TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

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Algorithmic Game Theory

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Exercises 2

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Problem 1.
From the lecture, you know that for *Rock-Paper-Scissors* and two players *Ann*, *Bob* with $\pi_{Ann} = \pi_{Bob} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, the mixed-strategy profile $\boldsymbol{\pi} = (\pi_{Ann} = \pi_{Bob})$ is a (strict) Nash-equilibrium in mixed strategies.

- Explain why it is, in fact, a Nash-equilibrium in mixed strategies.
- What happens if one player deviates from the strategy?
- Imagine you are competing in the *Rock-Paper-Scissors* world cup. How can you guarantee that you are playing the best strategy?

Problem 2.
Two bands, `BandA` and `BandB`, are competing in a Battle of the Bands competition. Each band has two strategies they can choose from: "Play `Loudly` " or "Play `Softly` ". `BandA` impresses more when both play the same style whereas `BandB` shines when both bands play differently.

(`BandA`, `BandB`)	`Loudly`	`Softly`
`Loudly`	(1,2)	(2,1)
`Softly`	(2,1)	(1,2)

Do the following:

a) Argue why there exists no Nash equilibrium in pure strategies for this game.

b) Compute the Nash equilibrium in mixed strategies. For this and all subsequent exercises, compute mixed equilibria by choosing strategies that make the opposing players indifferent among their strategies.

c) Argue whether there always exists an equilibrium in mixed strategies for finite games when there is no equilibrium in pure strategies.

Problem 3.
Find all mixed equilibria (which always includes any pure equilibria) of this 3×2 game.

$(Player_1, Player_2)$	l	r
T	(0,1)	(6,0)
M	(2,0)	(5,2)
B	(3,3)	(3,4)

Hint: Note that in any mixed equilibrium for this game, Player 1 plays only two of the three available strategies with positive probability.

Problem 4.
In this 2×2 game, A, B, C, D are the payoffs to player I, which are real numbers, no two of which are equal. Similarly, a, b, c, d are the payoffs to player II, which are real numbers, also no two of which are equal.

$(Player_1, Player_2)$	left	right
Top	(A, a)	(B, b)
Bottom	(C, c)	(D, d)

(a) Under which conditions does this game have a mixed equilibrium which is not a pure-strategy equilibrium? [Hint: Consider the possible patterns of best responses and resulting possible dominance relations, and express this by comparing the payoffs, as, for example, $A > C$.]

(b) Under which conditions in (a) is this the only equilibrium of the game?

Problem 5.
(Bonus)

As an alternative to the standard game theoretic notion of rationality, Douglas Hofstadter developed the concept of **superrationality** which can be informally defined as follows:

Definition 1: A player is considered to be superrational if she has perfect rationality (i.e. maximises her utility) and assumes that all other players are as rational as she is in that they reason in the same way (i.e. in symmetric games, all superrational players play the same strategy).

For example, when two superrational players play the Prisoner's dilemma, they both remain silent. (Both know they will play the same strategy, and out of the two options (`Cooperate`, `Cooperate`) and (`Defect`, `Defect`), the former yields the higher payoff.)

Consider the following two problems.

(a) In 1983, Douglas Hofstadter formulated the so-called *Platonia-Dilemma*: Imagine that you receive a letter from S.N. Platonia, an oil trillionaire, mentioning that you have been selected as one of 20 people to take part in a game. In that non-cooperative game, each person can send a telegram addressed to Platonia until the next day. However, if and only if exactly one person sends a telegram, that person receives a billion dollars. If Platonia receives more than one telegram, or no telegram at all, no one will receive money.

- What is the rational thing to do here under standard rationality?
- What is the rational thing to do here assuming that all players are superrational?

(b) Later in the 1980s, the popular science magazine, *Scientific American*, played a game referred to as *Luring Lottery*. The game is a lottery with a prize fund of one million dollars. Everyone can participate and enter the lottery as many times as she wants by sending in a postcard where the number of times she wants to enter the lottery is written down. In the end, all postcards are put into a hat, and one is drawn at random. However, there is a catch: the amount the winner gets is the million dollars, divided by the total number of entries.

- What would you do assuming standard rationality? What if you assume everyone to be superrational?
- How does that game relate to the *Prisoner's Dilemma*? What does it mean to cooperate and to defect here?
- What do you think happened when the game was actually played out?