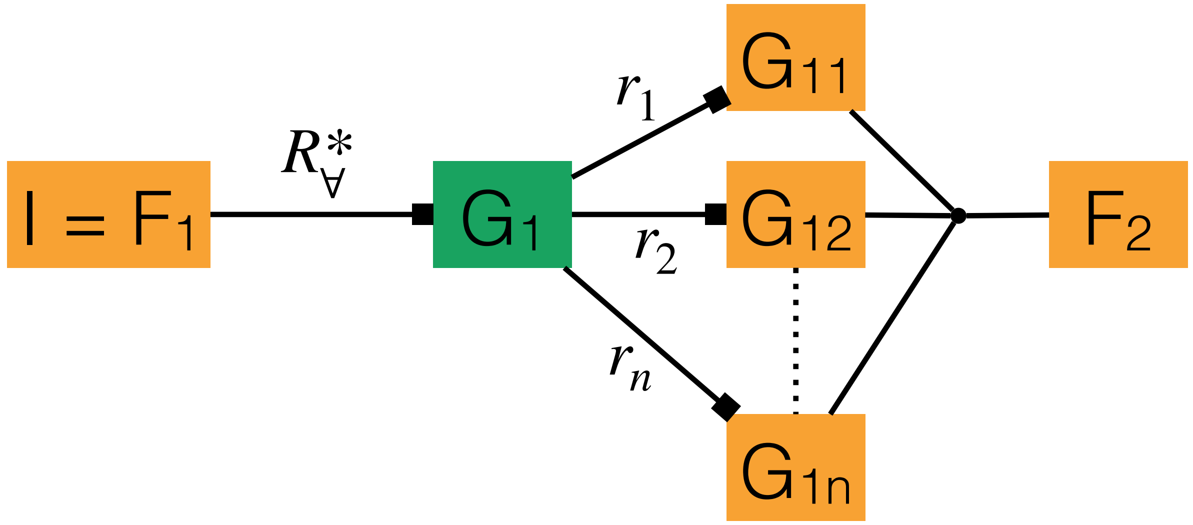

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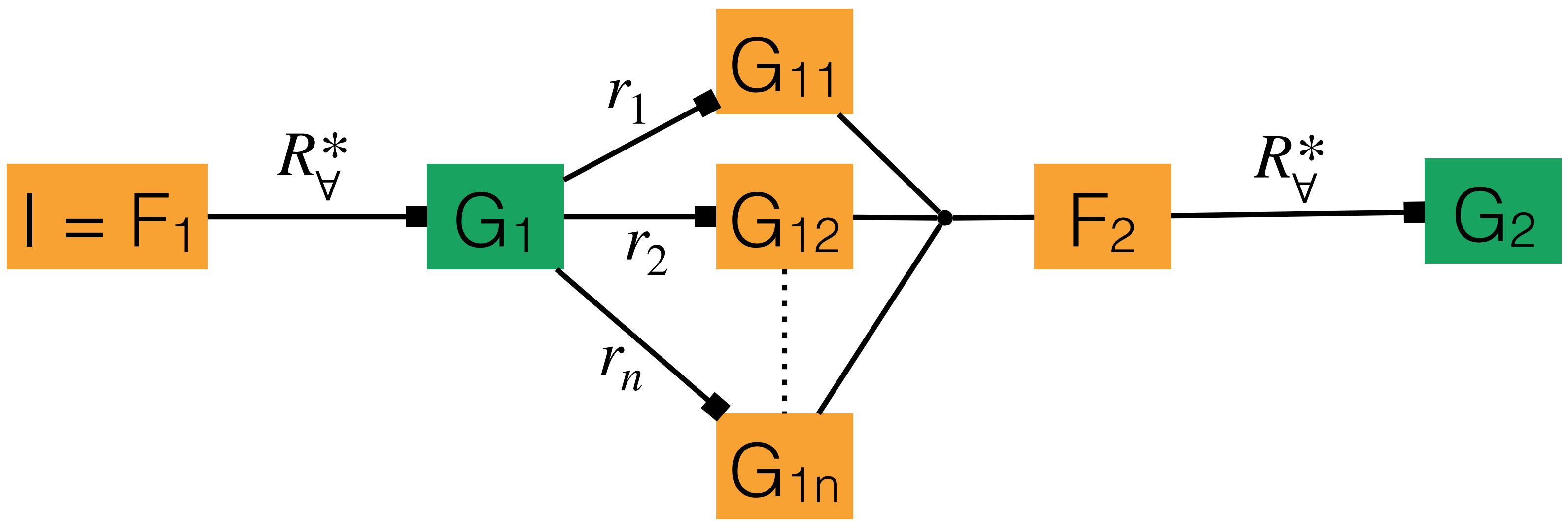

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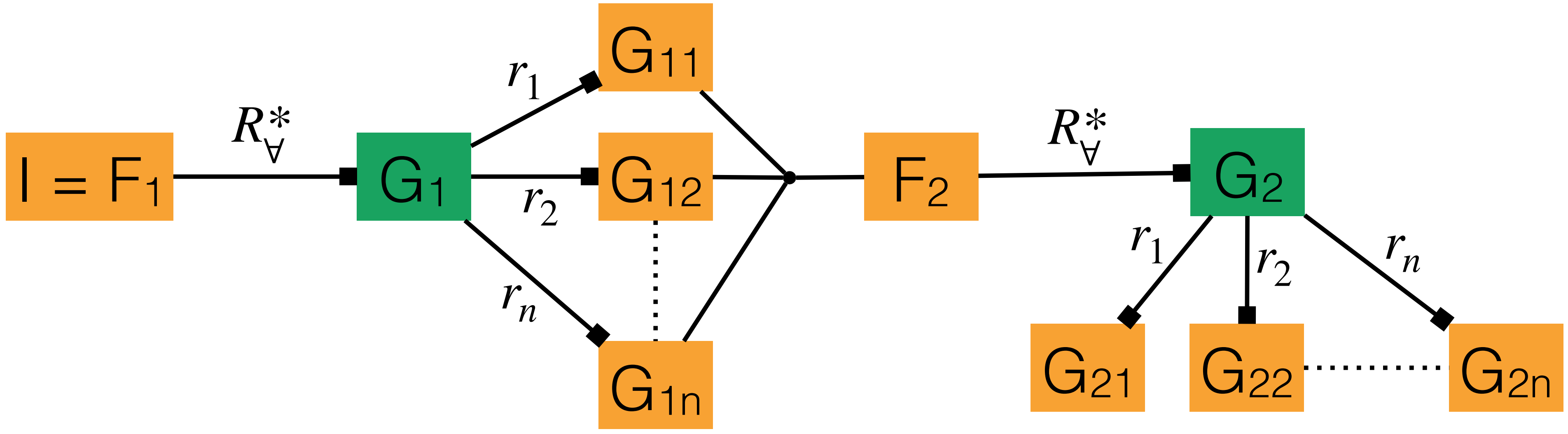

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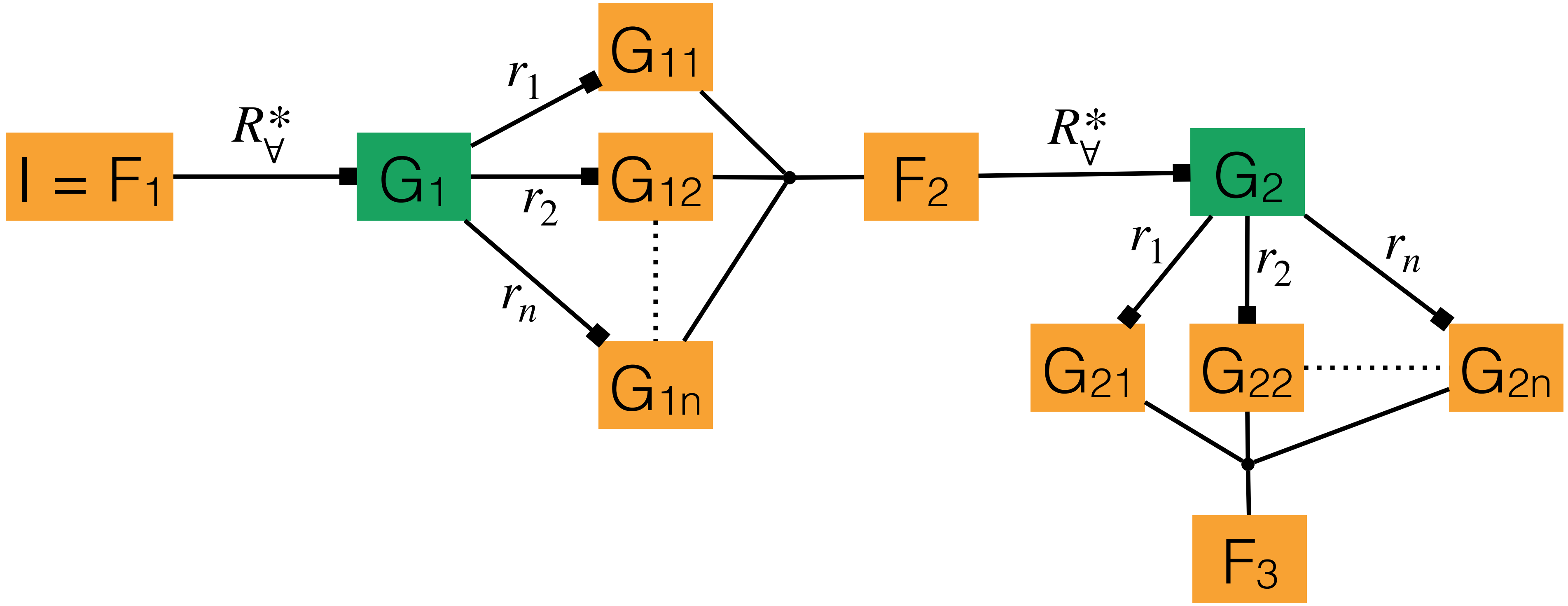

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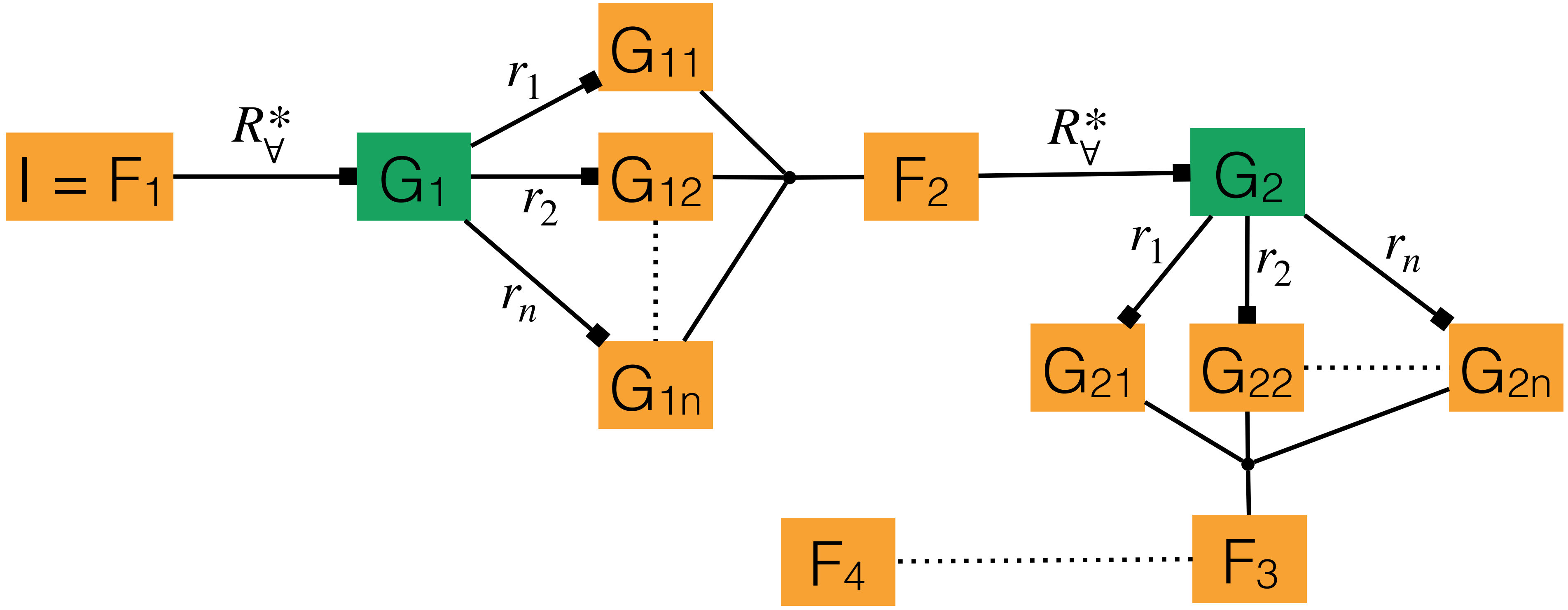

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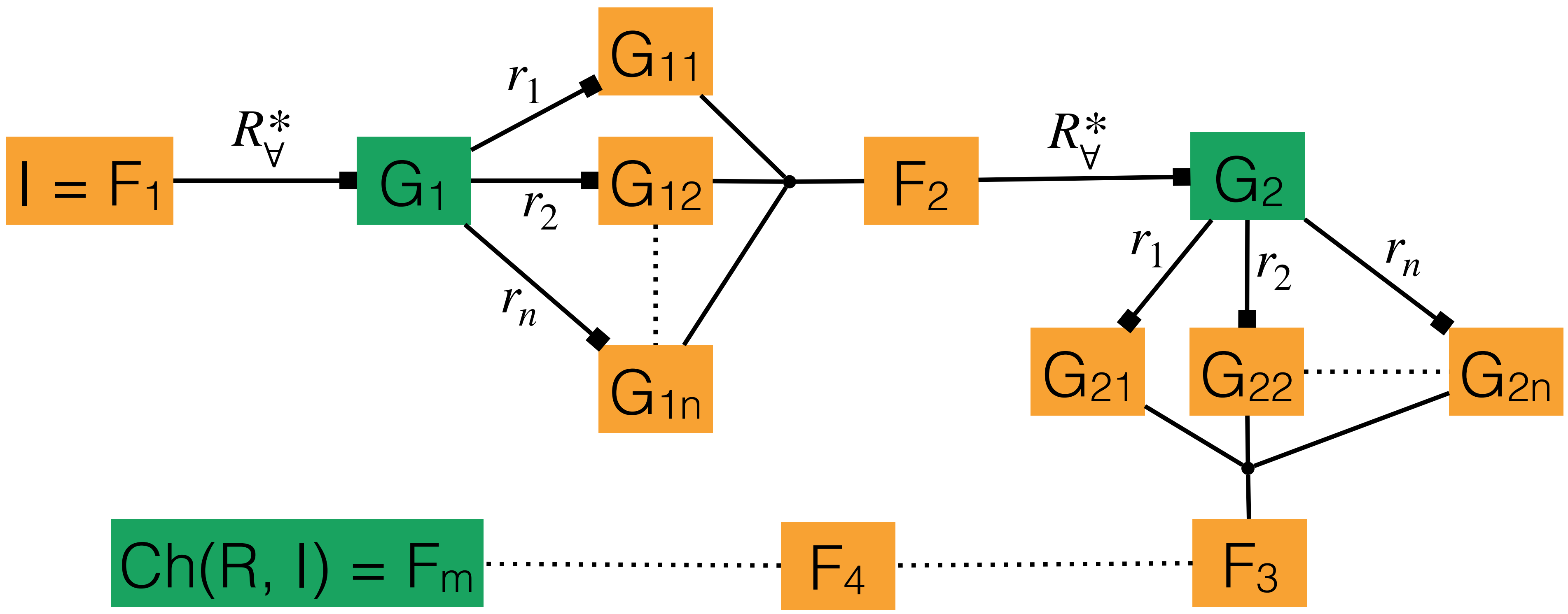

Performance: VLog vs RDFox

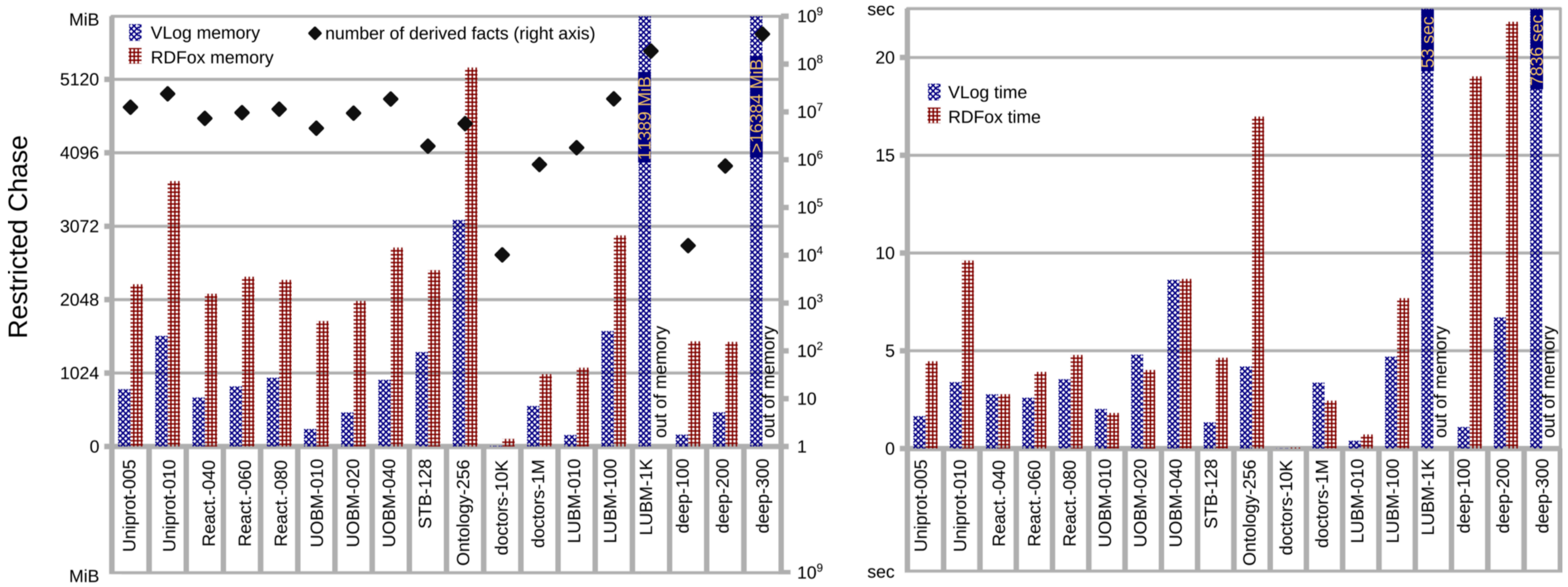

Fig. 1. Memory usage (left) and materialisation time (right) for VLog and RDFox

Conclusions

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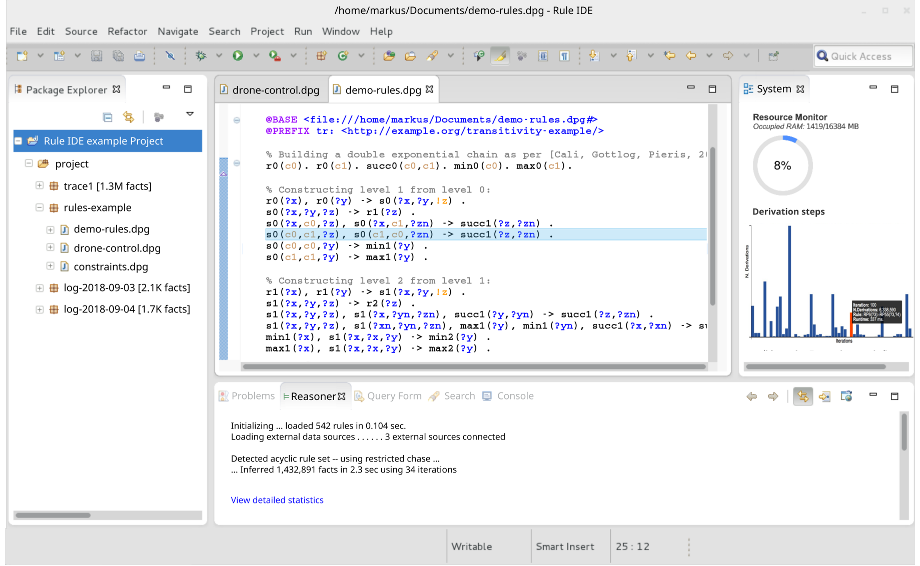

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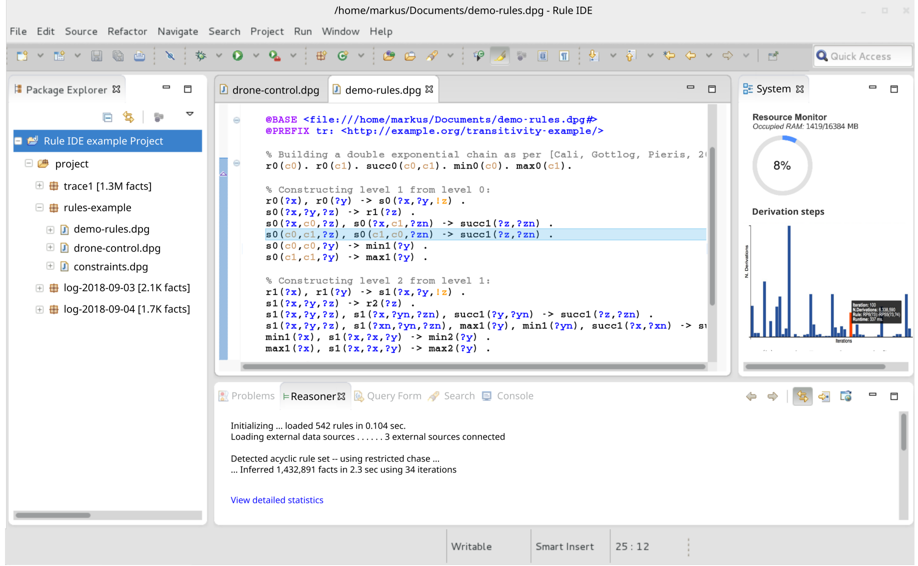

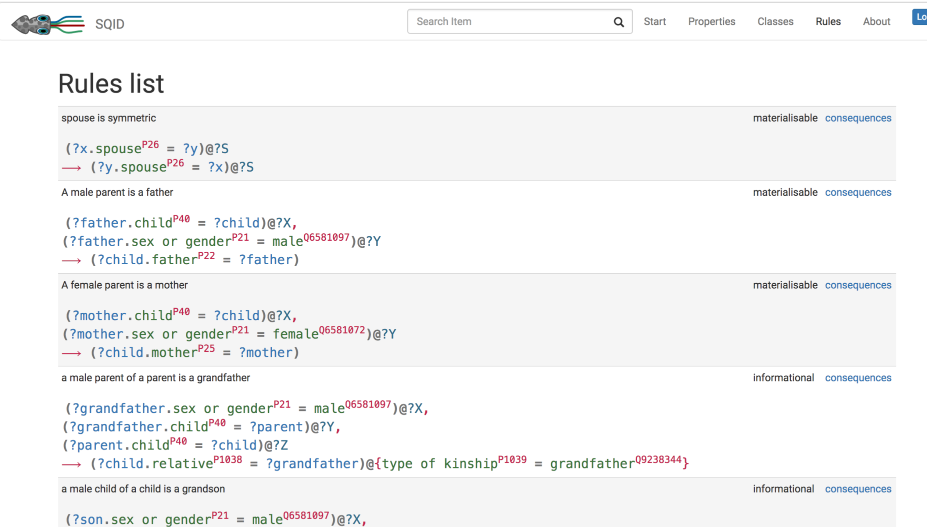

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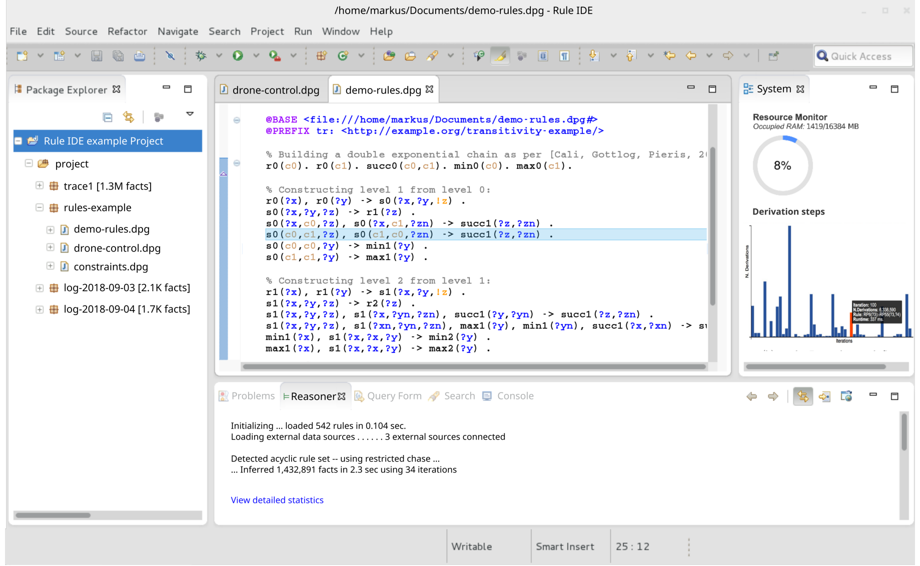

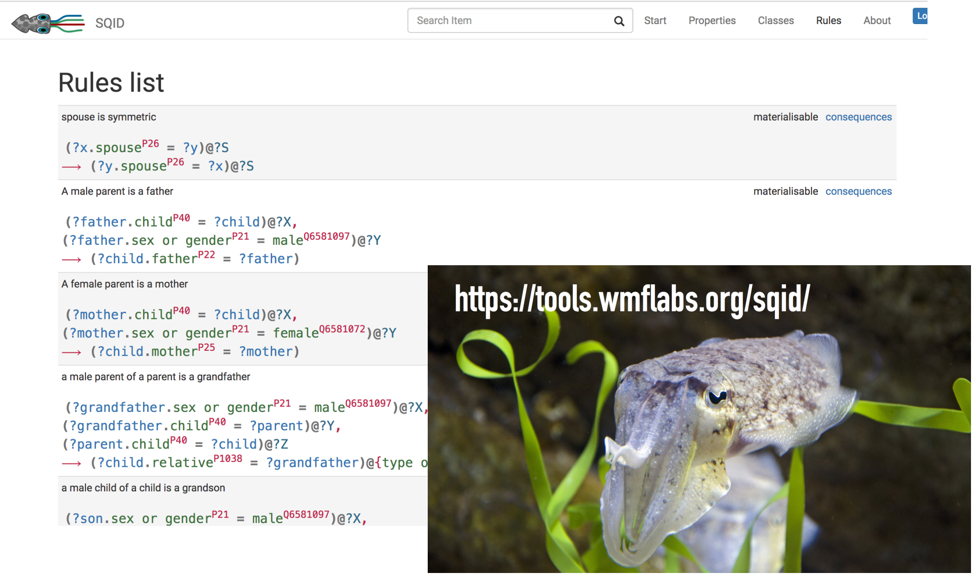

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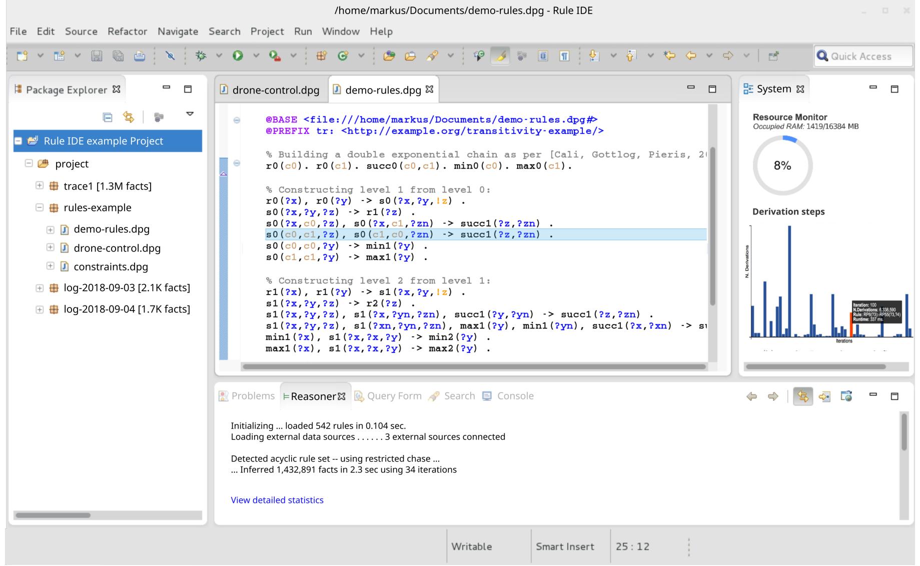

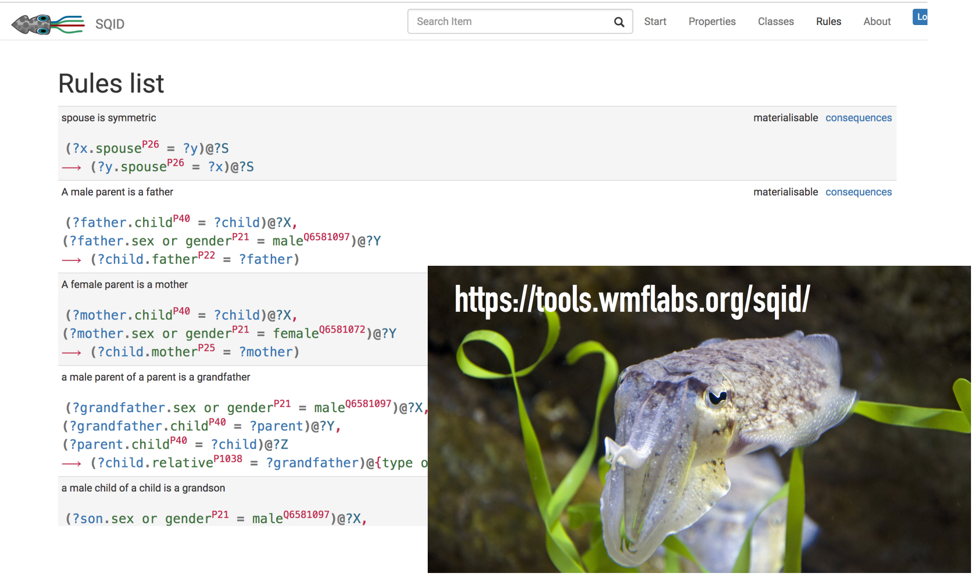

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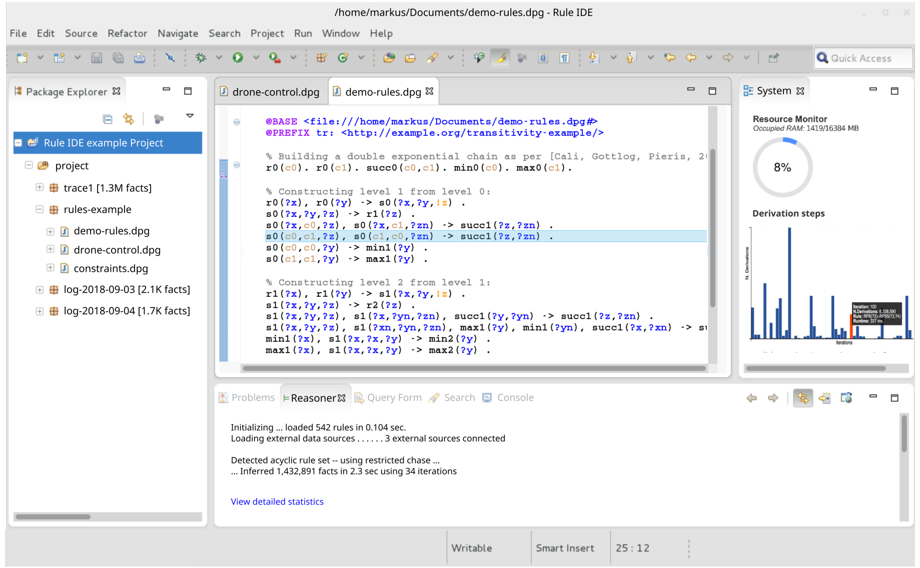

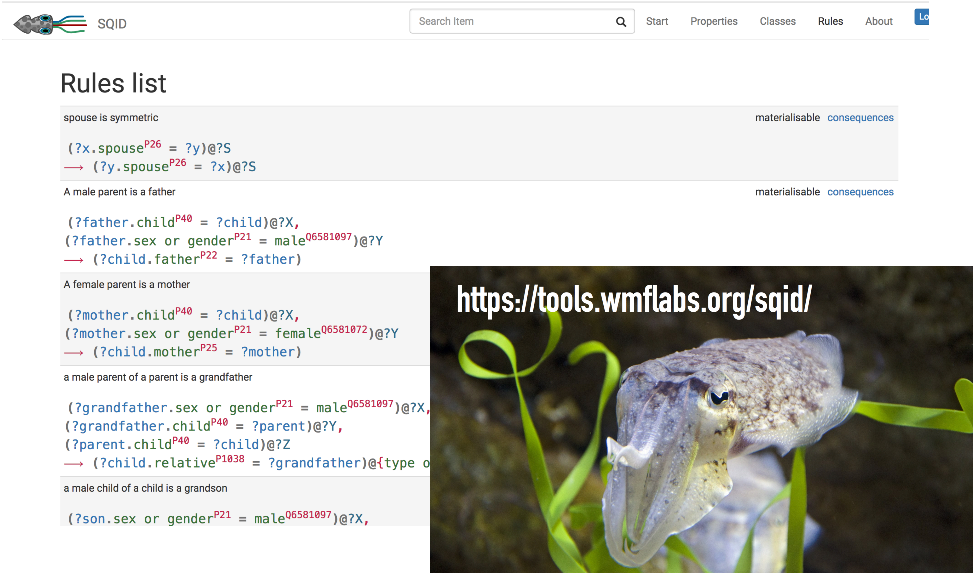

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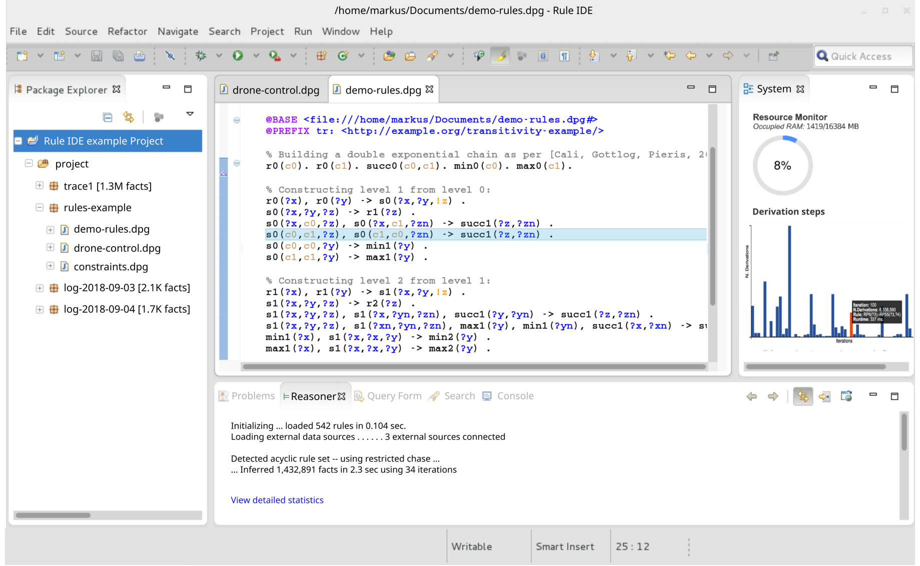

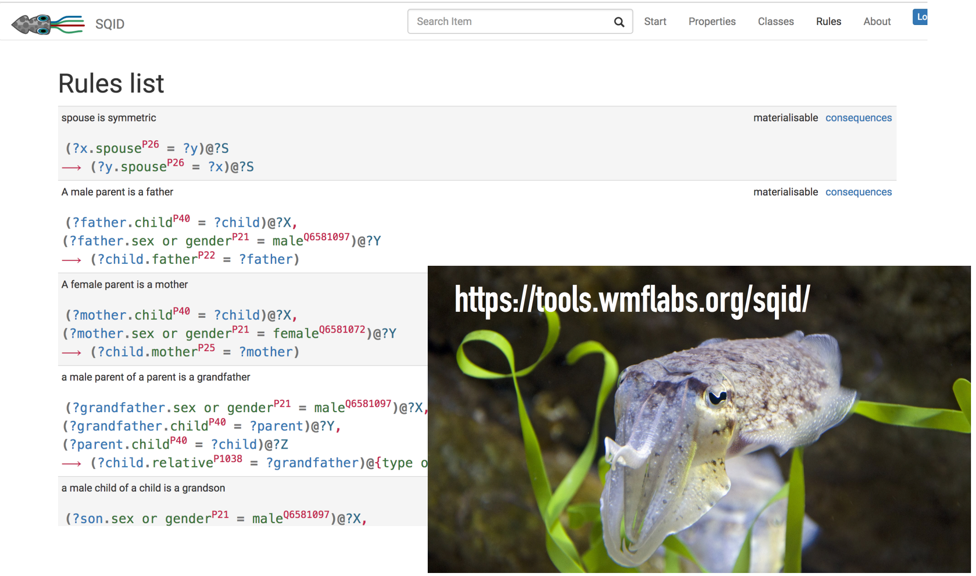

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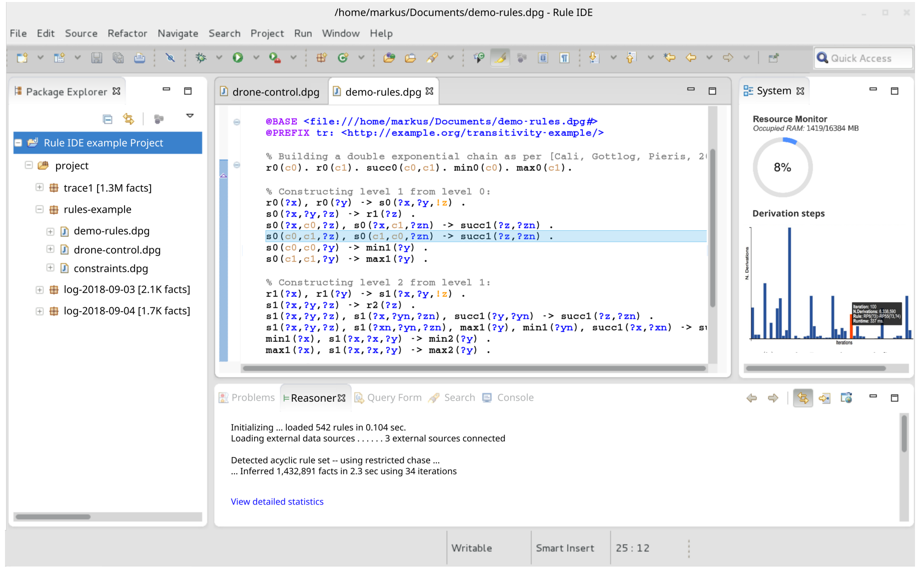

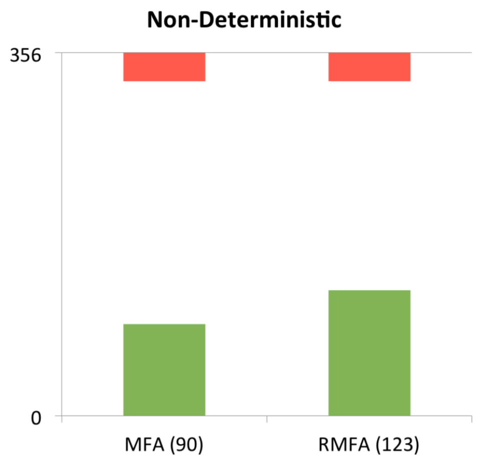

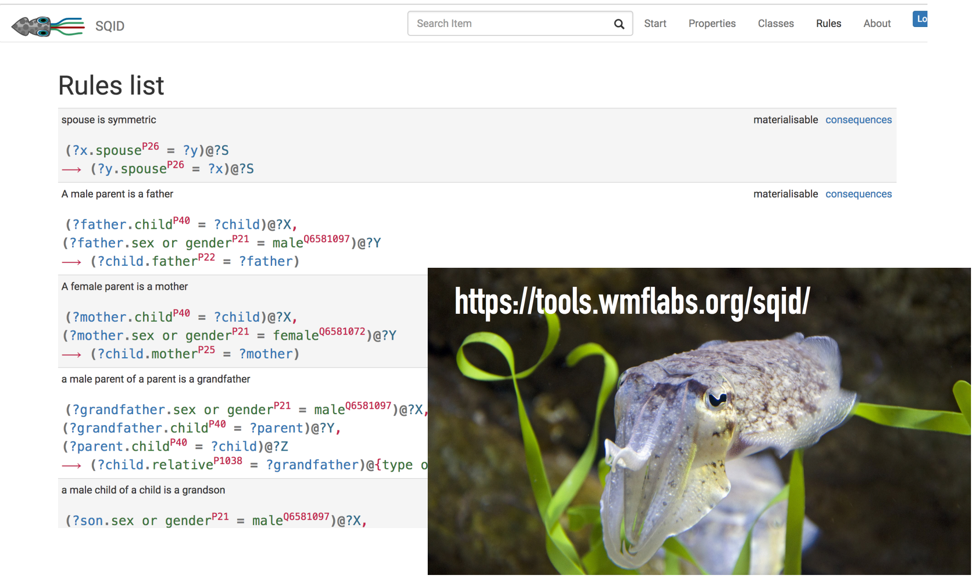

Bibliography

Reasoning over Existential Rules with Acyclicity Notions and the Datalog-first Restricted Chase

David Carral

TECHNISCHE UNIVERSITÄT DRESDEN

Slides available at https://iccl.inf.tu-dresden.de/web/Existential-rules-acyclicity

Bibliography: Sections

* **First section**
  Restricted Chase (Non)Termination for Existential Rules with Disjunctions [IJCAI 2017]
  https://iccl.inf.tu-dresden.de/web/Inproceedings3140/en

* **Second section**
  Tractable Query Answering for Expressive Ontologies and Rules [ISWC 2017]
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* **Third section**
  Efficient Model Construction for Horn Logic with VLog [IJCAR 2018]
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