

We first continue with exercises on pebble games that we omitted last time.

**Exercise 1**

Show that you cannot express in  $\text{FO}^2[\{E\}]$  that  $E$  is functional (i.e. each node has at most one outgoing  $E$ -edge).

**Exercise 2**

Show that you cannot express in  $\text{FO}^2[\{E\}]$  that  $E$  is a linear order.

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Employ *Hanf locality* to provide easy proofs of the fact that the following properties are not FO-definable (Hanf locality will be introduced during the first 30 minutes of the lecture on 18th of May). Hint: use last-week solutions...

**Exercise 3**

Give an easy proof that checking if a given graph is (a) two-colorable (b) acyclic (c) a complete binary tree is not  $\text{FO}[\{E\}]$ -definable.

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We define *monadic second order logic* **MSO** as an extension of FO, in which we can additionally quantify over unary predicates (a.k.a. sets of elements).

**Exercise 4**

Show that reachability query is expressible in  $\text{MSO}[\{E\}]$ .

**Exercise 5**

Prove that we can express in  $\text{MSO}[\{E\}]$  that the underlying graph is acyclic.

**Exercise 6**

Prove that we can express evenness the domain in  $\text{MSO}[\{<\}]$  under the assumption that  $<$  is interpreted as a linear order over the domain.

**Exercise 7**

Show how to express in  $\text{MSO}[\{E\}]$  that a structure is a directed tree.

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