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Game Description Language

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Previously ...

- In a **finite repeated game**, a two-player normal-form game is repeated for a fixed number of times; cooperation cannot be expected in this case.
- In a **random repeated game**, the end of interaction can not be predicted for sure; cooperation can emerge for large enough continuation probabilities, but equilibria make no specific predictions.
- A **noisy repeated game** may have implementation/perception errors.
- An **evolutionarily stable strategy** is a Nash equilibrium that performs better against "mutants" than the "mutants" against themselves.
- Deciding whether a game has an ESS is NP-hard and coNP-hard.

$(1,2)$	Hawk	Dove
Hawk	$\frac{V-C}{2}$	V
Dove	0	$\frac{V}{2}$

- If $V > C$, then $\frac{V-C}{2} > 0$ and Hawk is an ESS.
- If $V \leq C$ and $C > 0$, then $\pi = \left\{\text{Hawk} \mapsto \frac{V}{C}, \text{Dove} \mapsto 1 - \frac{V}{C}\right\}$ is an ESS.

Motivation: General Game Playing

- Game playing agents are a testbed for AI approaches and techniques.
- Programs playing specific games have limited value (for AI):
  - very narrow: can play the game(s) they are programmed for, but may not be able to learn to play other (not even simpler) games
  - most analysis and design work is done in advance by human programmers
- General Game Playing (GGP) systems use given descriptions of arbitrary games to play these games effectively without human intervention.
- A formal game description language (GDL) is used to compactly represent (state-based models of) games.
- Success of the general game player also depends on the "intelligence" of the system itself and not just the human programmer(s).

Overview

Game Description Language

Playing Games

Incomplete Information

Game Description Language

Game Description Language: Ideas

- Main idea: games can be *declaratively* described using logic
- Game rules are describe by a set of formulas (a normal logic program)
- A state in the game is represented by a logical interpretation
- GDL uses simultaneous moves (sequentiality is modelled via “no-ops”)
- GDL’s payoffs are scaled to values from [0, 100]
- During play, information is obtained from descriptions via reasoning:
  - Which moves are legal in a state
  - What the next state looks like after a joint move in a state
  - Which states are terminal
  - Players’ payoffs in terminal states
- But logical reasoning can in principle also be used to analyse the game.
- Thus GDL is also relevant for knowledge representation and reasoning.

Background: First-Order Logic (Syntax)

We start out from a logical vocabulary $(\mathcal{P}, \mathcal{F}, \mathcal{V})$ with

- $\mathcal{P}$ a set of **predicate** symbols $p, q, p_1, p_2, \ldots$, each with an **arity** $n \in \mathbb{N}$,
- $\mathcal{F}$ a set of **function** symbols $f, g, f_1, f_2, \ldots$, each with an arity $n \in \mathbb{N}$, and
- $\mathcal{V}$ a set of **variables** $x, y, x_1, x_2, \ldots$.

The set $T_{\mathcal{P},\mathcal{F},\mathcal{V}}$ of **terms** over $(\mathcal{P}, \mathcal{F}, \mathcal{V})$ is the smallest set such that:

- every variable $v \in \mathcal{V}$ is a term, and
- if $t_1, \ldots, t_n$ are terms and $f \in \mathcal{F}$ is a function symbol of arity n, then $f(t_1, \ldots, t_n)$ is a term.

The set $A_{\mathcal{P},\mathcal{F},\mathcal{V}}$ of **atoms** over $(\mathcal{P}, \mathcal{F}, \mathcal{V})$ contains all expressions of the form $p(t_1, \ldots, t_n)$ where p is a predicate symbol of arity n and $t_1, \ldots, t_n \in T_{\mathcal{P},\mathcal{F},\mathcal{V}}$.

- The **Herbrand universe** is $T_{\mathcal{P},\mathcal{F},\emptyset}$, the set of all variable-free terms.
- The **Herbrand base** is $A_{\mathcal{P},\mathcal{F},\emptyset}$, the set of all variable-free atoms.

Background: Logic Programs (Syntax)

Definition

Let $(\mathcal{P}, \mathcal{F}, \mathcal{V})$ be a logical vocabulary.

- A **definite** clause is an expression of the form (implicitly universally quantified)

  $$H \leftarrow B_1 \land \ldots \land B_m$$

  where $H, B_1, \ldots, B_m \in A_{\mathcal{P},\mathcal{F},\mathcal{V}}$; H is called the **head** and each B_i a **body** atom.
- A **normal** clause is an expression of the form

  $$H \leftarrow B_1 \land \ldots \land B_m \land {\sim}B_{m+1} \land \ldots \land {\sim}B_{m+n}$$

  where $H, B_1, \ldots, B_{m+n} \in A_{\mathcal{P},\mathcal{F},\mathcal{V}}$ and $0 \leq m, n$; the symbol $\sim$ is read as “not”.
- A (normal) **logic program** is a set of (normal) logic program clauses.
- A logic program D over vocabulary $(\mathcal{P}, \mathcal{F}, \mathcal{V})$ **defines** a predicate $p \in \mathcal{P}$ iff D contains a clause with head $p(t_1, \ldots, t_n)$ for some $t_1, \ldots, t_n \in T_{\mathcal{P},\mathcal{F},\mathcal{V}}$.

Intuition: A clause is a logical implication “body implies head”.

GDL by Example: Tic-Tac-Toe (1)

The Game Description Language uses logic programs to define games by requiring a number of special predicate symbols be used in a special way.

- Implication $\leftarrow$ is written as :- and conjunction $\land$ is written as &.
- Variables in terms are indicated by upper case identifiers.

There are two roles (players), x and o:

```
role(x)
role(o)
```

Cells are addressed by indices and can be either blank or marked:

```
base(cell(X, Y, M)) :- index(X) & index(Y) & marker(M)
index(1)
index(2)
index(3)
marker(P) :- role(P)
marker(b)
```

GDL by Example: Tic-Tac-Toe (2)

Available moves are “marking a cell” and “doing nothing”:

```
base(control(P)) :- role(P)
input(P, mark(X, Y)) :- role(P) & index(X) & index(Y)
input(P, noop) :- role(P)
```

Initially, all cells are blank and it is X’s turn:

```
init(cell(X, Y, b)) :- index(X) & index(Y)
init(control(x))
```

A player is allowed to mark a cell if that cell is blank and it is the player’s turn:

```
legal(P, mark(X, Y)) :- true(cell(X, Y, b)) & true(control(P))
```

If it is not the player’s turn, the only legal action is doing nothing:

```
legal(x, noop) :- true(control(o))
legal(o, noop) :- true(control(x))
```

GDL by Example: Tic-Tac-Toe (3)

If a player marks a cell, the cell gets that mark:

```
next(cell(X, Y, P)) :- does(P, mark(X, Y)) & true(cell(X, Y, b))
```

Any marked cell retains its mark for the rest of the game:

```
next(cell(X, Y, M)) :- true(cell(X, Y, M)), distinct(M, b)
```

Blank cells stay blank if not marked:

```
next(cell(X, Y, b)) :-
        does(P, mark(I, J)) & true(cell(X, Y, b)) & distinct(X, I)
next(cell(X, Y, b)) :-
        does(P, mark(I, J)) & true(cell(X, Y, b)) & distinct(Y, J)
```

Control alternates between the players:

```
next(control(o)) :- true(control(x))
next(control(x)) :- true(control(o))
```

GDL by Example: Tic-Tac-Toe (4)

The game terminates when one player has won or every cell is marked:

```
terminal :- line(P)
terminal :- ~open
open :- true(cell(X, Y, b))
```

The players' payoffs in terminal states are as expected:

```
goal(x, 100) :- line(x) & ~line(o)
goal(x, 50) :- ~line(x) & ~line(o)
goal(x, 0) :- ~line(x) & line(o)
goal(o, 100) :- line(o) & ~line(x)
goal(o, 50) :- ~line(o) & ~line(x)
goal(o, 0) :- ~line(o) & line(x)
```

Exercise: Define the predicate `line`, possibly using auxiliary predicates.

GDL Special Predicates: Overview

The **special predicates** of GDL are the following:

- `role`(r) ... r is a role (player) in the game
- `input`(r, m) ... player r has feasible move m in the game
- `base`(p) ... p is a base proposition in the game
- `init`(p) ... p is true in the initial state
- `true`(p) ... p is true in the current state
- `does`(r, m) ... player r makes move m in the current state
- `next`(p) ... p is true in the next state
- `legal`(r, m) ... it is legal for player r to make move m in the current state
- `goal`(r, u) ... the current state has utility u for player r
- `terminal` ... the current state is a terminal state

The pre-defined auxiliary predicate `distinct` defines syntactic inequality.

GDL Game Descriptions: Definition

Definition

A GDL **game description** is a logic program D over a vocabulary $(\mathcal{P}, \mathcal{F}, \mathcal{V})$ where $\mathcal{P}$ includes the special predicates of GDL. Furthermore:

1. D must give complete definitions for `role`, `base`, `input`, and `init`.
2. D must define `legal`, `terminal`, and `goal` in terms of `true`.
3. D must define `next` in terms of `true` and `does`.
4. D must not define `true` and `does`.

"Defining p in terms of $q_1, \ldots, q_n$" means:

For every clause with head predicate p, its body only contains:

- atoms with predicates among $q_1, \ldots, q_n$, or
- auxiliary predicates (in turn defined in terms of $q_1, \ldots, q_n$). (e.g. `line`)

Background: First-Order Logic (Semantics)

- An **interpretation** is a pair $\mathcal{I} = (\Delta, \cdot^{\mathcal{I}})$ where $\Delta \neq \emptyset$ and $\cdot^{\mathcal{I}}$ assigns:
- to each predicate symbol $p \in \mathcal{P}$ of arity n a relation $p^{\mathcal{I}} \subseteq \Delta^n$, and
- to each function symbol $f \in \mathcal{P}$ of arity n a function $f^{\mathcal{I}} \colon \Delta^n \to \Delta$.
- A **variable valuation** is a function $v : \mathcal{V} \to \Delta$.
- An **Herbrand interpretation** is an interpretation $(\Delta, \cdot^{\mathcal{I}})$ with $\Delta = T_{\mathcal{P},\mathcal{F},\emptyset}$ where every ground term $t \in T_{\mathcal{P},\mathcal{F},\emptyset}$ is interpreted by itself.
- The value of a term $t \in T_{\mathcal{P},\mathcal{F},\mathcal{V}}$ under an interpretation $\mathcal{I}$ and variable valuation v is

$$t^{\mathcal{I},v} := \begin{cases} v(x) & \text{if } t = x \in \mathcal{V}, \\ f^{\mathcal{I}}(t_1^{\mathcal{I},v}, \ldots, t_2^{\mathcal{I},v}) & \text{if } t = f(t_1, \ldots, t_n). \end{cases}$$

- An interpretation $\mathcal{I}$ with variable valuation v **satisfies** an atom $p(t_1, \ldots, t_n)$, written $\mathcal{I} \models p(t_1, \ldots, t_n)$, iff $(t_1^{\mathcal{I},v}, \ldots, t_n^{\mathcal{I},v}) \in p^{\mathcal{I}}$.

Background: Logic Programs (Semantics I)

Definition

Let D be a logic program under vocabulary $(\mathcal{P}, \mathcal{F}, \mathcal{V})$ and $\mathcal{I}$ be an interpretation for the vocabulary.

- $\mathcal{I}$ **satisfies** a clause $H \leftarrow B_1 \wedge \ldots \wedge B_m \wedge {\sim}B_{m+1} \wedge \ldots \wedge {\sim}B_{m+n}$ iff if $\mathcal{I} \models B_i$ for $1 \leq i \leq m$ and $\mathcal{I} \not\models B_{m+j}$ for $1 \leq j \leq n$, then $\mathcal{I} \models H$.
- $\mathcal{I}$ is a **model** of a logic program D iff $\mathcal{I}$ satisfies all clauses in D.
- An atom $A \in A_{\mathcal{P},\mathcal{F},\emptyset}$ is **entailed** by a logic program D, written $D \models A$, iff for every model $\mathcal{I}$ of D, we have $\mathcal{I} \models A$.

- Herbrand interpretations can be represented as sets $I \subseteq A_{\mathcal{P},\mathcal{F},\emptyset}$ of atoms.
- Definite logic programs (containing only definite clauses) have a unique $\subseteq$-least Herbrand model capturing the set of all atoms entailed by it.
- For normal logic programs, a (unique) model need not exist in general.

Background: Logic Programs (Semantics II)

For normal logic programs (using negation), a unique least Herbrand model exists only under special circumstances.

The program must be:

- safe (in every clause, every variable occurring in the head or in a negated body atom must also occur in a positive body atom)
- stratified (there must be no recursion through negation)
- recursion-restricted (positive recursion must be range-restricted)

Then, the intended semantics of the program is given by its standard model:

- We first consider the least model M_0 of the subset of rules for predicates $\mathcal{P}_0 \subseteq \mathcal{P}$ that do not depend negatively on another predicate.
- We next extend M_0 by all ground atoms derivable by clauses for predicates $\mathcal{P}_1 \subseteq \mathcal{P} \setminus \mathcal{P}_0$ that depend negatively only on predicates from $\mathcal{P}_0$.
- ...

For more details, see the lecture *Foundations of Logic Programming* (WS).

Game Description Language: Semantics

Definition

Given a GDL game description D, the resulting state-based game model is obtained as follows:

- The players are $P = \{r \mid D \models \texttt{role}(r)\}$. (Denote $n = |P|$.)
- The moves of each player $r \in P$ are $M_r = \{m \mid D \models \texttt{input}(r, m)\}$.
- The set of states is given by 2^Q with $Q = \{\texttt{true}(q) \mid D \models \texttt{base}(q)\}$.
- The initial state is given by $S_0 = \{\texttt{true}(q) \mid D \models \texttt{init}(q)\}$.
- The legal moves of $r \in P$ in state $S \subseteq Q$ are $\{m \mid D \cup S \models \texttt{legal}(r, m)\}$.
- Given a state $S \subseteq Q$ and a joint move $(m_1, \ldots, m_n)$, the next state is given by $\{\texttt{true}(q) \mid D \cup S \cup \{\texttt{does}(r_1, m_1), \ldots, \texttt{does}(r_n, m_n)\} \models \texttt{next}(q)\}$.
- The set of terminal states is given by $\{S \subseteq Q \mid D \cup S \models \texttt{terminal}\}$.
- The utility of player $r \in P$ in terminal state $S \subseteq Q$ is u for $D \cup S \models \texttt{goal}(r, u)$.

There are further technical requirements (playability, winnability) that we will not delve into.

Playing Games

Playing GDL Games

- A game manager coordinates the individual players (agents) via network using the game communication language.
- In the beginning, a `start`(*id*, *role*, *D*, *startclock*, *playclock*) message from the game manager to an agent signals that:
  - the match with *id* starts after *startclock* seconds have elapsed,
  - the agent receiving the message will play *role*, and
  - the agent will have *playclock* seconds to choose each move.
- Agents use the *startclock* time to understand the game rules, analyse the game and possibly start searching.
- For each subsequent round of the match, a `play`(*id*, *move*) message from the game manager to an agent indicates that:
  - the agent is supposed to submit a move for match *id*,
  - where the previous joint move (for non-initial states) is given in *move*.
- When the game is over, the game manager sends a `stop`(*id*, *move*) message to all agents, informing them about the last *move*.

Playing GDL Games: Example

Denote by D the GDL game description of Tic-Tac-Toe considered earlier.

- By description and definition, the initial state is

$$S_0 = \{\mathtt{true}(cell(1,1,b)), \mathtt{true}(cell(1,2,b)), \ldots, \mathtt{true}(cell(3,3,b)), \mathtt{true}(control(x))\}$$

- The legal moves of `x` in S_0 are

$$mark(1,1,x), mark(1,2,x), \ldots, mark(3,3,x)$$

- The only legal move of `o` in S_0 is *noop*.
- After the joint move $(mark(2,2,x), noop)$, the next state is

$$S_1 = \{\mathtt{true}(cell(2,2,x)), \mathtt{true}(cell(1,1,b)), \ldots, \mathtt{true}(cell(3,3,b)), \mathtt{true}(control(o))\}$$

- State S_1 is not yet terminal, as $D \cup S_1 \not\models \mathtt{terminal}$ because $D \cup S_1 \models open$.

Playing GDL Games: Move Selection

- Implement Monte Carlo or Minimax Tree Search on GDL descriptions: Consider turn-taking between own single and opponents' joint moves.
- For zero-sum games (can be checked in coNP), use alpha-beta pruning.
- Heuristics for depth-limited game tree search:
  - Analyse `goal` rules for goal proximity heuristics.
  - Analyse `legal` moves in states for mobility heuristics.
- Analyse `next` rules to find persistent propositions (e.g. markers in Tic-Tac-Toe).

Incomplete Information

GDL-II: GDL with Incomplete Information

Both imperfect information and incomplete information can be modelled using only two additional keywords:

- `percept`(r, q) ... player r has possible percept q in the game
- `sees`(r, q) ... player r perceives q in the next state

To model chance nodes (moves by `Nature`), a new role name is introduced:

- `random` ... special role that chooses a legal move uniformly at random

Definition

A **GDL-II game description** is a logic program D over vocabulary $(\mathcal{P}, \mathcal{F}, \mathcal{V})$ where $\mathcal{P}$ includes the GDL-II keywords and $\mathcal{F}$ includes the constant symbol `random`. Furthermore, D must obey the syntactic restrictions of GDL game descriptions where additionally predicate `sees` only appears as head of clauses and must be defined in terms of `true` and `does`.

GDL-II by Example: Simplified Poker (1)

There are three cards, two players, and the game begins with dealing:

```
card(1)     card(2)     card(3)
beats(3,2)     beats(3,1)     beats(2,1)
role(ann)     role(bob)     init(control(random))
```

`Nature` moves first and deals the cards (otherwise does nothing):

```
legal(random, deal(C, D)) :-
      true(control(random)) & card(C) & card(D) & distinct(C, D)
legal(random, noop) :- ~true(control(random))
```

Dealing has the expected effects and percepts:

```
next(hasCard(ann, C)) :- does(random, deal(C, D))
next(hasCard(bob, D)) :- does(random, deal(C, D))
sees(ann, yourCard(C)) :- does(random, deal(C, D))
sees(bob, yourCard(D)) :- does(random, deal(C, D))
```

GDL-II by Example: Simplified Poker (2)

Next comes Ann's turn to choose a move:

```
next(control(ann)) :- true(control(random))
legal(ann, check) :- true(control(ann))
legal(ann, raise) :- true(control(ann))
```

Bob can see Ann's decision and can move iff Ann did a `raise`:

```
sees(bob, annsMove(M)) :- does(ann, M)
next(control(bob)) :- true(control(ann)) & does(ann, raise)
next(showdown) :- does(ann, check)
next(hasCard(P, C)) :- true(hasCard(P, C))
```

Bob's moves are `fold` and `call`, with a showdown happening after `call`:

```
legal(bob, fold) :- true(control(bob))
legal(bob, call) :- true(control(bob))
next(showdown) :- does(bob, call)
```

GDL-II by Example: Simplified Poker (3)

If Bob folds, the game is over and Ann wins:

```
next(annWins) :- true(control(bob)) & does(bob, fold)
terminal :- true(annWins)
goal(bob, 0) :- true(annWins)
goal(ann, 100) :- true(annWins)
```

In a showdown, cards are revealed and the higher card wins:

```
sees(P, hasCard(O, C)) :-
  does(ann, check) & true(hasCard(O, C)) & role(P) & distinct(P, O)
sees(P, hasCard(O, C)) :-
  does(bob, call) & true(hasCard(O, C)) & role(P) & distinct(P, O)
terminal :- true(showdown)
goal(P, 100) :-
      true(hasCard(P, C)) & true(hasCard(O, D)) & beats(C, D)
goal(O, 0) :-
      true(hasCard(P, C)) & true(hasCard(O, D)) & beats(C, D)
```

GDL-II: Semantics via State Transitions

For a GDL-II game description D, the resulting state-based game model is:

- Players, (legal) moves, and initial/terminal state(s) are obtained as in GDL.
- The next state after joint move $\mathbf{m} := (m_1, \ldots, m_n)$ is obtained as usual:

$$n(\mathbf{m}, S) := \{\mathtt{true}(q) \mid D \cup S \cup \{\mathtt{does}(r_1, m_1), \ldots, \mathtt{does}(r_n, m_n)\} \models \mathtt{next}(q)\}$$

- An **information relation** $I \subseteq P \times M^n \times 2^Q \times Q$ models players' incomplete information: $(r, \mathbf{m}, S, q)$ indicates that player r perceives q after joint move $\mathbf{m}$ happens in state S.
- A probability distribution over possible resulting states models uncertainty induced by `random`'s moves: After joint move $\mathbf{m}$ in state $S \subseteq Q$, the probability of $T \subseteq Q$ being the resulting state is

$$\frac{|\{m \in L \mid n((\mathbf{m}; m), S) = T\}|}{|L|}$$

where $L = \{m \in M_{\mathtt{random}} \mid D \cup S \models \mathtt{legal}(\mathtt{random}, m, S)\}$,
and $(\mathbf{m}; m) := (m_1, \ldots, m_n, m)$ extends $\mathbf{m}$ by `random`'s move m.

Playing GDL-II Games

Game management can be adjusted to the incomplete information setting:

1. Send each agent the game description and inform them about their role.
2. Set S to the initial game state.
3. For every subsequent state S of the game:
   (a) Collect moves from all agents and (if applicable) choose a legal move for `random` with uniform probability.
   (b) To every agent $r \in P$, send percepts $\{q \mid (r, M, S, q) \in I\}$ for joint move M in S.
   (c) Update current state S to next state $n(M, S)$.
4. Repeat until S is terminal, then send utilities to agents.

Since the game manager has complete knowledge about the game state, it can compute all percepts and resulting states.

GDL-II: Developments

For a GDL-II game description D, it is also possible to define an extensive form game G_D. A first necessary ingredient is that of a development.

Definition

Consider the state-based game model of a GDL-II game description.

- A **development** is a finite sequence $\delta = \langle S_0, \mathbf{m}_1, S_1, \ldots, \mathbf{m}_d, S_d \rangle$ where
  - $d \geq 0$,
  - $S_0, \ldots, S_d \subseteq Q$ are states, in particular S_0 is the initial state,
  - $\mathbf{m}_j = (m_0, m_1, \ldots, m_n)$ is a joint move including a move m_0 for `random`,
  - every move in $\mathbf{m}_j$ is legal (for its player) in state S_{j-1}, for all $1 \leq j \leq d$,
  - the sequence obeys state update, i.e. $n(\mathbf{m}_j, S_{j-1}) = S_j$ for all $1 \leq j \leq d$, and
  - only S_d may be terminal.
- Two developments δ, δ' are **indistinguishable** for player $1 \leq i \leq n$ iff
  - $\left\{q \in Q \,\middle|\, (i, \mathbf{m}_j, S_{j-1}, q) \in I\right\} = \left\{q \in Q \,\middle|\, (i, \mathbf{m}'_j, S'_{j-1}, q) \in I\right\}$ for all $1 \leq j \leq d$, and
  - player i makes the same move in $\mathbf{m}_j$ and $\mathbf{m}'_j$, for all $1 \leq j \leq d$.

GDL-II: Quasi-Developments

- Main Idea: Sequentialise joint moves and keep individual moves private until joint move is complete.

Definition

Consider the state-based game model of a GDL-II game description.

- A **partial joint move** is a tuple $\mathbf{m}^{(i)} = (m_0, m_1, \ldots, m_i)$ with $0 \leq i < n$.
- A **quasi-development** is of the form $y = \langle \delta, \mathbf{m}^{(i)} \rangle$ where δ is a development and $\mathbf{m}^{(i)}$ is a partial joint move.

- Intuition: A partial joint move $\mathbf{m}^{(i)}$ serves to model the sequentialisation of a joint move where players $\{i + 1, \ldots, n\}$ are yet to move.
- The **history arising from a development** $\delta = \langle S_0, \mathbf{m}_1, S_1, \ldots, \mathbf{m}_d, S_d \rangle$ is then $h_\delta := [(\mathbf{m}_1)_0, (\mathbf{m}_1)_1, \ldots, (\mathbf{m}_1)_n, (\mathbf{m}_2)_0, \ldots, (\mathbf{m}_d)_n]$;
- the **history arising from a quasi-development** $\langle \delta, \mathbf{m}^{(i)} \rangle$ is then $h_{\langle \delta, \mathbf{m}^{(i)} \rangle} := [h_\delta; (\mathbf{m}^{(i)})_0, \ldots, (\mathbf{m}^{(i)})_i]$.

For a tuple $\mathbf{m} = (m_0, \ldots, m_n)$ we denote $(\mathbf{m})_i := m_i$ for $0 \leq i \leq n$.

GDL-II: Semantics via Extensive-Form Games

Definition

Consider the state-based game model of a GDL-II game description D.

The **associated extensive-form game** G_D is as follows:

- Its players are $\{0, 1, \ldots, n\}$, where 0 denotes `random`.
- Its moves and utilities are as in the state-based game model.
- Its histories are all those that arise from (quasi-)developments of D.
- Its terminal histories arise from developments δ with S_d terminal.
- Its player function assigns $p(h_\delta) = 0$ and $p(h_{\langle\delta,\mathbf{m}^{(i)}\rangle}) = i + 1$.
- Its probability distributions for chance nodes are always uniform.
- Its indistinguishability relation is as follows:

$$h_\delta \sim^{G_D} h_{\delta'} \quad \text{iff} \quad \delta \text{ and } \delta' \text{ are indistinguishable for some player}$$

$$h_{\langle\delta,\mathbf{m}^{(i)}\rangle} \sim^{G_D} h_{\langle\delta',\mathbf{m}^{(i')}\rangle} \quad \text{iff} \quad h_\delta \sim^{G_D} h_{\delta'} \text{ and } i = i'$$

Properties of GDL-II: Extension of GDL

Proposition

GDL-II is a proper extension of GDL.

Proof.

- Let D be a game description in GDL.
- To express the same game in GDL-II, we add one rule:

```
sees(P, move(O, M)) :- role(P) & does(O, M)
```

- Thus, every player knows every move of every other player. □

Properties of GDL-II: Universality (1)

Theorem (Thielscher, 2011)

GDL-II is universal, i.e. for every finite extensive-form game G there is a GDL-II game description D_G that formalises G.

Proof (Sketch, 1/3).

- We assume a game G given in extensive form (i.e. as explicit tree).
- Players are defined through `role`(`random`), `role`(1), . . . , `role`(n).
- Histories $h \in H$ are encoded as terms t_h via

$$t_{[]} := nil \quad \text{and} \quad t_{[h;m]} := cons(m, t_h).$$

- The initial state is encoded via `init`(nil).
- Terminal states are expressed via `terminal` :- `true`(t_h) for all $h \in T$.
- We declare utilities via `goal`($i, u_i(h)$) :- `true`(t_h) for $h \in T$.
  (Utilities are scaled to $[0, 100]$ using min/max $\{u_i(h) \mid h \in T, 1 \leq i \leq n\}$.)

Properties of GDL-II: Universality (2)

Proof (Sketch, 2/3).

- Legality and state update are defined as expected:

$$\texttt{legal}(i, m) \text{ :- } \texttt{true}(t_h)$$
$$\texttt{next}(t_{[h;m]}) \text{ :- } \texttt{true}(t_h) \;\&\; \texttt{does}(i, m)$$
$$\texttt{legal}(i', \texttt{noop}) \text{ :- } \texttt{true}(t_h)$$

  for all $[h; m] \in H$, $p(h) = i$ with $1 \leq i \leq n$, $m \in M_i$, and $0 \leq i' \leq n$ with $i' \neq i$.

- Information sets of the game lead to abstract percepts:

$$\texttt{sees}(i', j) \text{ :- } \texttt{true}(t_h), \texttt{does}(i, m)$$
$$\texttt{member}(t_{[h;m]}, j)$$

  for $[h; m] \in H$, $p(h) = i$, $[h; m] \in \mathcal{I}_j$, and $p(\mathcal{I}_j) = i'$, for $0 \leq i, i' \leq n$.

Properties of GDL-II: Universality (3)

Proof (Sketch, 3/3).

- For moves of Nature (random), we assume the probability distribution over moves is $\left\{m_1 \mapsto \frac{p_1}{q}, \ldots, m_\ell \mapsto \frac{p_\ell}{q}\right\}$ for some $h \in H$ with $p(h) = \texttt{Nature}$.
- For every $1 \leq k \leq \ell$, we now create p_k many copies of m_k and specify

$$\texttt{legal}(\texttt{random}, m_k^{(1)}) \text{ :- } \texttt{true}(t_h)$$

$$\texttt{next}(t_{[h;m_k]}) \text{ :- } \texttt{true}(t_h) \,\&\, \texttt{does}(\texttt{random}, m_k^{(1)})$$

$$\vdots$$

$$\texttt{legal}(\texttt{random}, m_k^{(p_k)}) \text{ :- } \texttt{true}(t_h)$$

$$\texttt{next}(t_{[h;m_k]}) \text{ :- } \texttt{true}(t_h) \,\&\, \texttt{does}(\texttt{random}, m_k^{(p_k)})$$

to express proportionality of probabilities. □

Playing GDL-II Games: Move Selection

Schofield, Cerexhe, & Thielscher [2012] propose a method called HyperPlay:

- Estimate the true history by a list of samples from the information set.
- Each sample is a complete history that is consistent with what is known.
- Initialise the list of samples as $\langle [\,], \ldots, [\,] \rangle$.
- Use "conventional" techniques to select a move for each complete history.
- An overall move is selected based on its expected utility weighted by the probability that its history h is the true match history given percepts Q:

$$P(h \mid Q) = \frac{P(Q \mid h) \cdot P(h)}{P(Q)}$$

- After each own move and received percepts, update the samples:
  - Randomly sample from other players' legal moves to obtain a full joint move.
  - Compute the next state and expected own percepts.
  - Remove those samples where received and expected percepts disagree.

Conclusion

Summary

- **General Game Playing** is concerned with computers learning to play previously unknown games without human intervention.
- The **game description language** (GDL) is used to declaratively specify (deterministic) games (with complete information about game states).
- The syntax of GDL game descriptions is that of **normal logic programs**; various restrictions apply to obtain a finite, unique interpretation.
- The semantics of GDL is given through a state transition system.
- GDL-II allows to represent moves by `Nature` and information sets.
- The semantics of GDL-II can be given through extensive-form games.
- Conversely, GDL-II can express any finite extensive-form game.

Exercise: Adapt the payoffs in the GDL model of simplified poker.