Reliance-Based Optimization of Existential Rule Reasoning

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TU Dresden

Introduction

$$\text{leadRole}(a, r, m) \to \text{stars}(a, m)$$
$$\text{stars}(a, m) \land \text{stars}(b, m) \to \text{costar}(a, b, m)$$
$$\text{bigBudget}(m) \to \exists a.\, \text{stars}(a, m) \land \text{famous}(a)$$

---

leadRole		
A_2	R_1	M_2

bigBudget
M_1
M_2

stars	
A_1	M_1
A_2	M_1
A_1	M_2

famous
A_2

costar

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$$h = \{m \mapsto M_1\}$$

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leadRole		
A_2	R_1	M_2

bigBudget
M_1
M_2

stars	
A_1	M_1
A_2	M_1
A_1	M_2

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A_2	R_1	M_2

bigBudget
M_1
M_2

stars	
A_1	M_1
A_2	M_1
A_1	M_2
n	M_2

famous
A_2
n

costar

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M_1
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---

stars	
A_1	M_1
A_2	M_1
A_1	M_2
n	M_2

$\bowtie$

stars	
A_1	M_1
A_2	M_1
A_1	M_2
n	M_2

$=$

costar		
A_1	A_1	M_1
A_1	A_2	M_1
A_2	A_1	M_1
	...	

Introduction

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leadRole		
A_2	R_1	M_2

bigBudget
M_1
M_2

stars	
A_1	M_1
A_2	M_1
A_1	M_2
n	M_2

famous
A_2
n

costar		
A_1	A_1	M_1
A_1	A_2	M_1
A_2	A_1	M_1
	...	

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leadRole		
A_2	R_1	M_2

bigBudget
M_1
M_2

stars	
A_1	M_1
A_2	M_1
A_1	M_2
n	M_2
A_2	M_2

famous
A_2
n

costar		
A_1	A_1	M_1
A_1	A_2	M_1
A_2	A_1	M_1
	...	

Introduction

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$$h^{\text{alt}} = \{m \mapsto M_1, a \mapsto A_2\}$$

leadRole		
A_2	R_1	M_2

bigBudget
M_1
M_2

stars	
A_1	M_1
A_2	M_1
A_1	M_2
n	M_2
A_2	M_2

famous
A_2
n

costar		
A_1	A_1	M_1
A_1	A_2	M_1
A_2	A_1	M_1
	...	

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$$\mathsf{costar}(a, b, m) \land \mathsf{famous}(a) \to \mathsf{famousCostar}(a, b)$$

costar		
A_1	A_1	M_1
A_1	A_2	M_1
	...	
A_2	A_2	M_2
A_2	n	M_2
	...	

$\bowtie$

famous
A_2
n

Optimization Goals

- Optimization 1: Minimize number of alternative Matches
  - Avoids introducing redundant facts
  - Might lead to core models
- Optimization 2: Minimize number of rule applications
  - Avoids splitting facts across multiple tables
  - Reduces number of temporary unions of blocks

Current approach by VLog: Cycle through rules but prefer datalog rules

Datalog First

$$\mathsf{book}(x) \rightarrow \exists v.\, \mathsf{writtenBy}(x, v) \wedge \mathsf{author}(v)$$

$$\mathsf{author}(x) \rightarrow \exists w.\, \mathsf{authorOf}(x, w) \wedge \mathsf{book}(w)$$

$$\mathsf{authorOf}(x, y) \rightarrow \mathsf{writtenBy}(y, x)$$

$$\mathsf{writtenBy}(x, y) \rightarrow \mathsf{authorOf}(y, x)$$

Datalog First

$$\text{book}(x) \rightarrow \exists v.\, \text{writtenBy}(x, v) \wedge \text{author}(v)$$

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book_1

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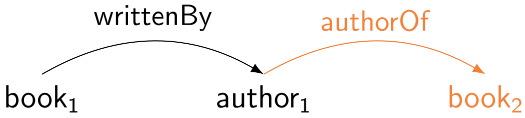

Datalog First

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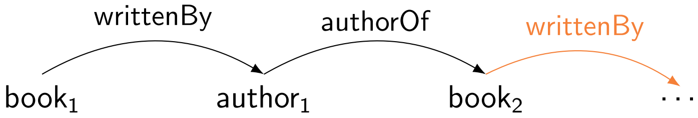

Datalog First

$$\text{book}(x) \rightarrow \exists v.\, \text{writtenBy}(x, v) \land \text{author}(v)$$

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book_1

Datalog First

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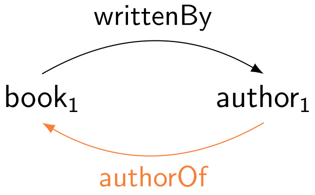


Datalog First

$$\mathsf{book}(x) \rightarrow \exists v, i.\, \mathsf{writtenBy}(x, v, i) \wedge \mathsf{author}(v)$$

$$\mathsf{author}(x) \rightarrow \exists w, i.\, \mathsf{authorOf}(x, w, i) \wedge \mathsf{book}(w)$$

$$\mathsf{authorOf}(x, y, j) \rightarrow \exists i.\, \mathsf{writtenBy}(y, x, i)$$

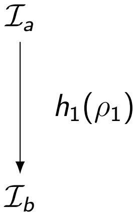
$$\mathsf{writtenBy}(x, y, j) \rightarrow \exists i.\, \mathsf{authorOf}(y, x, i)$$

Reliances

Positive Reliances: $\rho_1 \prec^{+} \rho_2$

Restraint Reliances: $\rho_1 \prec^{\square} \rho_2$

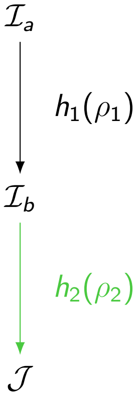
Positive Reliances: $\rho_1 \prec^{+} \rho_2$



Restraint Reliances: $\rho_1 \prec^{\square} \rho_2$

Reliances

Positive Reliances: $\rho_1 \prec^{+} \rho_2$



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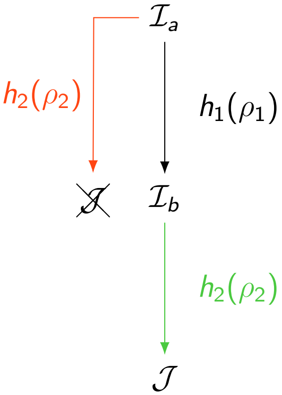

Restraint Reliances: $\rho_1 \prec^{\square} \rho_2$

Positive Reliances: $\rho_1 \prec^{+} \rho_2$

$\mathcal{I}_a = \{\text{leadRole}(A_2, R_1, M_2)\}$

$h_2(\rho_2)$

$\text{leadRole}(a, r, m) \rightarrow \text{stars}(a, m)$

$\not\mathcal{J}$ $\quad \mathcal{I}_b = \mathcal{I}_a \cup \{\text{stars}(A_2, M_2)\}$

$h_2(\rho_2)$

$\mathcal{J}$

Restraint Reliances: $\rho_1 \prec^{\square} \rho_2$

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$\mathcal{I}_a = \{\text{leadRole}(A_2, R_1, M_2)\}$

$h_2(\rho_2)$

$\text{leadRole}(a, r, m) \rightarrow \text{stars}(a, m)$

$\not\mathcal{J}$ $\quad \mathcal{I}_b = \mathcal{I}_a \cup \{\text{stars}(A_2, M_2)\}$

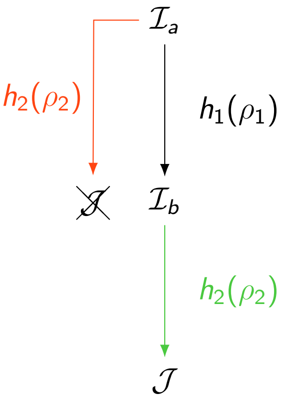
$\text{stars}(a, m) \wedge \text{stars}(b, m)$
$\rightarrow \text{costar}(a, b, m)$

$\mathcal{J}$

Restraint Reliances: $\rho_1 \prec^{\square} \rho_2$

Reliances

Positive Reliances: $\rho_1 \prec^{+} \rho_2$



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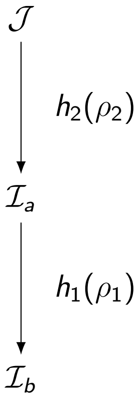

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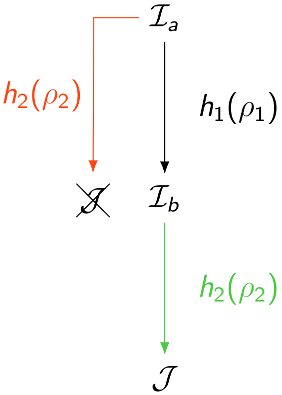

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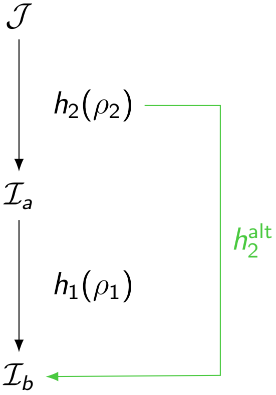
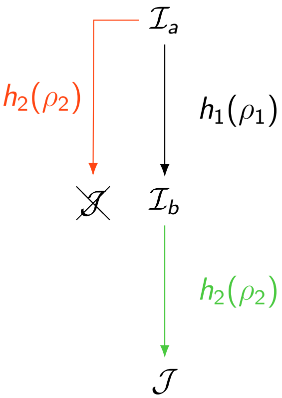


Positive Reliances: $\rho_1 \prec^{+} \rho_2$



Restraint Reliances: $\rho_1 \prec^{\square} \rho_2$

$$\mathcal{J} = \{\text{bigBudget}(M_2), \text{famous}(A_2)\}$$

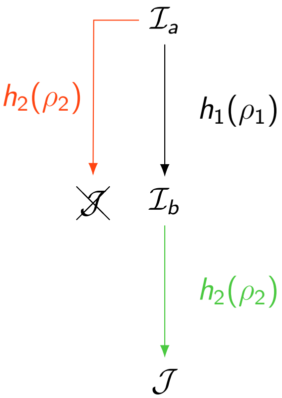
$$\text{bigBudget}(m) \rightarrow \exists a.\, \text{stars}(a, m) \wedge \text{famous}(a)$$

$$\mathcal{I}_a = \mathcal{J} \cup \{\text{stars}(n, M_2), \text{famous}(n)\}$$

$$\mathcal{I}_b$$

Reliances

Positive Reliances: $\rho_1 \prec^+ \rho_2$



Restraint Reliances: $\rho_1 \prec^{\Box} \rho_2$

$$\mathcal{J} = \{\text{bigBudget}(M_2), \text{famous}(A_2)\}$$

$$\downarrow \quad \text{bigBudget}(m) \rightarrow \exists a.\, \text{stars}(a, m) \land \text{famous}(a)$$

$$\mathcal{I}_a = \mathcal{J} \cup \{\text{stars}(n, M_2), \text{famous}(n)\}$$

$$\downarrow \quad \text{leadRole}(a, r, m) \rightarrow \text{stars}(a, m)$$

$$\mathcal{I}_b = \mathcal{I}_a \cup \{\text{stars}(A_2, M_2)\}$$

Calculating Reliances – Unification

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\, a(w, w) \wedge b(x_2)$$

Calculating Reliances – Unification

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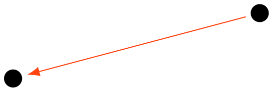

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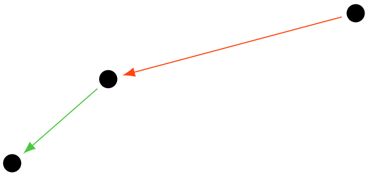

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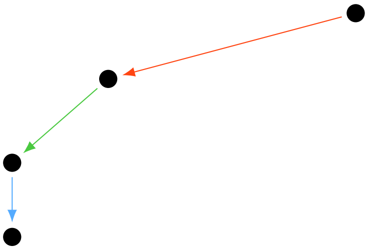

Calculating Reliances – Unification

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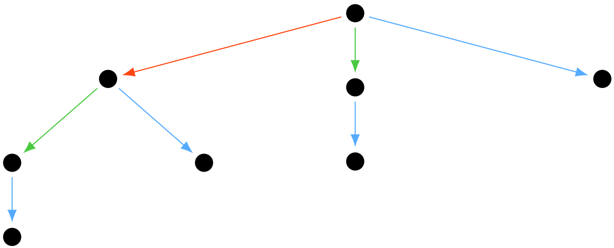

Calculating Reliances – Verification

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\, a(w, w) \wedge b(x_2)$$

Calculating Reliances – Verification

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\ p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\ a(w, w) \wedge b(x_2)$$

Calculating Reliances – Verification

$$a(x_1, y_1) \land b(z_1) \rightarrow \exists v.\, p(x_1, v) \land q(v, y_1)$$

$$p(x_2, x_2) \land p(x_2, y_2) \land q(y_2, x_2) \rightarrow \exists w.\, a(w, w) \land b(x_2)$$

1. Compute unifer η

Calculating Reliances – Verification

$$a(x_1, y_1) \land b(z_1) \rightarrow \exists v.\ p(x_1, v) \land q(v, y_1)$$

$$p(x_2, x_2) \land p(x_2, y_2) \land q(y_2, x_2) \rightarrow \exists w.\ a(w, w) \land b(x_2)$$

1. Compute unifer η $\qquad \eta = \{x_2/x_1/y_1 \mapsto c_{xy},$

Calculating Reliances – Verification

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\, a(w, w) \wedge b(x_2)$$

1. Compute unifer η $\quad \eta = \{x_2/x_1/y_1 \mapsto c_{xy}, v \mapsto n_v, y_2 \mapsto n_v,$

Calculating Reliances – Verification

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\, a(w, w) \wedge b(x_2)$$

1. Compute unifer η $\qquad \eta = \{x_2/x_1/y_1 \mapsto c_{xy}, v \mapsto n_v, y_2 \mapsto n_v, z_1 \mapsto c_z\}$

Calculating Reliances – Verification

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\, a(w, w) \wedge b(x_2)$$

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2. $\mathcal{I}_a = \varphi_1\eta \cup \varphi_{22}\eta$ $\qquad \mathcal{I}_a = \{a(c_{xy}, c_{xy}), b(c_z), p(c_{xy}, c_{xy})\}$

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3. $\mathcal{I}_b = \mathcal{I}_a \cup \psi_1\eta$ — $\mathcal{I}_b = \mathcal{I}_a \cup \{p(c_{xy}, n_v), q(n_v, c_{xy})\}$

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3. $\mathcal{I}_b = \mathcal{I}_a \cup \psi_1\eta$ — $\mathcal{I}_b = \mathcal{I}_a \cup \{p(c_{xy}, n_v), q(n_v, c_{xy})\}$
4. Check $\mathcal{I}_b \not\models \psi_2\eta$ — $\mathcal{I}_b \not\models \exists w.\, a(w, w) \wedge b(c_{xy})$

Calculating Reliances – Verification

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\, a(w, w)$$

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3. $\mathcal{I}_b = \mathcal{I}_a \cup \psi_1\eta$ — $\mathcal{I}_b = \mathcal{I}_a \cup \{p(c_{xy}, n_v), q(n_v, c_{xy})\}$
4. Check $\mathcal{I}_b \not\models \psi_2\eta$ — $\mathcal{I}_b \models \exists w.\, a(w, w)$

Calculating Reliances – Completeness

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\, a(w, w)$$

Calculating Reliances – Completeness

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

$$p(x_2, x_2) \wedge p(x_2, y_2) \wedge q(y_2, x_2) \rightarrow \exists w.\, a(w, w)$$

$$\mathcal{J}_a = \{r(7), a(1,1), b(2), p(1,1)\}$$

$$\downarrow h(\rho_1)$$

$$\mathcal{J}_b = \mathcal{J}_a \cup \{p(1, n_v), q(n_v, 1)\}$$

Calculating Reliances – Completeness

$$a(x_1, y_1) \wedge b(z_1) \rightarrow \exists v.\, p(x_1, v) \wedge q(v, y_1)$$

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$$\mathcal{I}_a = \{a(c_{xy}, c_{xy}), b(c_z), p(c_{xy}, c_{xy})\}$$

$$\downarrow \eta(\rho_1)$$

$$\mathcal{I}_b = \mathcal{I}_a \cup \{p(c_{xy}, n_v), q(n_v, c_{xy})\}$$

Calculating Reliances – Completeness

$$a(x_1, y_1) \land b(z_1) \rightarrow \exists v.\, p(x_1, v) \land q(v, y_1)$$

$$p(x_2, x_2) \land p(x_2, y_2) \land q(y_2, x_2) \rightarrow \exists w.\, a(w, w)$$

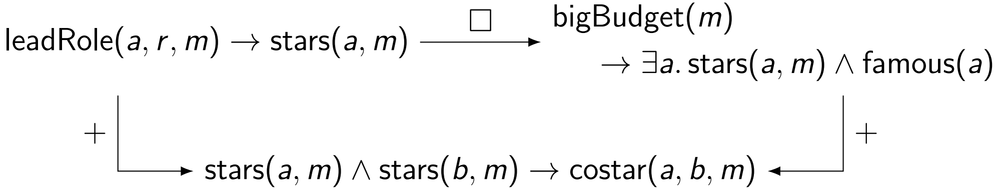
$$\mathcal{J}_a = \{r(7), a(1,1), b(2), p(1,1)\} \qquad \mathcal{I}_a = \{a(c_{xy}, c_{xy}), b(c_z), p(c_{xy}, c_{xy})\}$$

$$h(\rho_1) \qquad \eta(\rho_1)$$

$$\mathcal{J}_b = \mathcal{J}_a \cup \{p(1, n_v), q(n_v, 1)\} \qquad \mathcal{I}_b = \mathcal{I}_a \cup \{p(c_{xy}, n_v), q(n_v, c_{xy})\}$$

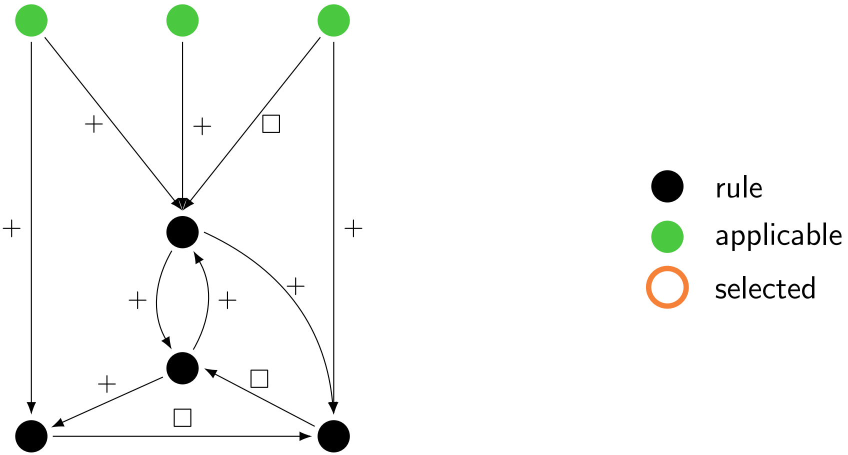
$$\tau = \{c_{xy} \mapsto 1, c_z \mapsto 2\}$$

Rule Order – Ideal Case

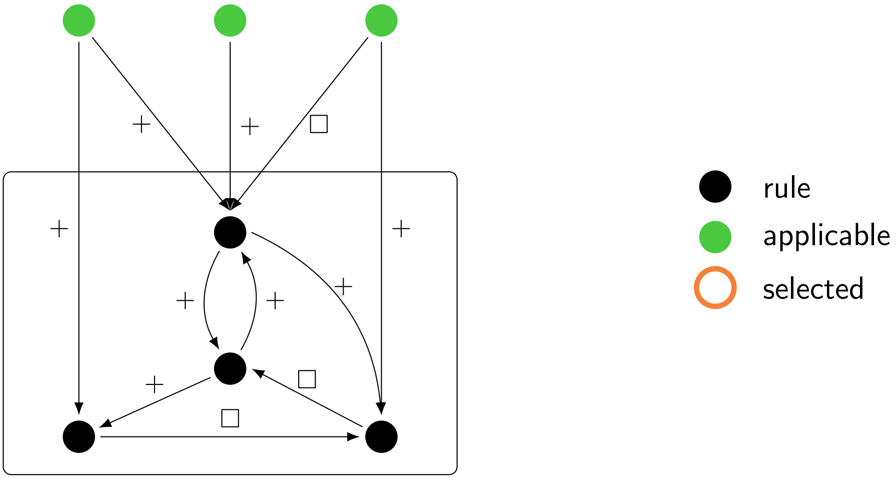


- Core-stratified = No cycle containing a $\prec^{\square}$-reliance
- Possible to compute a core model
- Possible to apply every rule only once

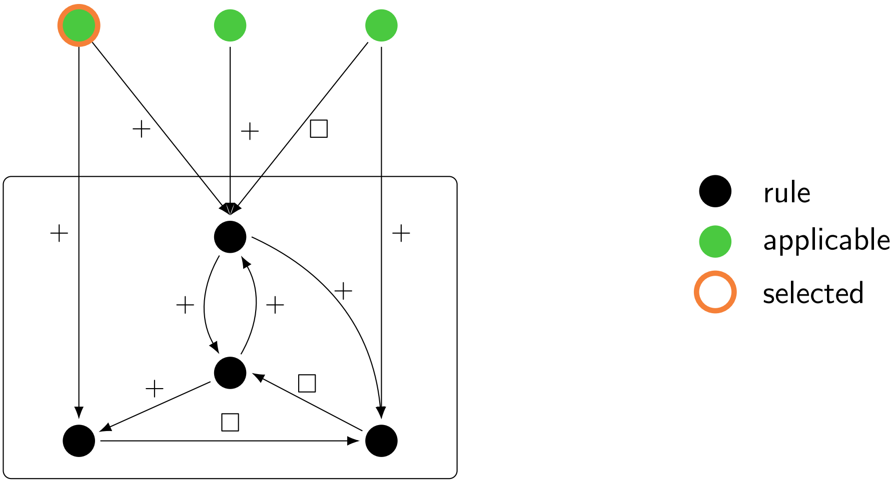
Rule Order – Non-Stratified Case



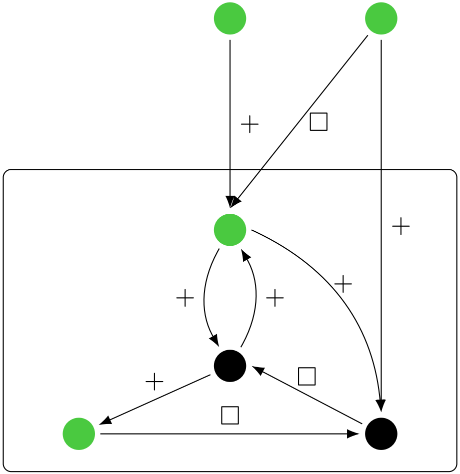
Rule Order – Non-Stratified Case

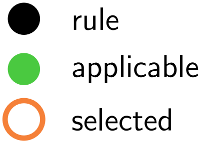


Rule Order – Non-Stratified Case

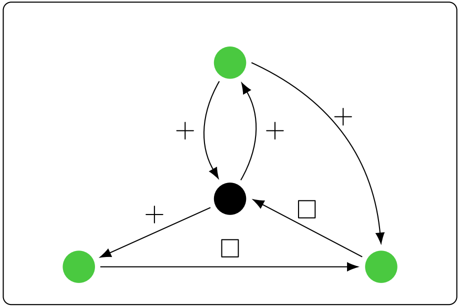


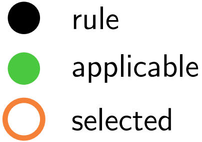
Rule Order – Non-Stratified Case



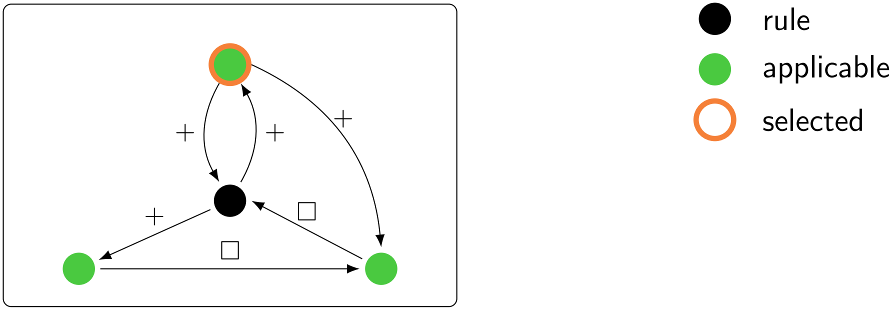


Rule Order – Non-Stratified Case

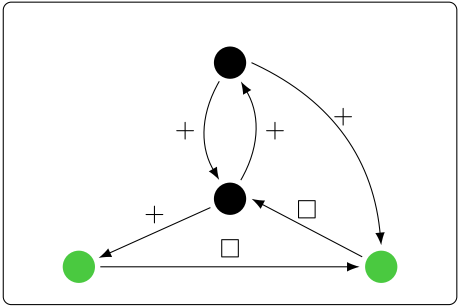




Rule Order – Non-Stratified Case

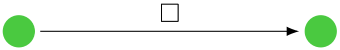


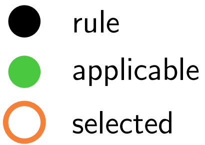
Rule Order – Non-Stratified Case



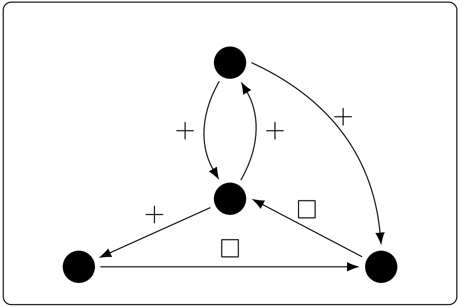
- rule
- applicable
- selected

Rule Order – Non-Stratified Case





Rule Order – Non-Stratified Case



- rule
- applicable
- selected

Unrestrained First

$$\text{book}(x) \rightarrow \exists v, i.\, \text{writtenBy}(x, v, i) \wedge \text{author}(v)$$

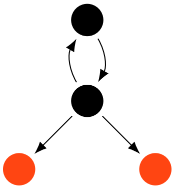
$$\text{author}(x) \rightarrow \exists w, i.\, \text{authorOf}(x, w, i) \wedge \text{book}(w)$$

$$\text{authorOf}(x, y, j) \rightarrow \exists i.\, \text{writtenBy}(y, x, i)$$

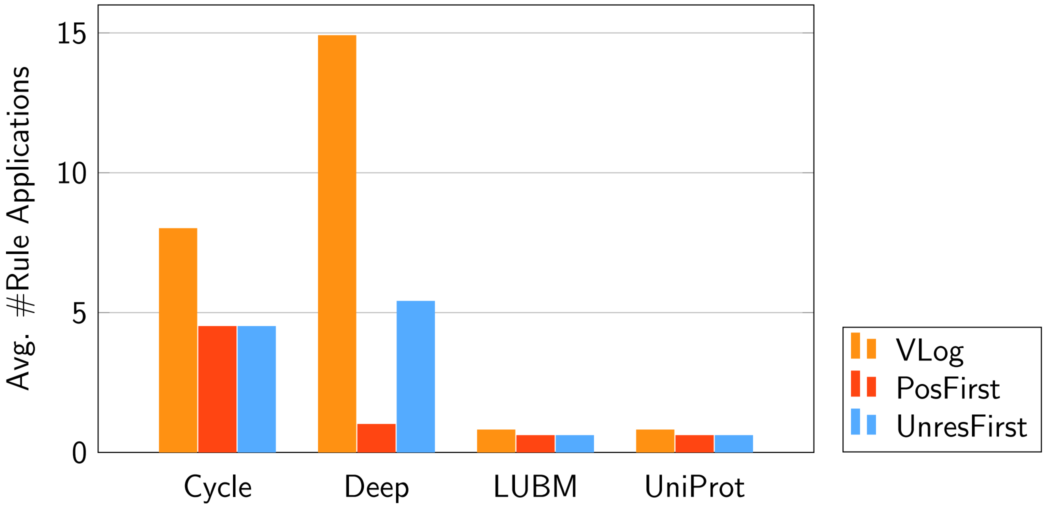
$$\text{writtenBy}(x, y, j) \rightarrow \exists i.\, \text{authorOf}(y, x, i)$$

Evaluation

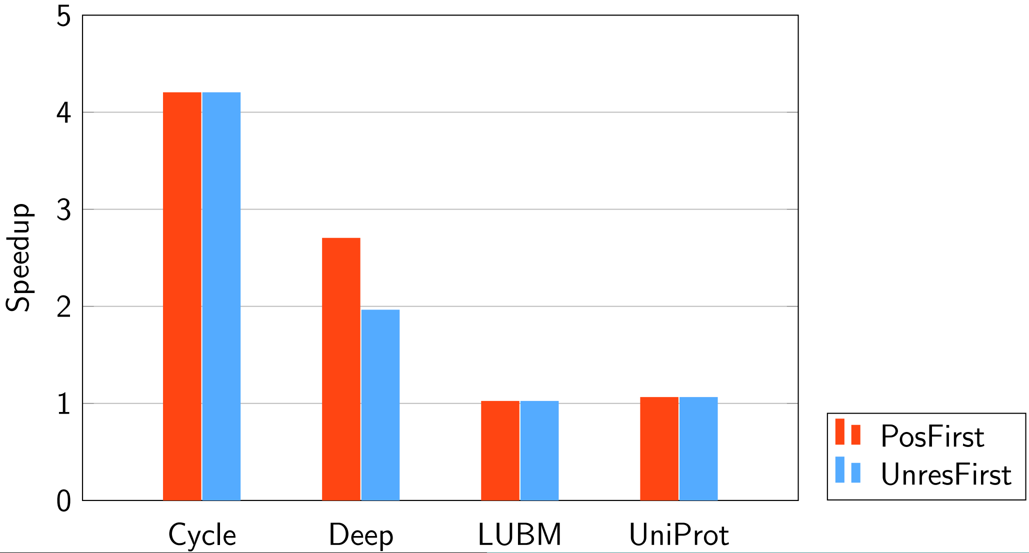
- Implemented reliance calculating and ordering strategy into VLog
- Measured performance on several knowledge bases
  - ChaseBench: DEEP, LUBM, ONTOLOGY-256, STB-128, DOCTORS, UOBM
  - Real World: UNIPROT, REACTOME
  - Created: CYCLE



Average Rule Applications



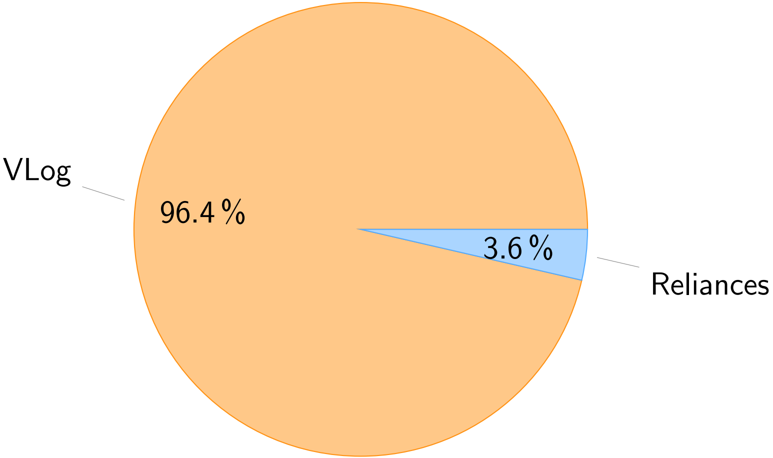
Time Measurements



Alternative Matches

Dataset	Strat.	Facts	PosFst		UnresFst	
			Sel.	Delta	Sel.	Delta
Cycle	✓	0.4M	0	0	0	0
Doctors	✓	0.8M	0	0	0	0
Reactome	✓	11.4M	0	0	0	0
STB	✓	1.9M	0	0	0	0
UniProt	✓	24.2M	0	0	0	0
LUBM	✗	18.7M	3	0	0	0
Deep	✗	0.9M	1148	+19K	2032	-0.5K
Ontology	✗	5.7M	123	0	64	0
UOMB	✗	18.6M	21	+35K	6	-12K

Cost of Computing Reliances



Summary

Summary:

- We optimized the order of rule execution by analysing the relationships between rules of a given program
  - Positive reliance: Avoids unnecessary rule executions
  - Restraint reliance: Avoids redundant facts
- Strategy shows potential for improving run times
- No significant reduction in the number of derived facts
- Computation reliances is cheap even on larger rule sets

Future work:

- Repeat experiments with larger input facts or other rule sets
- Problem: Strategy might exclude terminating runs
- Analyze factors that influence run time outside of reliances