Exercise 1: Relational Algebra

Database Theory

2023-04-11

Maximilian Marx, Markus Krötzsch

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

Cinema	Address	Phone
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
CinemaxX	Hüblerstr. 8	3158910
...	...	...

Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

1. Who is the director of “The Imitation Game”?

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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...	...	...
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1. Who is the director of "The Imitation Game"?

$$\pi_{Director}(\sigma_{Title=\text{“The Imitation Game”}}(Films))$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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1. Who is the director of "The Imitation Game"?

$$\pi_{Director}(\sigma_{Title=\text{“The Imitation Game”}}(Films))$$

2. Which cinemas feature "The Imitation Game"?

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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...	...	...
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1. Who is the director of "The Imitation Game"?

$$\pi_{Director}(\sigma_{Title=\text{“The Imitation Game”}}(Films))$$

2. Which cinemas feature "The Imitation Game"?

$$\pi_{Cinema}(\sigma_{Title=\text{“The Imitation Game”}}(Program))$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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3. What are the address and phone number of “Schauburg”?

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Films

Title	Director	Actor
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The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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3. What are the address and phone number of "Schauburg"?

$$\pi_{Address,Phone}(\sigma_{Cinema=\text{"Schauburg"}}(Venues))$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
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3. What are the address and phone number of "Schauburg"?

$$\pi_{Address,Phone}(\sigma_{Cinema=\text{“Schauburg”}}(Venues))$$

4. *Boolean query:* Is a film directed by "Smith" playing in some cinema?

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
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3. What are the address and phone number of "Schauburg"?

$$\pi_{Address,Phone}(\sigma_{Cinema=\text{“Schauburg”}}(Venues))$$

4. *Boolean query:* Is a film directed by "Smith" playing in some cinema?

$$\pi_{\emptyset}(\sigma_{Director=\text{“Smith”}}(Films) \bowtie Program)$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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5. List the pairs of persons such that the first directed the second in a film, and vice versa.

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\pi_{Director,D}(\sigma_{Director=A}(\sigma_{Actor=D}(\delta_{Title,Director,Actor \rightarrow T,D,A}(Films) \bowtie Films)))$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\pi_{Director,D}(\sigma_{Director=A}(\sigma_{Actor=D}(\delta_{Title,Director,Actor \to T,D,A}(Films) \bowtie Films)))$$

6. List the names of directors who have acted in a film they directed.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\pi_{Director,D}(\sigma_{Director=A}(\sigma_{Actor=D}(\delta_{Title,Director,Actor \to T,D,A}(Films) \bowtie Films)))$$

6. List the names of directors who have acted in a film they directed.

$$\pi_{Director}(\sigma_{Actor=Director}(Films))$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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7. Always return $\{\text{Title} \mapsto \texttt{"Apocalypse Now"}, \text{Director} \mapsto \texttt{"Coppola"}\}$ as the answer.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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7. Always return {Title ↦ "`Apocalypse Now`", Director ↦ "`Coppola`"} as the answer.

$$\{\{\mathit{Title} \mapsto \text{"Apocalypse Now"}\}\} \bowtie \{\{\mathit{Director} \mapsto \text{"Coppola"}\}\}$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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7. Always return $\{\text{Title} \mapsto \texttt{"Apocalypse Now"}, \text{Director} \mapsto \texttt{"Coppola"}\}$ as the answer.

$$\{\{\mathit{Title} \mapsto \text{"Apocalypse Now"}\}\} \bowtie \{\{\mathit{Director} \mapsto \text{"Coppola"}\}\}$$

8. Find the actors cast in at least one film by “Smith”.

Exercise 1

Films

Title	Director	Actor
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7. Always return {Title ↦ `"Apocalypse Now"`, Director ↦ `"Coppola"`} as the answer.

$$\{\{\mathit{Title} \mapsto \text{"Apocalypse Now"}\}\} \bowtie \{\{\mathit{Director} \mapsto \text{"Coppola"}\}\}$$

8. Find the actors cast in at least one film by “Smith”.

$$\pi_{\mathit{Actor}}(\sigma_{\mathit{Director}=\text{“Smith”}}(\mathit{Films}))$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
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9.1 Find the actors for which there is a movie directed by “Smith” that they are not cast in.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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9.1 Find the actors for which there is a movie directed by “Smith” that they are not cast in.

$$q = \pi_{Actor}[(\pi_{Actor}(Films) \bowtie \pi_{Title}(\sigma_{Director=\text{“Smith”}}(Films))) - \pi_{Actor,Title}(\sigma_{Director=\text{“Smith”}}(Films))]$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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9.1 Find the actors for which there is a movie directed by "Smith" that they are not cast in.

$$q = \pi_{Actor}[\Big(\pi_{Actor}(Films) \bowtie \pi_{Title}(\sigma_{Director=\text{"Smith"}}(Films))\Big) - \pi_{Actor,Title}(\sigma_{Director=\text{"Smith"}}(Films))]$$

9 Find the actors cast in every film by "Smith."

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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9.1 Find the actors for which there is a movie directed by "Smith" that they are not cast in.

$$q = \pi_{Actor}[(\pi_{Actor}(Films) \bowtie \pi_{Title}(\sigma_{Director=\text{"Smith"}}(Films))) - \pi_{Actor,Title}(\sigma_{Director=\text{"Smith"}}(Films))]$$

9 Find the actors cast in every film by "Smith."

$$\pi_{Actor}(Films) - q$$

Exercise 1

Films

Title	Director	Actor
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The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
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10 Find the actors cast only in films by "Smith."

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Films

Title	Director	Actor
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The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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10 Find the actors cast only in films by "Smith."

$$\pi_{Actor}(Films) - \pi_{Actor}[Films - \sigma_{Director=\text{“Smith”}}(Films)]$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
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10 Find the actors cast only in films by "Smith."

$$\pi_{Actor}(Films) - \pi_{Actor}[Films - \sigma_{Director=\text{“Smith”}}(Films)]$$

11 Find all pairs of actors who act together in at least one film.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

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UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
CinemaxX	Hüblerstr. 8	3158910
...	...	...

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Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

10 Find the actors cast only in films by "Smith."

$$\pi_{Actor}(Films) - \pi_{Actor}[Films - \sigma_{Director=\text{“Smith”}}(Films)]$$

11 Find all pairs of actors who act together in at least one film.

$$\pi_{RA,Actor}[\delta_{Actor \rightarrow RA}(Films) \bowtie Films]$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a'.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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CinemaxX	The Imitation Game	19:30

12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a'.

$$q_1 = \pi_{Actor,RA}\left[\left(\pi_{Actor}(Films) \bowtie \delta_{Actor \to RA}(Films)\right) - \left(Films \bowtie \delta_{Actor \to RA}(\pi_{Actor}(Films))\right)\right]$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a'.

$$q_1 = \pi_{Actor,RA}\big[\big(\pi_{Actor}(Films) \bowtie \delta_{Actor \to RA}(Films)\big) - \big(Films \bowtie \delta_{Actor \to RA}(\pi_{Actor}(Films))\big)\big]$$

If $\{\{Actor \mapsto a'\}, \{RA \mapsto a\}\} \in q_1(\mathcal{D})$, then a acts in a movie that does not feature a'.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Program

Cinema	Title	Time
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Schauburg	Dogma	20:45
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CinemaxX	The Imitation Game	19:30

12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a'.

$$q_1 = \pi_{Actor,RA}\Big[\Big(\pi_{Actor}(Films) \bowtie \delta_{Actor\to RA}(Films)\Big) - \Big(Films \bowtie \delta_{Actor\to RA}(\pi_{Actor}(Films))\Big)\Big]$$

If $\{\{Actor \mapsto a'\}, \{RA \mapsto a\}\} \in q_1(\mathcal{D})$, then a acts in a movie that does not feature a'.

12.2 Find all pairs of actors a and a' such that a acts in all the movies that feature a'.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Program

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12.1 Find all pairs of actors *a* and *a'* such that *a* acts in a movie that does not feature *a'*.

$$q_1 = \pi_{Actor,RA}[(\pi_{Actor}(Films) \bowtie \delta_{Actor \to RA}(Films)) - (Films \bowtie \delta_{Actor \to RA}(\pi_{Actor}(Films)))]$$

If $\{\{Actor \mapsto a'\}, \{RA \mapsto a\}\} \in q_1(\mathcal{D})$, then *a* acts in a movie that does not feature *a'*.

12.2 Find all pairs of actors *a* and *a'* such that *a* acts in all the movies that feature *a'*.

$$q_2 = (\pi_{Actor}(Films) \bowtie \delta_{Actor \to RA}(\pi_{Actor}(Films))) - q_1$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
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12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a'.

$$q_1 = \pi_{Actor,RA}\Big[\big(\pi_{Actor}(Films) \bowtie \delta_{Actor\to RA}(Films)\big) - \big(Films \bowtie \delta_{Actor\to RA}(\pi_{Actor}(Films))\big)\Big]$$

If $\{\{Actor \mapsto a'\}, \{RA \mapsto a\}\} \in q_1(\mathcal{D})$, then a acts in a movie that does not feature a'.

12.2 Find all pairs of actors a and a' such that a acts in all the movies that feature a'.

$$q_2 = (\pi_{Actor}(Films) \bowtie \delta_{Actor\to RA}(\pi_{Actor}(Films))) - q_1$$

If $\{\{Actor \mapsto a\}, \{RA \mapsto a'\}\} \in q_2(\mathcal{D})$, then a acts in all the movies that feature a'.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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12 Find all pairs of actors cast in exactly the same films.

$$q_1 = \pi_{Actor,RA}\Big[\Big(\pi_{Actor}(Films) \bowtie \delta_{Actor\to RA}(Films)\Big) - \Big(Films \bowtie \delta_{Actor\to RA}(\pi_{Actor}(Films))\Big)\Big]$$

$$q_2 = (\pi_{Actor}(Films) \bowtie \delta_{Actor\to RA}(\pi_{Actor}(Films))) - q_1$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
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12 Find all pairs of actors cast in exactly the same films.

$$q_1 = \pi_{Actor,RA}\Big[\Big(\pi_{Actor}(Films) \bowtie \delta_{Actor \to RA}(Films)\Big) - \Big(Films \bowtie \delta_{Actor \to RA}(\pi_{Actor}(Films))\Big)\Big]$$

$$q_2 = (\pi_{Actor}(Films) \bowtie \delta_{Actor \to RA}(\pi_{Actor}(Films))) - q_1$$

$$q_2 \bowtie \delta_{Actor,RA \to RA,Actor}(q_2)$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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CinemaxX	The Imitation Game	19:30

13.1 Find all pairs of directors *d* and actors *a* such that *d* directs some movie that features *a*.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
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13.1 Find all pairs of directors *d* and actors *a* such that *d* directs some movie that features *a*.

$$q_1 = \pi_{Director,Actor}(Films)$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
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CinemaxX	The Imitation Game	19:30

13.1 Find all pairs of directors *d* and actors *a* such that *d* directs some movie that features *a*.

$$q_1 = \pi_{Director,Actor}(Films)$$

13.2 Find the directors who do not direct all actors.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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13.1 Find all pairs of directors *d* and actors *a* such that *d* directs some movie that features *a*.

$$q_1 = \pi_{Director,Actor}(Films)$$

13.2 Find the directors who do not direct all actors.

$$q_2 = \pi_{Director}((\pi_{Director}(Film) \bowtie \pi_{Actor}(Film)) - q_1)$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
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13 Find the directors such that every actor is cast in one of their films.

$$q_1 = \pi_{Director,Actor}(Films)$$

$$q_2 = \pi_{Director}((\pi_{Director}(Film) \bowtie \pi_{Actor}(Film)) - q_1)$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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13 Find the directors such that every actor is cast in one of their films.

$$q_1 = \pi_{Director,Actor}(Films)$$
$$q_2 = \pi_{Director}((\pi_{Director}(Film) \bowtie \pi_{Actor}(Film)) - q_1)$$

$$\pi_{Director}(Film) - q_2$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

(1) $R \bowtie R$ (2) $R \bowtie \emptyset$ (3) $R \bowtie \{\varepsilon\}$

Solution.

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

(1) $R \bowtie R$ (2) $R \bowtie \emptyset$ (3) $R \bowtie \{\varepsilon\}$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{\, f : U \cup V \to \textbf{dom} \mid f_U \in R \text{ and } f_V \in S \,\},$$

where f_U and f_V are the restriction of f to elements in U and V, respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

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$$R \bowtie S = \{\, f : U \cup V \to \textbf{dom} \mid f_U \in R \text{ and } f_V \in S \,\},$$

where f_U and f_V are the restriction of f to elements in U and V, respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(1)

$$R \bowtie R = \{\, f : U \cup U \to \textbf{dom} \mid f_U \in R \text{ and } f_U \in R \,\}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

(1) $R \bowtie R$ (2) $R \bowtie \emptyset$ (3) $R \bowtie \{\varepsilon\}$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{ f : U \cup V \to \textbf{dom} \mid f_U \in R \text{ and } f_V \in S \},$$

where f_U and f_V are the restriction of f to elements in U and V, respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(1)

$$\begin{aligned} R \bowtie R &= \{ f : U \cup U \to \textbf{dom} \mid f_U \in R \text{ and } f_U \in R \} \\ &= \{ f : U \to \textbf{dom} \mid f_U \in R \} \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

(1) $R \bowtie R$ (2) $R \bowtie \emptyset$ (3) $R \bowtie \{\varepsilon\}$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{\, f : U \cup V \to \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S \,\},$$

where f_U and f_V are the restriction of f to elements in U and V, respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(1)

$$\begin{aligned} R \bowtie R &= \{\, f : U \cup U \to \mathbf{dom} \mid f_U \in R \text{ and } f_U \in R \,\} \\ &= \{\, f : U \to \mathbf{dom} \mid f_U \in R \,\} \\ &= \{\, f_U \mid f_U \in R \,\} \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

(1) $R \bowtie R$ (2) $R \bowtie \emptyset$ (3) $R \bowtie \{\varepsilon\}$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{\, f : U \cup V \to \textbf{dom} \mid f_U \in R \text{ and } f_V \in S \,\},$$

where f_U and f_V are the restriction of f to elements in U and V, respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(1)

$$\begin{aligned} R \bowtie R &= \{\, f : U \cup U \to \textbf{dom} \mid f_U \in R \text{ and } f_U \in R \,\} \\ &= \{\, f : U \to \textbf{dom} \mid f_U \in R \,\} \\ &= \{\, f_U \mid f_U \in R \,\} \\ &= R \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

$$(1)\ R \bowtie R \qquad (2)\ R \bowtie \emptyset \qquad (3)\ R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{ f : U \cup V \to \textbf{dom} \mid f_U \in R \text{ and } f_V \in S \},$$

where f_U and f_V are the restriction of f to elements in U and V, respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(2)

$$R \bowtie \emptyset = \{ f : U \cup \emptyset \to \textbf{dom} \mid f_U \in R \text{ and } f_\emptyset \in \emptyset \}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

$$(1)\ R \bowtie R \qquad (2)\ R \bowtie \emptyset \qquad (3)\ R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{\, f : U \cup V \to \textbf{dom} \mid f_U \in R \text{ and } f_V \in S \,\},$$

where f_U and f_V are the restriction of f to elements in U and V, respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(2)

$$\begin{aligned} R \bowtie \emptyset &= \{\, f : U \cup \emptyset \to \textbf{dom} \mid f_U \in R \text{ and } f_\emptyset \in \emptyset \,\} \\ &= \emptyset \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.
Now for a table R, what are the results of the following expressions?

(1) $R \bowtie R$ (2) $R \bowtie \emptyset$ (3) $R \bowtie \{\varepsilon\}$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{\, f : U \cup V \to \textbf{dom} \mid f_U \in R \text{ and } f_V \in S \,\},$$

where f_U and f_V are the restriction of f to elements in U and V, respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

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Exercise 3

Exercise. Express the following operations using other operations presented in the lecture:

1. Intersection $R \cap S$.
2. Cartesian product $R \times S$.
3. Selection $\sigma_{n=a}(R)$ with a a constant.
4. Arbitrary constant tables in queries.

Solution.

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Solution.

1. Note that $R \cap S$ is well-defined only if the attributes of R and S coincide. Suppose that the common set of attributes is U. Then we have

$$R \cap S = \{ f : U \to \mathbf{dom} \mid f \in R \text{ and } f \in S \}$$

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&= \{ f : U \cup U \to \mathbf{dom} \mid f_U \in R \text{ and } f_U \in S \}
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2. Suppose R has attributes U and S has attributes V. Let W be a set of fresh attributes with $|W| = |V|$ and $W \cap U = \emptyset$. Then, $R \times S = R \bowtie \delta_{\vec{V} \to \vec{W}}(S)$.

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3. $\sigma_{n=a}(R) = R \bowtie \{\{n \mapsto a\}\}$
4. To create a constant table with a single row and many attribute-value pairs, simply join several single attribute-value pair constant tables (cf. query 7 in Exercise 1). Then use union to create a table with several rows.

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Exercise. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

1. $R \bowtie S = S \bowtie R$
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&= \{\, f : (U \cup V) \to \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S \,\} \bowtie T \\
&= (R \bowtie S) \bowtie T
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$$\pi_X(R \cup S) = \pi_X(R) \cup \pi_X(S)$$

Let $f \in \pi_X(R \cup S)$. Then there is some $f' \in R \cup S$ with $f'_X = f$ and hence $f \in \pi_X(R) \cup \pi_X(S)$.

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Let $f \in \pi_X(R \cup S)$. Then there is some $f' \in R \cup S$ with $f'_X = f$ and hence $f \in \pi_X(R) \cup \pi_X(S)$.
Conversely, let $f \in \pi_X(R) \cup \pi_X(S)$. Then $f \in \pi_X(R)$ or $f \in \pi_X(S)$, and there is some $f' \in R \cup S$ such that $f'_X = f$.
Thus $f \in \pi_X(R \cup S)$.

Exercise 4

Exercise. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

1. $R \bowtie S = S \bowtie R$
2. $R \bowtie (S \bowtie T) = (R \bowtie S) \bowtie T$
3. $\pi_X(R \circ S) = \pi_X(R) \circ \pi_X(S)$ for all $\circ \in \{\cup, \cap, -, \bowtie\}$
4. $\sigma_{n=m}(R \circ S) = \sigma_{n=m}(R) \circ \sigma_{n=m}(S)$ for all $\circ \in \{\cup, \cap, -\}$.
5. $\sigma_{n=m}(R \bowtie S) = \sigma_{n=m}(R) \bowtie S$, for n and m attributes of R only.

Why are these identities of interest?

Solution. These identities can be used to optimise queries, e.g., by pushing selection inwards so that joins receive smaller inputs.

3.2

$$\pi_X(R \cap S) = \pi_X(R) \cap \pi_X(S)$$

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$$\pi_X(R \cap S) = \pi_X(R) \cap \pi_X(S)$$

Consider tables $R = \{\{\, A \mapsto 1, B \mapsto 2 \,\}\}$ and $S = \{\{\, A \mapsto 1, B \mapsto 3 \,\}\}$.

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Consider tables $R = \{\!\{\, A \mapsto 1, B \mapsto 2 \,\}\!\}$ and $S = \{\!\{\, A \mapsto 1, B \mapsto 3 \,\}\!\}$.
Then $\pi_A(R \cap S) = \emptyset \subsetneq \pi_A(R) \cap \pi_A(S) = \{\!\{\, A \mapsto 1 \,\}\!\}$.

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Exercise. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

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$$\pi_X(R \bowtie S) = \pi_X(R) \bowtie \pi_X(S)$$

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Consider tables $R = \{\!\{ A \mapsto 1, B \mapsto 2 \}\!\}$ and $S = \{\!\{ A \mapsto 1, B \mapsto 3 \}\!\}$.
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$$\pi_X(R - S) = \pi_X(R) - \pi_X(S)$$

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Then $\pi_A(R - S) = \{\!\{ A \mapsto 1 \}\!\} \supsetneq \pi_A(R) - \pi_A(S) = \emptyset$.

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True, proof is analogous to 3.1.

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Exercise 5

Exercise. Let $R^{\mathcal{I}}$ and $S^{\mathcal{I}}$ be tables of schema $R[U]$ and $S[V]$, respectively. The *division* of $R^{\mathcal{I}}$ by $S^{\mathcal{I}}$, written as $(R^{\mathcal{I}} \div S^{\mathcal{I}})$, is defined to be the maximal table over the attributes $U \setminus V$ that satisfies $(R^{\mathcal{I}} \div S^{\mathcal{I}}) \bowtie S^{\mathcal{I}} \subseteq R^{\mathcal{I}}$. Note that the joined tables here do not have any attributes in common, so the natural join works as a cross product.
Consider the following table and use the division operator to (1) express a query for the cities that have been visited by all people.

Visited

Person	*City*
Tomas	Berlin
Markus	Santiago
Markus	Berlin
Fred	New York
Fred	Berlin

Then, (2) express division using the standard relational algebra operators.

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Then, (2) express division using the standard relational algebra operators.
Solution.

(1)

$$Visited \div \pi_{Person}(Visited)$$

(2) Let X be the set of all attributes of R that are not attributes of S (i.e., $X = U \setminus V$).

$$R \div S = \pi_X(R) - \pi_X[(\pi_X(R) \bowtie S) - R]$$

Exercise 6

Exercise. Suggest how to write the relational algebra operations for using the unnamed perspective. What changes?

Solution.

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- Order matters in projections.

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Solution.

- Natural join becomes cartesian product $\times$.
- No renaming.
- Order matters in projections.
- New set of operators: $\{\sigma, \pi, \cup, -, \times\}$.

Exercise 7

Exercise. The set of operations $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$ can express all queries of relational algebra. Show that it is not possible to reduce this set any further.
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Solution.

- Operator δ is the only one that can rename attributes in tables.

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Solution.

- Operator δ is the only one that can rename attributes in tables.
- Operator π is the only one that can produce tables with less attributes.

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  1. Let $\mathcal{D}$ be the database containing the tables $R = \{\{A \mapsto 1\}\}$ and $S = \{\{A \mapsto 2\}\}$.

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  3. Let q be a query constructed using only $\{\sigma, \pi, -, \bowtie, \delta\}$.
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  5. Then, $q(\mathcal{D}) \neq \{\{A \mapsto 1\}, \{A \mapsto 2\}\}$.
  6. The query language $\{\sigma, \pi, -, \bowtie, \delta\}$ is less expressive than $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$.

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Exercise. The set of operations $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$ can express all queries of relational algebra. Show that it is not possible to reduce this set any further.

Solution.

- Operator δ is the only one that can rename attributes in tables.
- Operator π is the only one that can produce tables with less attributes.
- Operator $\bowtie$ is the only one that can produce tables with more attributes.
- Operator $\cup$ cannot be removed.
- Operator σ cannot be removed:
  1. Let $\mathcal{D}$ be the database containing the table $R = \{\{A \mapsto 1\}, \{A \mapsto 2\}\}$.
  2. Then, $\sigma_{A=B}(R \bowtie \delta_{A\to B}(R))(\mathcal{D}) = \{\{A \mapsto 1, B \mapsto 1\}, \{A \mapsto 2, B \mapsto 2\}\}$.
  3. Let q be a query constructed using only $\{\pi, \cup, -, \bowtie, \delta\}$.
  4. Via induction, we can show that every intermediate table T produced in the evaluation of q over $\mathcal{D}$ satisfies the following property: if T has n attributes, then T contains 2^n rows featuring every single combination of the symbols 1 and 2.

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