Exercise 3: Complexity of First-Order Queries

Database Theory

2022-04-26

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Exercise. We consider three problems related to query answering in the lecture:

Boolean Query Entailment Given a Boolean query q and a database instance $\mathcal{I}$, does $\mathcal{I} \models q$ hold?

Query Answering Given an n-ary query q, a database instance $\mathcal{I}$, and an n-ary tuple $\mathbf{c}$, does $\mathbf{c} \in M[q](\mathcal{I})$ hold?

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  2. Otherwise, $\mathcal{M}'$ simulates $\mathcal{M}$ on all inputs $\langle \mathcal{I}, q[\mathbf{x}], \mathbf{c} \rangle$ with $\mathbf{c} \in \mathbf{adom}(\mathcal{I}, q)^n$ and accepts if any simulation accepts.

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Show that these problems are equivalent, i.e., show that any algorithm solving one of these problems, it can also be used to solve the others.

Solution.

- We restate the problems as decision problems:

$$\mathrm{BQE} = \left\{ \langle \mathcal{I}, q \rangle \,\middle|\, q \text{ a BCQ with } \mathcal{I} \models q \right\} \quad \mathrm{QA} = \left\{ \langle \mathcal{I}, q[\mathbf{x}], \mathbf{c} \rangle \,\middle|\, \mathbf{c} \in M[q](\mathcal{I}) \right\} \quad \mathrm{QE} = \left\{ \langle \mathcal{I}, q[\mathbf{x}] \rangle \,\middle|\, M[q](\mathcal{I}) \neq \emptyset \right\}$$

- We show that using a TM deciding BQE, we can construct a TM deciding QA, and
- that using a TM deciding QA we can construct a TM deciding QE:
- Let $\mathcal{M}$ be a TM deciding QA.
- Construct the TM $\mathcal{M}'$ that, on input $\langle \mathcal{I}, q[\mathbf{x}] \rangle$ with $\mathbf{x} = \langle x_1, \ldots, x_n \rangle$:
  1. If $n = 0$, then $\mathcal{M}'$ simulates $\mathcal{M}$ on input $\langle \mathcal{I}, q, \langle\rangle \rangle$ and accept iff the simulation accepts.
  2. Otherwise, $\mathcal{M}'$ simulates $\mathcal{M}$ on all inputs $\langle \mathcal{I}, q[\mathbf{x}], \mathbf{c} \rangle$ with $\mathbf{c} \in \mathbf{adom}(\mathcal{I}, q)^n$ and accepts if any simulation accepts.
  3. If no simulation accepts, $\mathcal{M}'$ rejects.

Exercise 1

Exercise. We consider three problems related to query answering in the lecture:

Boolean Query Entailment Given a Boolean query q and a database instance $\mathcal{I}$, does $\mathcal{I} \models q$ hold?

Query Answering Given an n-ary query q, a database instance $\mathcal{I}$, and an n-ary tuple $\mathbf{c}$, does $\mathbf{c} \in M[q](\mathcal{I})$ hold?

Query Emptiness Given a query q and a database instance $\mathcal{I}$, is $M[q](\mathcal{I}) \neq \emptyset$?

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  3. If no simulation accepts, $\mathcal{M}'$ rejects.
- Then $\mathcal{M}'$ decides QE.

Exercise 2

Exercise. It was shown in the lecture that joins can be computed in logarithmic space. Outline algorithms that implement *selection*, and *projection* in logarithmic space.

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Definition (Lecture 3, Slides 20–21)

A LogSpace transducer is a deterministic TM with three tapes:

- a read-only input tape
- a read/write working tape of size $O(\log n)$
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The output of a LogSpace transducer is the contents of its output tape when it halts, i.e., LogSpace transducers compute partial functions $\Sigma^* \to \Sigma^*$.

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- We describe a LogSpace transducer $\mathcal{M}$ that, given a table R with schema $R[a_1, \ldots, a_n]$ and some $a_i, a_j \in \{ a_1, \ldots, a_n \}$, computes $\sigma_{a_i=a_j}(R)$:

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  3. Initially, p_r points to the first \$ symbol, and we repeat:

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  2. We use three pointers p_r, p_i, and p_j.
  3. Initially, p_r points to the first \$ symbol, and we repeat:
     3.1 point p_i at the beginning of the i-th constant of the row;

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     - 3.4 if the constants are equal, copy the row to the output tape (using p_r); and

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     - 3.3 using p_i and p_j compare the two constants.
     - 3.4 if the constants are equal, copy the row to the output tape (using p_r); and
     - 3.5 point p_r to the next \$, if there is any, otherwise halt.

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     - 2.3 write \$ to the output.

Exercise 3

Exercise. Expressions of relational algebra under named perspective can be translated into Boolean circuits, in a similar fashion to the translation illustrated for FO queries in the lecture. Show how each operator of relational algebra gives rise to a corresponding circuit by describing the circuits for the following expressions:

$\sigma_{i=c}(R)$ (c a constant)	$\sigma_{i=j}(R)$ (j an attribute)
$\pi_{a_1,\ldots,a_\ell}(R)$	$R \bowtie S$
$\delta_{a_1,\ldots,a_\ell \to b_1,\ldots,b_\ell}(R)$	$R - S$
$R \cup S$	$R \cap S$

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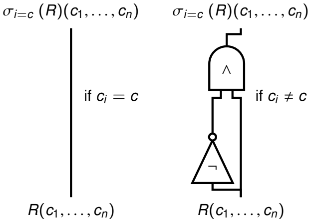
$\sigma_{i=c}(R)$ for each tuple $\langle c_1, \dots, c_n \rangle$ in R, we add one of these two circuits:



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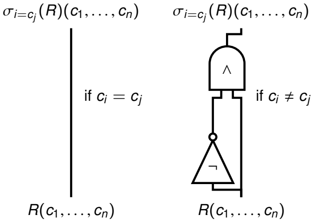

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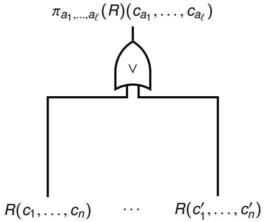

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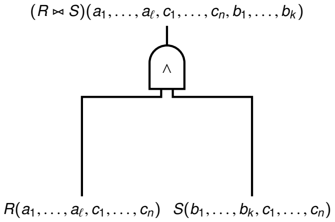

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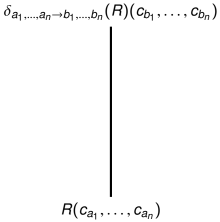

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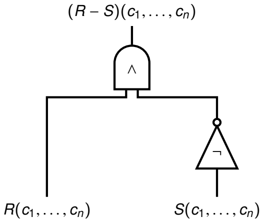

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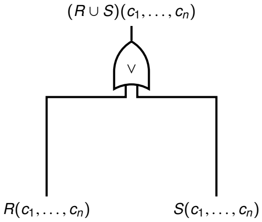

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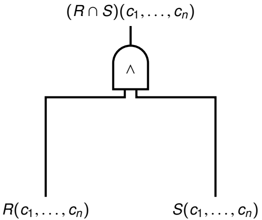
$R \cap S$ analogous to $R \bowtie S$.



Exercise 4

Exercise. Decide whether the following statements are true or false:

1. The combined complexity of a query language is at least as high as its data complexity.
2. The query complexity of a query language is at least as high as its data complexity.

If true, explain why, otherwise give a counter-example.

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Combined complexity given BCQ q and database instance $\mathcal{I}$ does $\mathcal{I} \models q$ hold?

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1. True (*why?*).
2. False: Consider $L = \{q\}$ with q a non-trivial BCQ, i.e., a BCQ such that there are database instances $\mathcal{I}$ and $\mathcal{J}$ with $\mathcal{I} \models q$ and $\mathcal{J} \not\models q$. Then the query complexity is constant, yet the data complexity of L is still in AC^0.

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  5. Both simulations can be performed in logarithmic space, and thus, $\mathcal{M}$ runs in logarithmic space.

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A TM decides a decision problem $\mathcal{L}$ if it halts on all inputs and accepts exactly the words in $\mathcal{L}$.

Solution.

- Let $\mathcal{L}$ be the decision problem for “$\mathrm{P} = \mathrm{NP}$?”, i.e., let $\mathcal{L} = \Sigma^*$ if $\mathrm{P} = \mathrm{NP}$, and let $\mathcal{L} = \emptyset$ otherwise.

Exercise 6

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- Let $\mathcal{M}_A$ and $\mathcal{M}_R$ be two terminating TMs that accept and reject every input, respectively.
- One of these two TMs decides $\mathcal{L}$.
- Thus, $\mathcal{L}$ is decidable, and hence, so is "$\mathrm{P} = \mathrm{NP}$?".