Concurrency Theory

Lecture 7: Abstraction from Internal Activities

Stephan Mennicke
Knowledge-Based Systems Group

June 6, 2023

Recap: CCS

$\mathcal{N} = \{a, b, c, \dots\}$ ... set of names ($\tau \notin \mathcal{N}$)

$\overline{\mathcal{N}} = \{\overline{\alpha} \mid \alpha \in \mathcal{N}\}$ ... set of conames

$Act = \mathcal{N} \cup \overline{\mathcal{N}} \cup \{\tau\}$ (note, there is no $\overline{\tau}$ and for $\alpha \in Act \setminus \{\tau\}$, $\overline{\overline{\alpha}} = \alpha$)

The set of (CCS) processes *Pr* is defined by

$$P \quad ::= \quad \mathbf{0} \;\Big|\; \mu.P \;\Big|\; P + P \;\Big|\; P \mid P \;\Big|\; (\nu a)(P) \;\Big|\; K$$

where $\mu \in Act$, $a \in \mathcal{N}$, and $K \in \mathcal{K}$.

Define the language CCS parameterized over Act, $\mathcal{K}$, and $\mathcal{T}_{\mathcal{K}} \subseteq \mathcal{K} \times Act \times Pr$.

$$\mathrm{CCS}(Act, \mathcal{K}, \mathcal{T}_{\mathcal{K}})$$

Recap: SOS of CCS

$\text{CCS}(Act, \mathcal{K}, \mathcal{T}_{\mathcal{K}})$ specifies an LTS $(Pr, Act, \rightarrow \cup \mathcal{T}_{\mathcal{K}})$ where $\rightarrow \subseteq (Pr \setminus \mathcal{K}) \times Act \times Pr$ is the smallest relation satisfying the following rules:

$$\text{(Pref)}\ \frac{}{\mu.P \xrightarrow{\mu} P}$$

$$\text{(SumL)}\ \frac{P \xrightarrow{\mu} P'}{P + Q \xrightarrow{\mu} P'} \qquad \text{(SumR)}\ \frac{Q \xrightarrow{\mu} Q'}{P + Q \xrightarrow{\mu} Q'}$$

$$\text{(ParL)}\ \frac{P \xrightarrow{\mu} P'}{P \mid Q \xrightarrow{\mu} P' \mid Q} \qquad \text{(ParR)}\ \frac{Q \xrightarrow{\mu} Q'}{P \mid Q \xrightarrow{\mu} P \mid Q'}$$

$$\text{(Com)}\ \frac{P \xrightarrow{\mu} P' \quad Q \xrightarrow{\overline{\mu}} Q'}{P \mid Q \xrightarrow{\tau} P' \mid Q'} \qquad \text{(Res)}\ \frac{P \xrightarrow{\mu} P'}{(\nu a)\,(P) \xrightarrow{\mu} (\nu a)\,(P')}\ \text{if } a \notin \{\mu, \overline{\mu}\}$$

An Example

Let $P = (\nu a)\,(b.\overline{a} \mid a.c)$ and consider processes $Q_1 = b.\tau.c$ and $Q_2 = b.c$.

Referring to τ as a silent – unobservable or internal – action, let bisimilarity appear as insufficient.

In analogy, consider the programs

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print(5)
```

and

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if true then print(5) else skip
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Resolution: weak LTSs, weak transitions, weak bisimilarity

Abstraction from Internal Activities

Definition 1 (Weak Transitions)

- Relation $\Longrightarrow$ is the reflexive and transitive closure of $\xrightarrow{\tau}$.
- For all $\mu \in Act$, relation $\stackrel{\mu}{\Longrightarrow}$ is the composition of the relations $\Longrightarrow$, $\xrightarrow{\mu}$, and $\Longrightarrow$; meaning $P \stackrel{\mu}{\Longrightarrow} P'$ holds if there are P_1 and P_2 such that $P \Longrightarrow P_1 \xrightarrow{\mu} P_2 \Longrightarrow P'$.

Definition 2
An LTS is *image-finite under weak transitions* if $\stackrel{\mu}{\Longrightarrow}$ is image-finite.

Consider CCS with every constant having only finitely many transitions. Is the resulting LTS image-finite under weak transitions?

Weak Bisimulation

Weak bisimilarity is defined in terms of weak transitions. The bisimilarity we already know ($\leftrightarroweq$) is called *strong bisimilarity*, in analogy.

Definition 3 (Weak Bisimilarity)
A process relation $\mathcal{R}$ is a *weak bisimulation* if, whenever $P \mathrel{\mathcal{R}} Q$, we have

1. for all P' and $l \in Act \setminus \{\tau\}$ with $P \stackrel{l}{\Rightarrow} P'$, there is Q' such that $Q \stackrel{l}{\Rightarrow} Q'$ and $P' \mathrel{\mathcal{R}} Q'$;
2. for all P' with $P \stackrel{\tau}{\Rightarrow} P'$, there is Q' such that $Q \Longrightarrow Q'$ and $P' \mathrel{\mathcal{R}} Q'$;
3. the converse of 1 and 2 on actions from Q.

The union of all weak bisimulations is called *weak bisimilarity* ($\leftrightarroweq_w$).

Weak Bisimulation: Alternative Definitions

Define

$$\overset{\hat{\mu}}{\Longrightarrow} := \begin{cases} \overset{\mu}{\Longrightarrow} & \text{if } \mu \neq \tau \\ \Longrightarrow & \text{otherwise.} \end{cases}$$

Lemma 4
A process relation $\mathcal{R}$ is a weak bisimulation if, and only if, if $P \; \mathcal{R} \; Q$ implies

1. *whenever $P \overset{\hat{\mu}}{\Longrightarrow} P'$, there is Q' such that $Q \overset{\hat{\mu}}{\Longrightarrow} Q'$ and $P' \; \mathcal{R} \; Q'$;*
2. *the converse on actions from Q.*

Weak Bisimulation: The Classic

Lemma 5
A process relation $\mathcal{R}$ *is a weak bisimulation if, and only if, if* $P \mathrel{\mathcal{R}} Q$ *implies*

1. *whenever* $P \xrightarrow{\mu} P'$*, there is* Q' *such that* $Q \xRightarrow{\hat{\mu}} Q'$ *and* $P' \mathrel{\mathcal{R}} Q'$*;*
2. *the converse on actions from* Q*.*

Definition 6
We say that $n \in \mathbb{N}$ is the *weight of* $P \Longrightarrow P'$ if there are $P_1, \ldots, P_n$ with $P_n = P'$ and $P \xrightarrow{\tau} P_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} P_n$.

Weak Bisimilarity & Congruence

Lemma 7
$\leftrightarroweq_w$ *is preserved by the operators of parallel composition, restriction, and prefixing.*

$\leftrightarroweq_w$ is not preserved by the choice operator.

Definition 8 (Rooted Weak Bisimilarity)
Two processes P and Q are *rooted weakly bisimilar*, denoted $P \leftrightarroweq_r Q$, if for all $\mu \in Act$, (1) for all P' with $P \xrightarrow{\mu} P'$ there is Q' such that $Q \overset{\mu}{\Longrightarrow} Q'$ and $P' \leftrightarroweq_w Q'$; (2) the converse on actions from Q.

Lemma 9
$\leftrightarroweq \subsetneq \leftrightarroweq_r \subsetneq \leftrightarroweq_w$.

Theorem 10
$\leftrightarroweq_r$ *is a congruence for CCS.*

Detour: Divergence

Definition 11 (Divergent Process)
A process P *diverges*, written $P \Uparrow$, if it has an ω-trace under τ. That is, divergence is the largest predicate $\Uparrow$ on processes such that $P \Uparrow$ implies $P \xrightarrow{\tau} P'$ and $P' \in \Uparrow$.

An LTS is divergence-free of no process of the LTS diverges.

Outlook

- Alternative approach to behavioral equivalence: testing
- Alternative model: Carl Adam Petri and his Nets
- What is decidable about Petri nets?
- Enhancing CCS: the π-calculus