Algorithmic Game Theory
Introduction to Strategic Games - Problems 1

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Problem 1. *Consider the games **the battle of sexes** as well as **the prisoner's dilemma** with the payoff tables displayed below. Although you already know the answers to the questions below from the lecture, work closely with the central definitions to answer them again.*

- *What strategy profiles provide Nash equilibria in both games?*

- *Is there a difference between the equilibria in both games?*

- *Are the equilibria Pareto optimal in both games?*

- *Are there dominant strategies in both games?*

The battle of the sexes

		Helena	
		Soccer	Concert
George	Soccer	**(10,1)**	(0,0)
	Concert	(0,0)	**(1,10)**

The prisoners' dilemma

		Wesson	
		Confession	Silence
Smith	Confession	$(-4,-4)$	$(0,-10)$
	Silence	$(-10,0)$	$(-2,-2)$

Problem 2. *Consider the following two problems.*

(a) *Imagine that you are a fully rational decision maker facing a (one-shot) prisoner's dilemma. Explain why the outcome of the game will not be affected if you are allowed to meet and discuss with the other player before you make your move.*

(b) *By definition, a non-cooperative game is a game in which the players are not able to form binding agreements. Why can we not just say that a non-cooperative game is a game in which the players do not actually cooperate?*

Problem 3. *Consider the following 3×3 game.*

I \ II	l	c	r
T	1, 0	3, 1	1, 1
M	1, 1	3, 0	0, 1
B	2, 2	3, 3	0, 2

(a) *Identify all pairs of strategies where one strategy weakly dominates the other.*

(b) *Assume you are allowed to remove a weakly dominated strategy of some player. Do so, and repeat this process (of iterated elimination of weakly dominated strategies) until you find a single strategy pair of the original game.*

(c) *Find such an iterated elimination of weakly dominated strategies that results in a strategy pair other than the one found in (b), where both strategies, and the payoffs to the players, are different.*

(d) *What are the equilibria (in pure strategies) of the game?*

Problem 4. *Consider the following three-player game in strategic form.*

I \ II	l	r
T	$3,4,4$	$1,3,3$
B	$8,1,4$	$2,0,6$

III: L

I \ II	l	r
T	$4,0,5$	$0,1,6$
B	$5,1,3$	$1,2,5$

III: R

(a) *Identify all pairs of strategies where one strategy strictly, or weakly, dominates the other.*

(b) *Apply iterated elimination of strictly dominated strategies to this game. What are the Nash equilibria of the game?*