

# Formal Concept Analysis

## III Knowledge Discovery

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# Agenda

## 6 Triadic Formal Concept Analysis

- Motivation
- Folksonomies
- Triadic Formal Concepts
- Concept-Tri-Lattice
- Visualization of Tri-Lattices
- Iceberg Tri-Lattices
- Computing Tri-Concepts
- Evaluation
- Neighborhoods

# Motivation: Collaborative Tagging Systems

**flickr** von YAHOO!

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[Startseite](#) [Die Tour](#) [Registrieren](#) [Entdecken](#) ▾

Suchen ▾

## Entdecken / Tags / nature

Diashow

Sortieren nach:  
Neueste • [Interessanteste](#)

**Cluster mit dem Tag nature**  
Entdecken und filtern Sie diese Liste mit dem Tag nature mit unserem tollen Cluster-Feature!

Dazu passende Tags:  
[macro](#) [flower](#) [green](#) [landscape](#) [trees](#) [sky](#) [water](#) [insect](#) [flowers](#) [leaves](#)

Ähnliche Inhalte mit [der Yahoo! Bildersuche suchen](#)

Sponsoren-Links  
[PureNature Versand](#)  
Hier finden Sie alles für ein gesundes, allergiefreies Leben!  
[www.PureNature.de](#)  
[Natururlaub in Frankreich](#)  
Natur pur und nachhaltige Konzepte  
[de.francequids.com/bakoteurismus](#)

Von [andy.v](#) Von [Tony Reilly1959](#) Von [Tony Reilly1959](#) Von [andy.v](#)

Von [Tony Reilly1959](#) Von [Detch](#) Von [Giorgio](#) Von [Carme M.V.](#)

Von [Vanquard1219](#) Von [Vanquard1219](#) Von [Diane S...](#) Von [Claytona](#)

# Motivation: Collaborative Tagging Systems

The screenshot shows a web browser displaying search results on the Vimeo platform. On the left, a sidebar from flickr is partially visible. The main content area features the Vimeo logo, a search bar with the query 'nature', and a result count of 12,812 videos. Below the search bar are filters for 'Show me' (set to 'newest'), 'videos in' (set to 'thumbnail'), and 'format'. Three video thumbnails are displayed in a row, each with a title and upload time: 'Time' (2 hours ago), 'Flies' (2 hours ago), and 'Nurture' (4 hours ago). The 'Time' video thumbnail shows a close-up of a textured surface, 'Flies' is a solid black image, and 'Nurture' shows a brick wall. To the right of the videos is a 'Sponsored Links' section with two entries: 'Human Nature Explained' and 'Visit Lapland'. Further right is an advertisement for Vimeo+ and a section titled 'Do more with tags'.

flickr  
Startseite Di

Entdeck

Sortieren nach:  
Neueste • [Interes](#)

[Cluster mit de](#)  
Entdecken und fil  
mit dem Tag natu  
tollen Cluster-Fea

Dazu passende T  
[macro](#) [flower](#) [g](#)  
[trees](#) [sky](#) [water](#)  
[leaves](#)

Ähnliche Inha  
[der Yahoo! B](#)

Sponsore  
[PureNature Ve](#)  
Hier finden Sie all  
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[www.PureNature](#)

[Natururlaub in](#)  
[Natur pur und na](#)  
[ds.francequids.c](#)

**vimeo**

Join **vimeo** Log In Explore Help

Videos tagged:

Videos 1-12 of 12,812

Show me  videos in  format

**Human Nature Explained** Sponsored Links  
The breakthrough world-transforming explanation of the human condition  
[www.WorldTransformation.com/](#)

**Visit Lapland**  
Watch the Polar Lights From the Untouched Wilderness  
[www.HemavanTarnaby.se](#)

**Time** 2 hours ago

**Flies** 2 hours ago

**Nurture** 4 hours ago

Advertisement

Vimeo + a  
Vim

Do more with tags

Hey, did you know yo  
and all these videos s  
Just click on the "Sub  
never miss a video wi

# Motivation: Collaborative Tagging Systems

The image displays three side-by-side screenshots of collaborative tagging systems:

- flickr:** Shows a sidebar with navigation links like "Startseite", "Entdecken", and "Sortieren nach: Neueste". It features a "Cluster mit dem Tag" section and a "Sponsor" section for "PureNature".
- vimeo:** Displays "Videos tagged" with "Videos 1-12 of 12,812". It includes a "Show me" dropdown set to "newest" and a video titled "Human Nature Explained" with a thumbnail image.
- delicious:** Shows a "Recent nature Bookmarks" section with a "Tags" input field containing "nature". It lists several bookmarked items with their respective tags, such as "yosemite digitalphoto's Photos- powered by SmugMug" (tags: penb, blog, blogs, california, photo) and "Albino Alligator Photo, Animal Wallpaper – National Geographic Photo" (tags: animals, images, photography).

# Motivation: Collaborative Tagging Systems

http://www.bibsonomy.org/user/jaeschke

**BibSonomy** user :: jaeschke ::  search (Robert Jäschke) CV

THE BLUE SOCIAL BOOKMARK AND PUBLICATION SHARING SYSTEM

home myBibSonomy add post groups popular

logged in as jaeschke logout

### BOOKMARKS (1141)

- Twitter Calendar**  
http://statuscalendar.cs.washington.edu/  
17 hours and 29 minutes ago by jaeschke  
Information named entity ner calendar twitter e...  
★★★★★ (0)
- Feature of the week: CSL via REST-API**  
http://blog.bibsonomy.org/2013/01/feature-of-week-csl-via-rest-a...  
3 days and an hour ago by jaeschke  
bibsonomynews fofw csl bibsonomy rest  
★★★★★ (0)
- prisma.de: It Might Get Loud**  
http://www.prisma.de/film/2008\_it\_might\_get\_loud/fernsehen.html  
4 days ago by jaeschke as **private**  
program tv music  
★★★★★ (0)
- ORCID: Robert Jäschke**  
http://orcid.org/0000-0003-3271-9853  
7 days ago by jaeschke  
orcid science myown research publication  
★★★★★ (0)
- BibSonomy**  
http://www.jibs.uni-hannover.de/~jaeschke/bibsonomy/  
7 days ago by jaeschke  
taoqino collaborative social bookmarking bibso...

### PUBLICATIONS (726)

- The Wiki way: quick collaboration on the Web** 21  
Bo Leuf, and Ward Cunningham. Addison-Wesley, London, (Marc...  
7 days ago by jaeschke  
collaboration wiki management web knowledge  
★★★★★ (0)
- Best Practices for Scientific Computing** 3  
Greg Wilson, D. A. Aruliah, C. Titus Brown, Neil P. Chue Hong, Ma...  
9 days ago by jaeschke  
programming science computing research  
★★★★★ (0)
- Understanding the internet: a socio-cultural ..** 4  
Bridgette Wessets. Palgrave Macmillan, Houndmills, Basingstok...  
9 days ago by jaeschke  
stair understanding culture internet  
★★★★★ (0)
- The no-nonsense guide to equality** 2  
Daniel Döring, Kate Pickle, and Richard G. Wilkinson. New Intern...  
9 days ago by jaeschke  
stair income justice guide equality  
★★★★★ (0)
- So You Think You Know About Britain?** 3  
Daniel Döring. Constable & Robinson, London, (2011)  
9 days ago by jaeschke  
stair britain georaphy

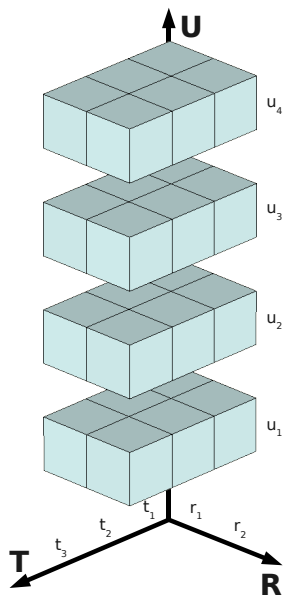
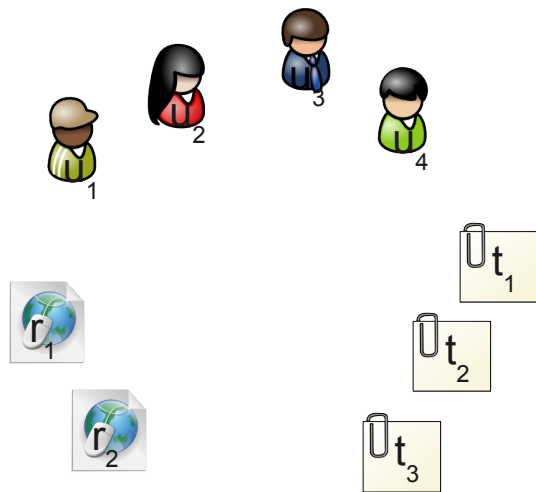
### concepts (show all | hide all)

- author ← newman
- conference ← ecal ecmpldki gvd loes lodm icfca recays
- folksonomy ← bookmarking tagging
- geo ← gps map um
- howto ← manual reference tutorial
- location ← anhalt berlin bitterfeld blad cele dagstuhl dresden europe frankfurt hannover hessen kalen kassel london magdeburg ort saarland sachsen sachsen\_anhalt toulouse tübingen wadmern wittenberg wüzburg
- ort ← location
- programming ← ada c fortran java jsp perl prog python ruby
- protocol ← ftp http smtp
- researcher ← devaultze stammon turing
- science ← chemistry math
- software ← apache beagle cocoa debian eclipse firefox haystack nextstep photoshop protege thunderbird weka wine word x11 zope

- manage your web bookmarks and publication references
- open for the public since beginning of 2006, > 5 000 active users
- developed and operated at L3S Research Center

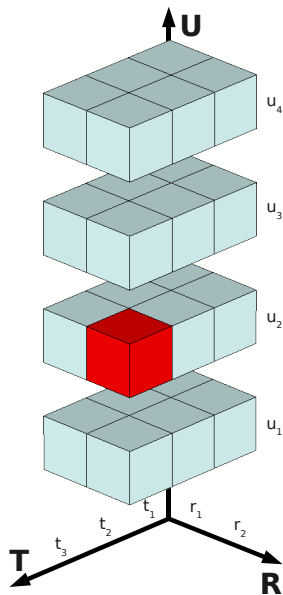
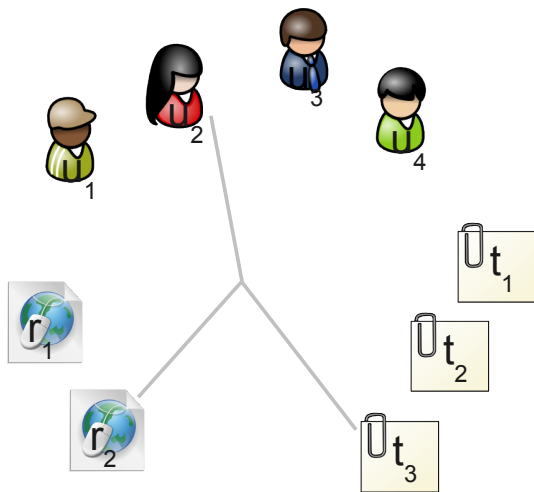


# Folksonomies: Hypergraph, Tensor

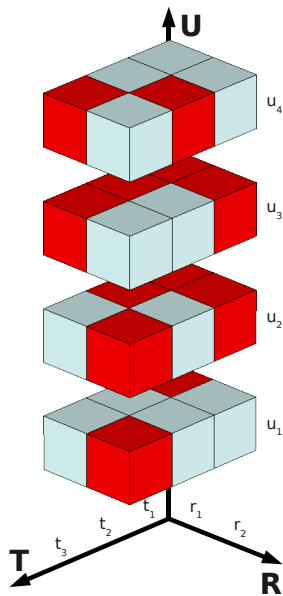
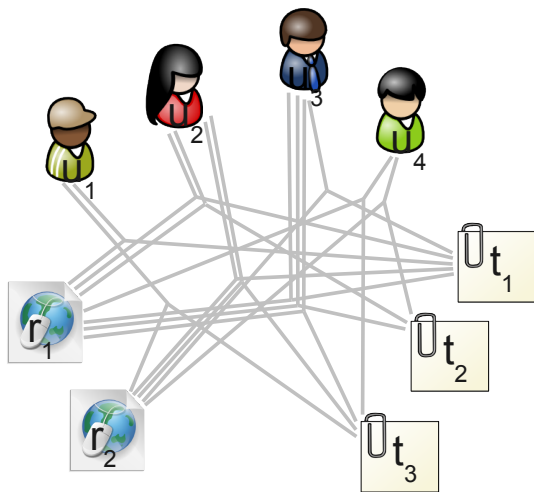




# Folksonomies: Hypergraph, Tensor



# Folksonomies: Hypergraph, Tensor

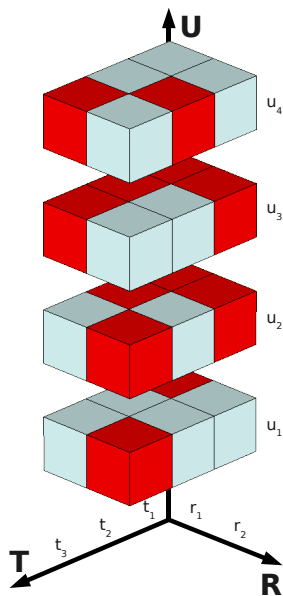
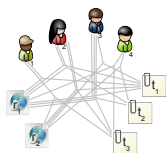


# Folksonomies

## Definition (Folksonomy)

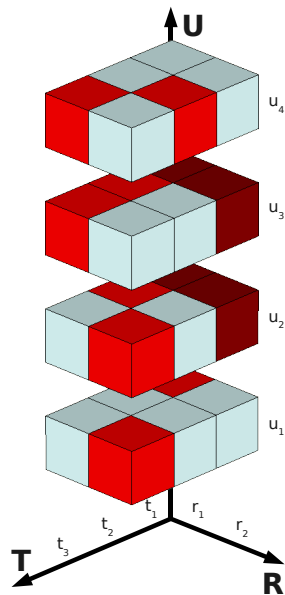
$\mathbb{F} := (U, T, R, Y)$  with

- $U, T, R$  finite sets of users, tags, and resources, resp.
- $Y \subseteq U \times T \times R$  ternary relation
- tripartite hypergraph
- boolean 3-dimensional tensor
- triadic formal context



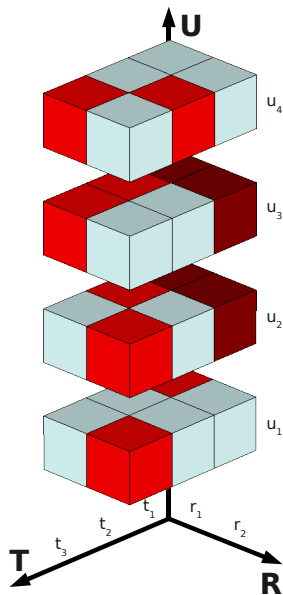
# Folksonomies

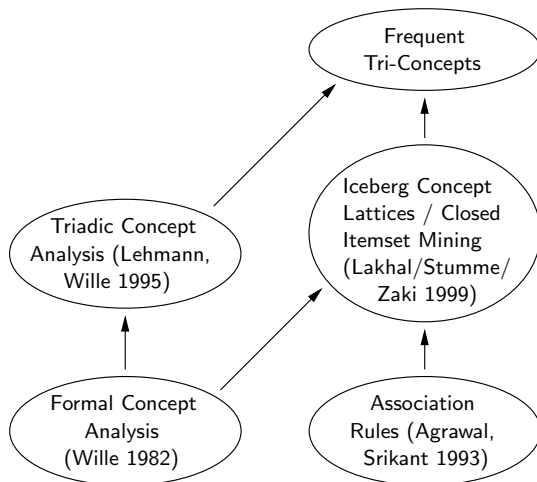
- conceptual clustering of folksonomies
  - ▶ find interesting concepts/clusters
  - ▶ support browsing, community detection, recommendations
  - ▶ get an overview into the structure of a folksonomy



# Folksonomies

- conceptual clustering of folksonomies
  - find interesting concepts/clusters
  - support browsing, community detection, recommendations
  - get an overview into the structure of a folksonomy
- *tri-concept*  $(A, B, C) \subseteq U \times T \times R$ :  
maximal cuboid in which every user from  $A$  has tagged every resource from  $C$  with all tags from  $B$ 
  - shared conceptualization

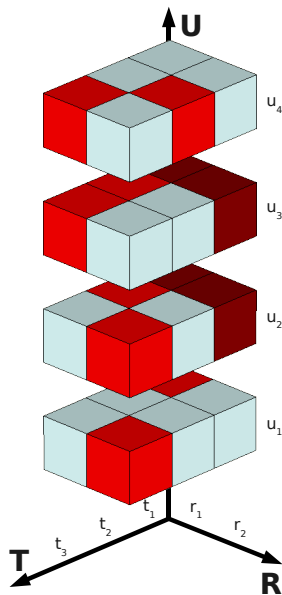




# Folksonomies

We regard  $\mathbb{F} = (U, T, R, Y)$  as *triadic formal context*.

In general, the elements of  $U$ ,  $T$  and  $R$  are then called *objects*, *attributes* and *conditions* and  $(u, t, r) \in Y$  is read as “object  $u$  has the attribute  $t$  under condition  $r$ ”.

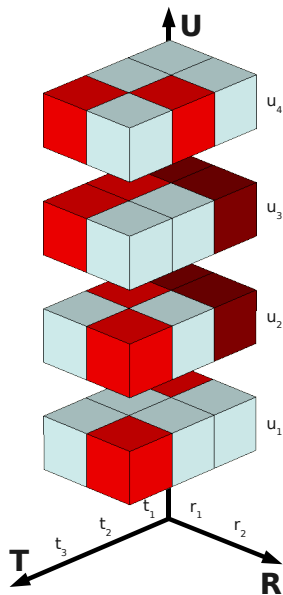


# Triadic Formal Concepts

## Definition (tri-concept)

triple  $(A, B, C)$  with  $A \subseteq U$ ,  $B \subseteq T$ ,  $C \subseteq R$  and  $A \times B \times C \subseteq Y$ , such that none of the three components can be enlarged without violating the condition  $A \times B \times C \subseteq Y$ . We call  $A$  the *extent*,  $B$  the *intent* and  $C$  the *modus* of the formal tri-concept.

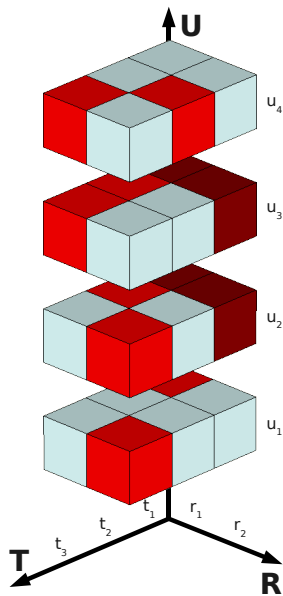
→ natural extension of formal concepts





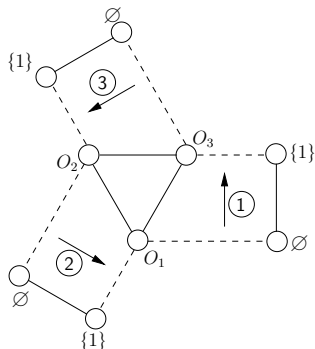
# Concept-Tri-Lattice

- three quasi orders  $\lesssim_1, \lesssim_2, \lesssim_3$ :  
 $(A_1, A_2, A_3) \lesssim_i (B_1, B_2, B_3)$   
 $:\Leftrightarrow A_i \subseteq B_i$ , for  $i = 1, 2, 3$ .
- *not antisymmetric*, i. e. from  
 $(A_1, A_2, A_3) \lesssim_i (B_1, B_2, B_3)$  and  
 $(B_1, B_2, B_3) \lesssim_i (A_1, A_2, A_3)$  does not  
follow  $(A_1, A_2, A_3) = (B_1, B_2, B_3)$
- *concept tri-lattice*  $\underline{\mathfrak{B}}(\mathbb{K})$  of the triadic  
context  $\mathbb{K}$
- not a real (mathematical) lattice!

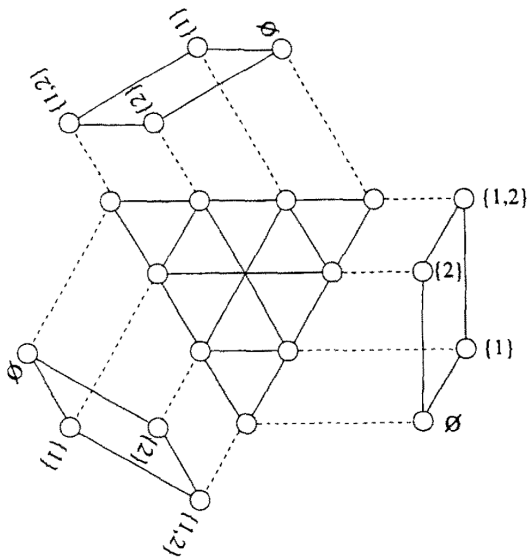


# Visualization of Tri-Lattices

- Since it is not really a lattice, we can not draw a lattice diagram
- Alternative:
  - every quasi-order is written along the edge of a virtual triangle
  - the tri-concepts are drawn into the triangle
- example to the right: smallest non-trivial tri-lattice  
 $\mathbb{B}_3 = \underline{\mathfrak{B}}(\{1\}, \{1\}, \{1\}, \emptyset)$
- visualization not always possible
  - satisfied tetrahedron condition
  - violated Thomson condition

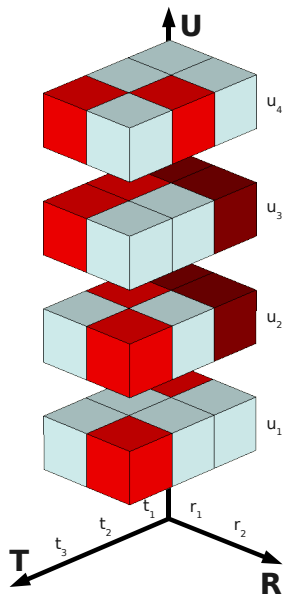


# Visualization of Tri-Lattices



# Iceberg Tri-Lattices

- Given support constraints  $\tau_u, \tau_t, \tau_r$  :  
tri-concept  $(A, B, C)$  frequent  
 $:\Leftrightarrow |A| \geq \tau_u, |B| \geq \tau_t, \text{ and } |C| \geq \tau_r$   
  
 $\rightarrow$  iceberg tri-lattice

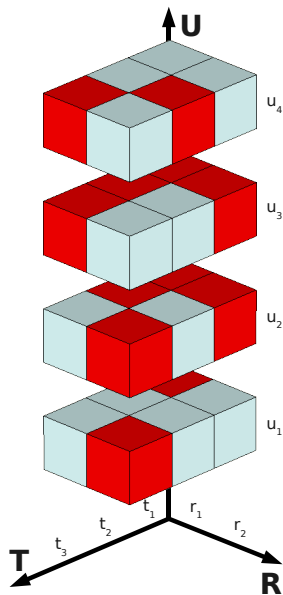


# Computing Tri-Concepts

- Given
  - sets  $U, T, R$
  - ternary relation  $Y \subseteq U \times T \times R$
  - support constraints  $\tau_u, \tau_t, \tau_r$
- Find  $(A, B, C)$  with
  - $A \subseteq U, B \subseteq T, C \subseteq R$
  - $|A| \geq \tau_u, |B| \geq \tau_t, |C| \geq \tau_r$
  - $A \times B \times C \subseteq Y$
  - such that none of the sets  $A, B$  or  $C$  can be enlarged without violating the former condition

# Computing Tri-Concepts

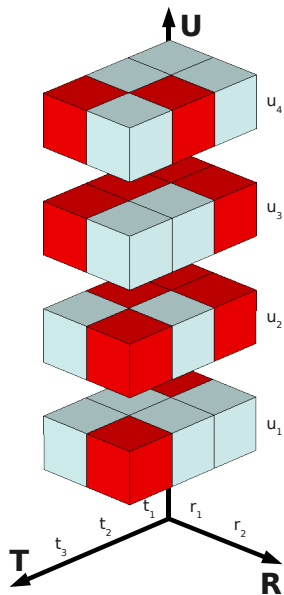
computes the iceberg tri-lattice of a triadic formal context



# Computing Tri-Concepts

computes the iceberg tri-lattice of a triadic formal context

## Algorithm

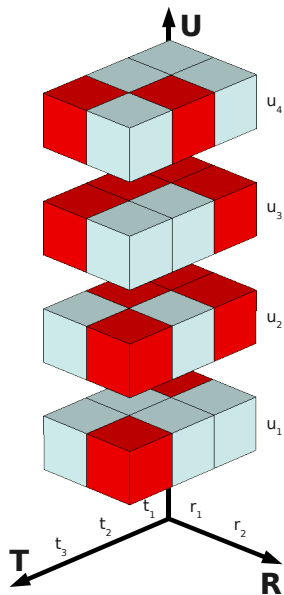


# Computing Tri-Concepts

computes the iceberg tri-lattice of a triadic formal context

## Algorithm

- Let  $\tilde{Y} := \{(u, (t, r)) \mid (u, t, r) \in Y\}$



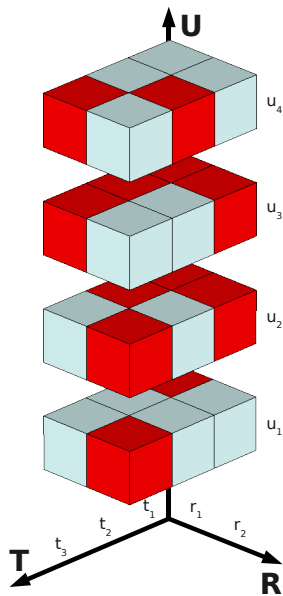


# Computing Tri-Concepts

computes the iceberg tri-lattice of a triadic formal context

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- Let  $\tilde{Y} := \{(u, (t, r)) \mid (u, t, r) \in Y\}$
- Loop: Find (frequent) concepts  $(\mathbf{A}, I)$  in  $(U, T \times R, \tilde{Y})$



# Computing Tri-Concepts

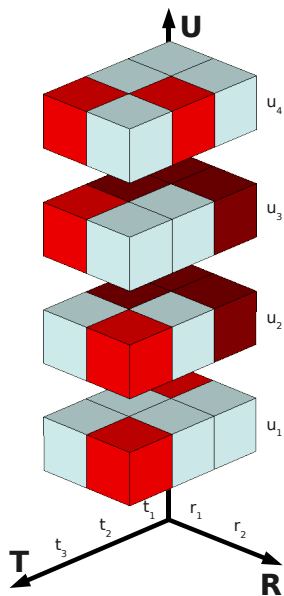
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In the example:

$$(\mathbf{A}, I) = (\{u_2, u_3\}, \{(t_1, r_1), (t_1, r_2), (t_2, r_1)\})$$



# Computing Tri-Concepts

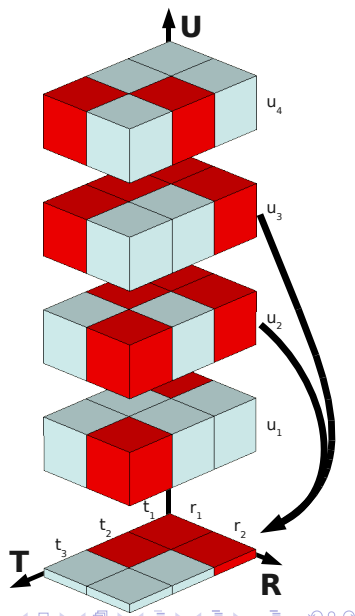
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  - Loop: Find (frequent) concepts  $(\mathbf{B}, C)$  in  $(T, R, I)$

In the example:

$(T, R, I) = (T, R, \{(t_1, r_1), (t_1, r_2), (t_2, r_1)\})$



# Computing Tri-Concepts

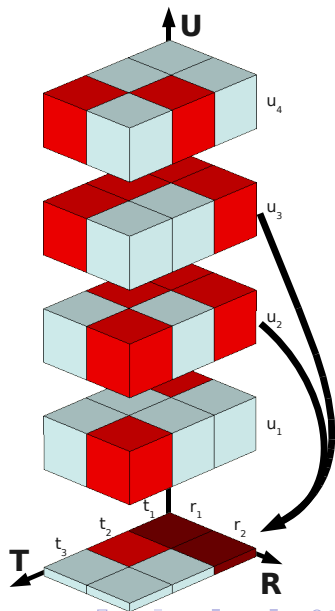
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  - Loop: Find (frequent) concepts  $(\mathbf{B}, C)$  in  $(T, R, I)$

In the example:

$$(B, C) = (\{t_1\}, \{r_1, r_2\})$$



# Computing Tri-Concepts

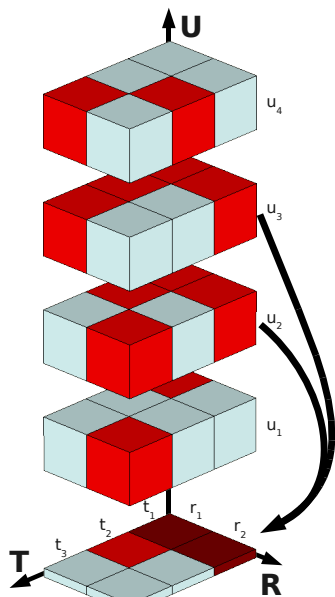
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  - Loop: Find (frequent) concepts  $(\mathbf{B}, C)$  in  $(T, R, I)$ 
    - ★ If  $\mathbf{A} = (\mathbf{B} \times \mathbf{C})^{\tilde{Y}}$ , then output  $(\mathbf{A}, \mathbf{B}, \mathbf{C})$

In the example:

$$(\mathbf{B} \times \mathbf{C})^{\tilde{Y}} = (\{t_1\} \times \{r_1, r_2\})^{\tilde{Y}}$$



# Computing Tri-Concepts

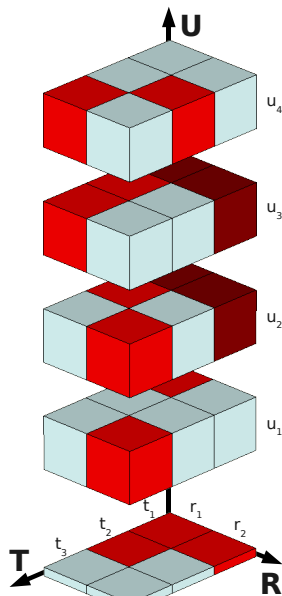
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    - ★ If  $\mathbf{A} = (\mathbf{B} \times \mathbf{C})^{\tilde{Y}}$ , then output  $(\mathbf{A}, \mathbf{B}, \mathbf{C})$

In the example:

$$\begin{aligned}(B \times C)^{\tilde{Y}} &= (\{t_1\} \times \{r_1, r_2\})^{\tilde{Y}} \\ &= \{u_2, u_3\} = A\end{aligned}$$



# Computing Tri-Concepts

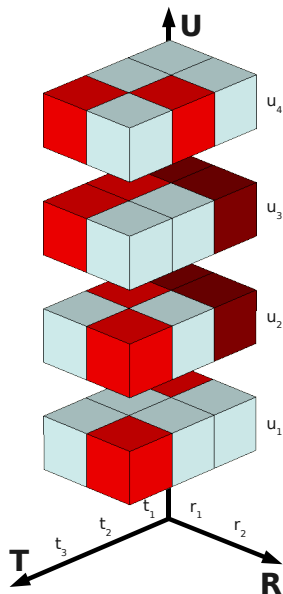
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## Algorithm

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    - ★ If  $\mathbf{A} = (\mathbf{B} \times \mathbf{C})^{\tilde{Y}}$ , then output  $(\mathbf{A}, \mathbf{B}, \mathbf{C})$

In the example:

$$(A, B, C) = (\{u_2, u_3\}, \{t_1\}, \{r_1, r_2\})$$



# Computing Tri-Concepts

**Require:**  $U, T, R, Y, \tau_u, \tau_t, \tau_r$

- 1:  $\tilde{Y} := \{(u, (t, r)) \mid (u, t, r) \in Y\}$
- 2:  $(A, I) := \text{FirstFrequentConcept}((U, T \times R, \tilde{Y}), \tau_u)$
- 3: **repeat**
- 4:   **if**  $|I| \geq \tau_t \cdot \tau_r$  **then**
- 5:      $(B, C) := \text{FirstFrequentConcept}((T, R, I), \tau_t)$
- 6:     **repeat**
- 7:       **if**  $|C| \geq \tau_r$  **then**
- 8:         **if**  $A = (B \times C)^{\tilde{Y}}$  **then**
- 9:         **print** A,B,C
- 10:        **end if**
- 11:       **end if**
- 12:     **until not**  $\text{NextFrequentConcept}((B, C), (T, R, I), \tau_t)$
- 13:   **end if**
- 14: **until not**  $\text{NextFrequentConcept}((A, I), (U, T \times R, \tilde{Y}), \tau_u)$



# Computing Tri-Concepts

The *FirstFrequentConcept* method:

**Require:**  $(G, M, I), \tau$

- 1:  $A := \emptyset^I$
- 2:  $B := A^I$
- 3: **if**  $|A| < \tau$  **then**
- 4:     $NextFrequentConcept((A, B), (G, M, I), \tau)$
- 5: **end if**
- 6: **return**  $(A, B)$

# Computing Tri-Concepts

the *NextFrequentConcept* method:

**Require:**  $(A, B), (G, M, I), \tau$

```
1:  $i := \max(M)$ 
2: while  $\text{defined}(i)$  do
3:    $A := (B \bullet i)^I$ 
4:   if  $|A| \geq \tau$  then
5:      $D := A^I$ 
6:     if  $B <_i D$  then
7:        $B := D$ 
8:       return true
9:     end if
10:  end if
11:   $i := \max(M \setminus B \cap \{1, \dots, i - 1\})$ 
12: end while
13: return false
```

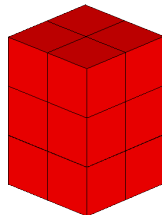
# Evaluation

## BibSonomy Dataset:

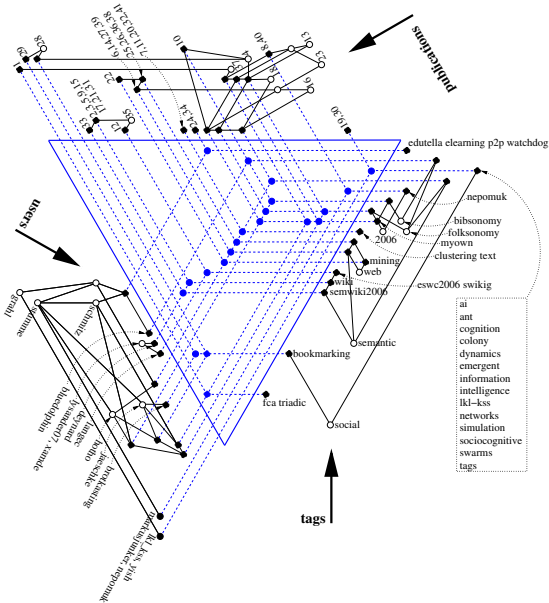
- all publication records until November 23rd, 2006
- removed: DBLP, posts with the tag “imported”
- $|U| = 262$ ,  $|T| = 5\,954$ ,  $|R| = 11\,101$ ,  $|Y| = 44\,944$

## Result:

- 13 992 tri-concepts (75 minutes on a 2 GHz PC)
- with support constraints  $\tau_u = 3$ ,  $\tau_t = 2$ ,  $\tau_r = 2$ :
  - 21 tri-concepts
  - contain 41 publications, 15 users and 36 tags

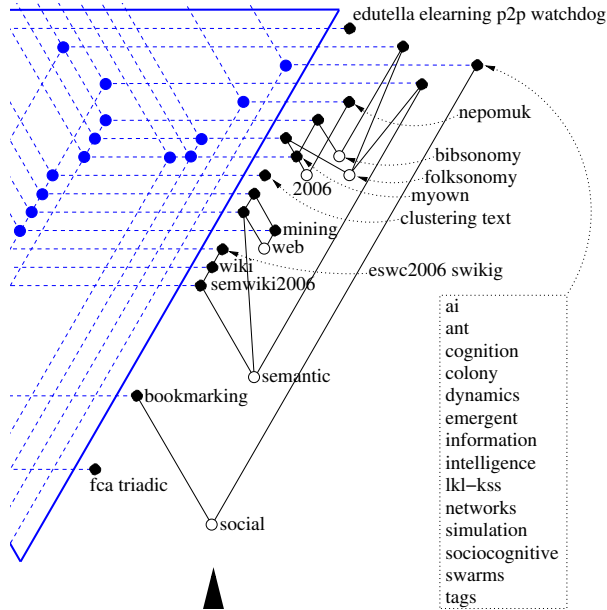


# Evaluation



visualisation of the iceberg  
tri-lattice for  $\tau_u = 3$ ,  $\tau_t = 2$ ,  
 $\tau_r = 2$

# Evaluation



two topical groups:

- semantic
- social

*semantic* further divided:

- wiki
- web
- folksonomy

# Neighborhoods

The visualization of tri-lattices is . . .

- at the moment manual work,
- time-intensive and pretty complicated,
- or even impossible (cf. *tetrahedron condition* and *Thomson condition*).

Thus: easier visualization option desirable

# Neighborhoods

## Idea:

- We regard tri-concepts as nodes in a graph.
- We connect two tri-concepts with an edge, when they contain the same tags, users, or resources.

## More formally:

- Two tri-concepts  $(A_1, A_2, A_3)$  and  $(B_1, B_2, B_3)$  are *neighbors*, if for an  $i \in \{1, 2, 3\}$  it holds  $A_i = B_i$ .
- neighbor relation  $\sim \subseteq (\underline{\mathfrak{B}}(\mathbb{F}) \times \underline{\mathfrak{B}}(\mathbb{F}))$
- The *neighborhood graph* then is  $(\underline{\mathfrak{B}}(\mathbb{F}), \sim)$ .

# Neighborhoods

neighborhood graph for the tri-concept

$(\{jaeschke, schmitz, stumme\}, \{fca, triadic\}, \{1, 37\})$

bluedolphin, grahl, schmitz, stumme  
bibsonomy, folksonomy  
4, 10

hotho, schmitz, stumme  
2006, bibsonomy, myown  
4, 10

hotho, schmitz, stumme  
2006, folksonomy, myown  
4, 13, 16, 18

hotho, schmitz, stumme  
2006, myown  
4, 10, 13, 16, 18, 23

jaeschke, schmitz, stumme  
2006, myown  
4, 8, 13, 18, 23, 37, 40

jaeschke, schmitz, stumme  
fca, triadic  
1, 37



hotho, jaeschke, schmitz, stumme  
2006, folksonomy, myown  
4, 13, 18

hotho, jaeschke, schmitz, stumme  
2006, myown  
4, 13, 18, 23