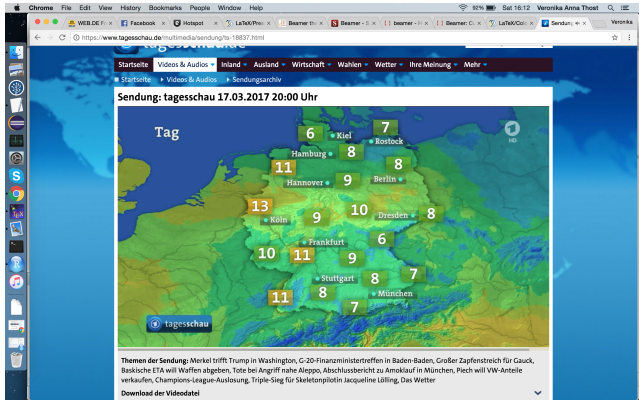
# Using Ontology-Based Data Access to Enable Context Recognition in the Presence of Incomplete Information

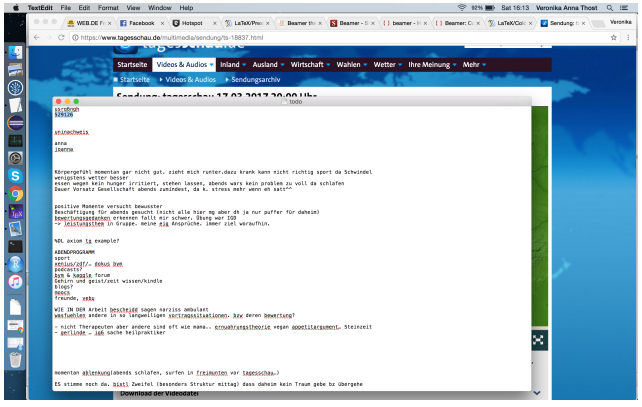
Verteidigung der Dissertation

Veronika Thost

June 19, 2017

# Context Recognition Example



## Example context

User Bob watches a video, but then starts working with a text editor and the video window is not visible anymore

Possible system optimization:
Save resources by decreasing quality parameters of the video

Components in user focus?

| ID | APP | TYPE |
|---|---|---|
| w1 | a1 | mov |
| w4 | a2 | text |

*Window*

| SENSOR | TYPE | USER | ITEM | TIME |
|---|---|---|---|---|
| s3 | cam | ann | book5 | 20:10 |
| s1 | cam | bob | w1 | 20:13 |

*Observation*

Components in user focus?

**SELECT** ID **FROM** *Win* **WHERE** *Win*.ID=*Obs*.ITEM & *Obs*.TYPE=`cam`

| ID | APP | TYPE |
|---|---|---|
| w1 | a1 | mov |
| w4 | a2 | text |

*Window*

| SENSOR | TYPE | USER | ITEM | TIME |
|---|---|---|---|---|
| s3 | cam | ann | book5 | 20:10 |
| s1 | cam | bob | w1 | 20:13 |

*Observation*

# Ontology-Based Data Access

## Ontology: Domain Terminology

Components in user focus?

```
VideoPlayer   Application  EnergyIntensive
                SystemCritical
     Window   Component                    NotVisible
    LooksAt   FocusesOn                    User
```

| ID | APP | TYPE |
|---|---|---|
| w1 | a1 | mov |
| w4 | a2 | text |

*Window*

| SENSOR | TYPE | USER | ITEM | TIME |
|---|---|---|---|---|
| s3 | cam | ann | book5 | 20:10 |
| s1 | cam | bob | w1 | 20:13 |

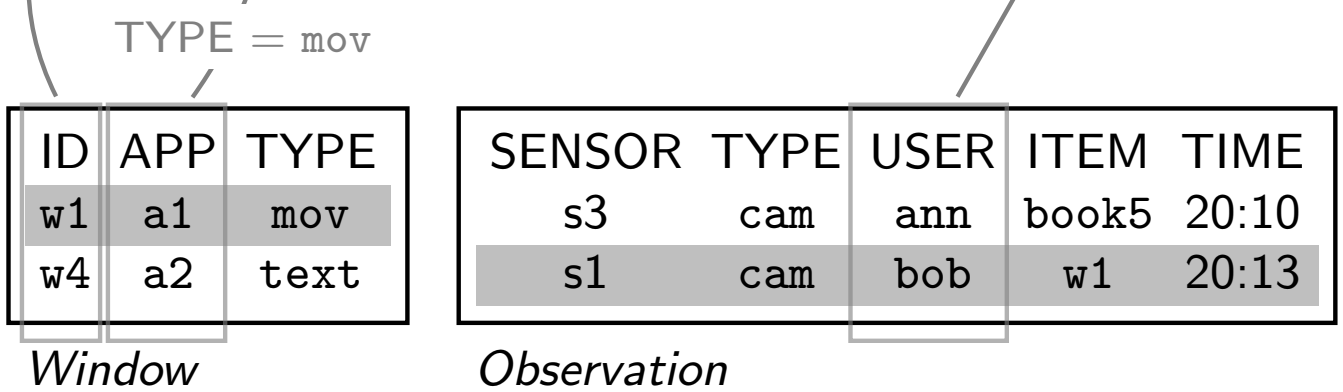
*Observation*

# Ontology-Based Data Access

## Ontology: Domain Terminology

Components in user focus?

VideoPlayer Application EnergyIntensive
SystemCritical
Window Component NotVisible
LooksAt FocusesOn User

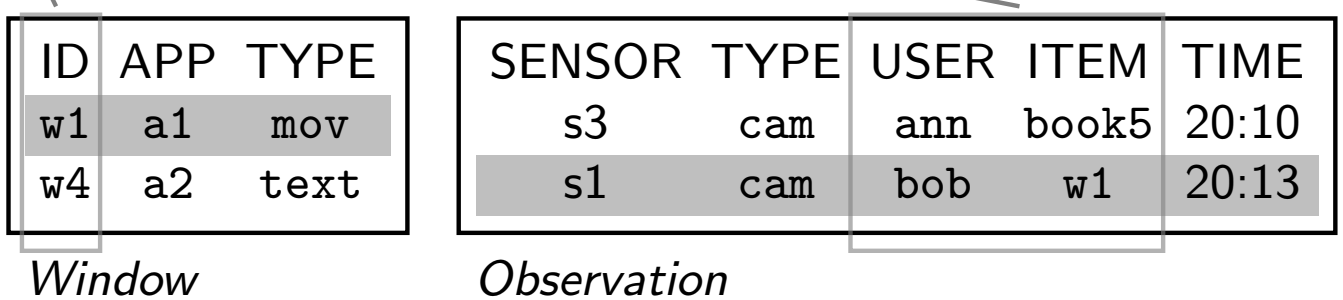


# Ontology-Based Data Access

## Ontology: Domain Terminology

Components in user focus?

```
VideoPlayer   Application   EnergyIntensive
                SystemCritical
     Window   Component                    NotVisible
    LooksAt   FocusesOn                   User
```



| ID | APP | TYPE |
|---|---|---|
| w1 | a1 | mov |
| w4 | a2 | text |

*Window*

| SENSOR | TYPE | USER | ITEM | TIME |
|---|---|---|---|---|
| s3 | cam | ann | book5 | 20:10 |
| s1 | cam | bob | w1 | 20:13 |

*Observation*

# Ontology-Based Data Access

## Ontology: Domain Terminology

Components in user focus?

VideoPlayer  Application  EnergyIntensive
SystemCritical
Window  Component  NotVisible
LooksAt  FocusesOn  User

User(bob)
VideoPlayer(a1)
Window(w1)
HasPart(a1, w1)  LooksAt(bob, w1)  NotVisible(w1)

# Ontology-Based Data Access

## Ontology: Domain Terminology

Components in user focus?

$\exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)$ Answer: $x$

| | | | |
|---|---|---|---|
| VideoPlayer | Application | EnergyIntensive | |
| | SystemCritical | | |
| Window | Component | | NotVisible |
| LooksAt | FocusesOn | | User |

User(bob)

VideoPlayer(a1)

Window(w1)

HasPart(a1, w1) LooksAt(bob, w1) NotVisible(w1)

# Ontology-Based Data Access

## Ontology: Domain Terminology

Components in user focus?

$\exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)$ Answer: $x$

$\texttt{VideoPlayer} \sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \texttt{SystemCritical}$

$\texttt{Window} \sqsubseteq \texttt{Component} \sqcap \texttt{NotVisible}$

$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn} \sqcap \texttt{User}$

$\texttt{User}(\texttt{bob})$

$\texttt{VideoPlayer}(\texttt{a1})$

$\texttt{Window}(\texttt{w1})$

$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$ $\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$ $\texttt{NotVisible}(\texttt{w1})$

# Ontology-Based Data Access

## Ontology: Domain Terminology

Components in user focus?

$\exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)$ Answer: $x$

$$\texttt{VideoPlayer} \sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical}$$

$$\texttt{Window} \sqsubseteq \texttt{Component} \qquad \texttt{NotVisible}$$

$$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn} \qquad \texttt{User}$$

$\texttt{User}(\texttt{bob})$

$\texttt{VideoPlayer}(\texttt{a1})$

$\texttt{Window}(\texttt{w1})$

$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$ $\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$ $\texttt{NotVisible}(\texttt{w1})$

# Ontology-Based Data Access

## Query Answering

Components in user focus?

$\exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)$ Answer: $x = \texttt{w1}$

$\texttt{VideoPlayer} \sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical}$

$\texttt{Window} \sqsubseteq \texttt{Component}$ $\quad$ $\texttt{NotVisible}$

$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn}$ $\quad$ $\texttt{User}$

$\texttt{User(bob)}$

$\texttt{VideoPlayer(a1)}$

$\texttt{Window(w1)}$

$\texttt{HasPart(a1, w1)}$ $\quad$ $\texttt{LooksAt(bob, w1)}$ $\quad$ $\texttt{NotVisible(w1)}$

# Ontology-Based Data Access

## Query Answering

Components in user focus in the past, and not visible anymore (now)?

$$\begin{aligned}
\texttt{VideoPlayer} &\sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical} \\
\texttt{Window} &\sqsubseteq \texttt{Component} \qquad\qquad \texttt{NotVisible} \\
\texttt{LooksAt} &\sqsubseteq \texttt{FocusesOn} \qquad\qquad \texttt{User}
\end{aligned}$$

$$\texttt{User(bob)} \quad \texttt{VideoPlayer(a1)} \quad \texttt{Window(w1)}$$

$$\texttt{HasPart(a1, w1)} \qquad \texttt{LooksAt(bob, w1)} \qquad \texttt{NotVisible(w1)}$$

# Ontology-Based Data Access

## Query Answering

Components in user focus in the past, and not visible anymore (now)?

$$\begin{aligned}
\texttt{VideoPlayer} &\sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical} \\
\texttt{Window} &\sqsubseteq \texttt{Component} \qquad\qquad \texttt{NotVisible} \\
\texttt{LooksAt} &\sqsubseteq \texttt{FocusesOn} \qquad\qquad \texttt{User}
\end{aligned}$$

20 : 11

$\texttt{User}(\texttt{bob})$

$\texttt{VideoPlayer}(\texttt{a1})$

$\texttt{Window}(\texttt{w1})$

$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$

20 : 13

$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$

20 : 15

$\texttt{NotVisible}(\texttt{w1})$

# Ontology-Based Data Access

## Query Answering

Components in user focus in the past, and not visible anymore (now)?

$(\Diamond_P \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)) \wedge \texttt{NotVisible}(x)$

$$\begin{aligned}
\texttt{VideoPlayer} &\sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \\
&\quad \neg\texttt{SystemCritical} \\
\texttt{Window} &\sqsubseteq \texttt{Component} \qquad\qquad \texttt{NotVisible} \\
\texttt{LooksAt} &\sqsubseteq \texttt{FocusesOn} \qquad\qquad \texttt{User}
\end{aligned}$$

20 : 11

$\texttt{User}(\texttt{bob})$

$\texttt{VideoPlayer}(\texttt{a1})$

$\texttt{Window}(\texttt{w1})$

$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$

20 : 13

$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$

20 : 15

$\texttt{NotVisible}(\texttt{w1})$

# Ontology-Based Data Access

## Query Answering with Rigid Names

Components in user focus in the past, and not visible anymore (now)?

$(\Diamond_P \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)) \wedge \texttt{NotVisible}(x)$

$$\texttt{VideoPlayer} \sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical}$$

$$\texttt{Window} \sqsubseteq \texttt{Component} \qquad \texttt{NotVisible}$$

$$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn} \qquad \texttt{User}$$

20 : 11

$\texttt{User}(\texttt{bob})$

$\texttt{VideoPlayer}(\texttt{a1})$

$\texttt{Window}(\texttt{w1})$

$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$

20 : 13

$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$

20 : 15

$\texttt{NotVisible}(\texttt{w1})$

# Ontology-Based Data Access

## Query Answering with Rigid Names

Components in user focus in the past, and not visible anymore (now)?

$(\Diamond_P \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)) \wedge \texttt{NotVisible}(x)$

$$\texttt{VideoPlayer} \sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical}$$

$$\texttt{Window} \sqsubseteq \texttt{Component} \qquad \texttt{NotVisible}$$

$$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn} \qquad \texttt{User}$$

20 : 11

$\texttt{User}(\texttt{bob})$

$\texttt{VideoPlayer}(\texttt{a1})$

$\texttt{Window}(\texttt{w1})$

$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$

20 : 13

$\texttt{User}(\texttt{bob})$

$\texttt{Component}(\texttt{w1})$

$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$

20 : 15

$\texttt{User}(\texttt{bob})$

$\texttt{Component}(\texttt{w1})$

$\texttt{NotVisible}(\texttt{w1})$

# Ontology-Based Data Access

## Query Answering with Rigid Names

Components in user focus in the past, and not visible anymore (now)?

$(\Diamond_P \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)) \wedge \texttt{NotVisible}(x)$

$$\texttt{VideoPlayer} \sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical}$$

$$\texttt{Window} \sqsubseteq \texttt{Component} \qquad \texttt{NotVisible}$$

$$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn} \qquad \texttt{User}$$

20 : 11

$\texttt{User}(\texttt{bob})$

$\texttt{VideoPlayer}(\texttt{a1})$

$\texttt{Window}(\texttt{w1})$

$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$

20 : 13

$\texttt{User}(\texttt{bob})$

$\texttt{Component}(\texttt{w1})$

$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$

20 : 15

$\texttt{User}(\texttt{bob})$

$\texttt{Component}(\texttt{w1})$

$\texttt{NotVisible}(\texttt{w1})$

# Outline

- **Temporal data:** sequence of fact bases
- **Ontology:** lightweight description logics (DLs)
- **Temporal queries:** linear temporal logic (LTL) + conjunctive queries (CQs)

# Outline

- **Temporal data:** sequence of fact bases
- **Ontology:** lightweight description logics (DLs)
- **Temporal queries:** linear temporal logic (LTL) + conjunctive queries (CQs)

## I

| | |
|---|---|
| **Problem:** | Temporal query satisfiability |
| **Results:** | Computational complexity |
| **Application:** | Choose languages according to available resources (time and memory) |

# Outline

- **Temporal data:** sequence of fact bases
- **Ontology:** lightweight description logics (DLs)
- **Temporal queries:** linear temporal logic (LTL) + conjunctive queries (CQs)

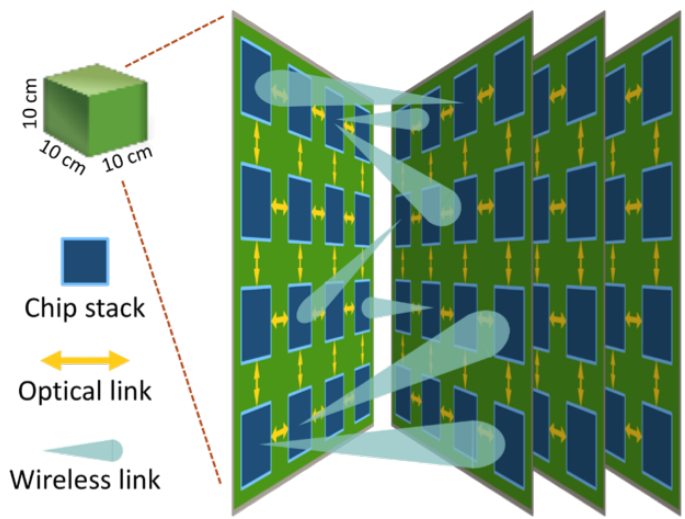
| | I | II |
|---|---|---|
| **Problem:** | Temporal query satisfiability | Temporal query answering |
| **Results:** | Computational complexity | Rewritability |
| **Application:** | Choose languages according to available resources (time and memory) | Hints for implementation (use existing tools) |

# Outline

- **Temporal data:** sequence of fact bases
- **Ontology:** lightweight description logics (DLs)
- **Temporal queries:** linear temporal logic (LTL) + conjunctive queries (CQs)

**Why . . .**

- **no temporal ontology language?** expensive $\Diamond_{\mathrm{P}}\mathtt{User} \sqsubseteq \mathtt{User}$
- **DLs?** user-friendly, well investigated, basis for W3C OWL standard
- **lightweight DLs?** allow for efficient atemporal reasoning
- **CQs?** describe complex networks



### Symbols

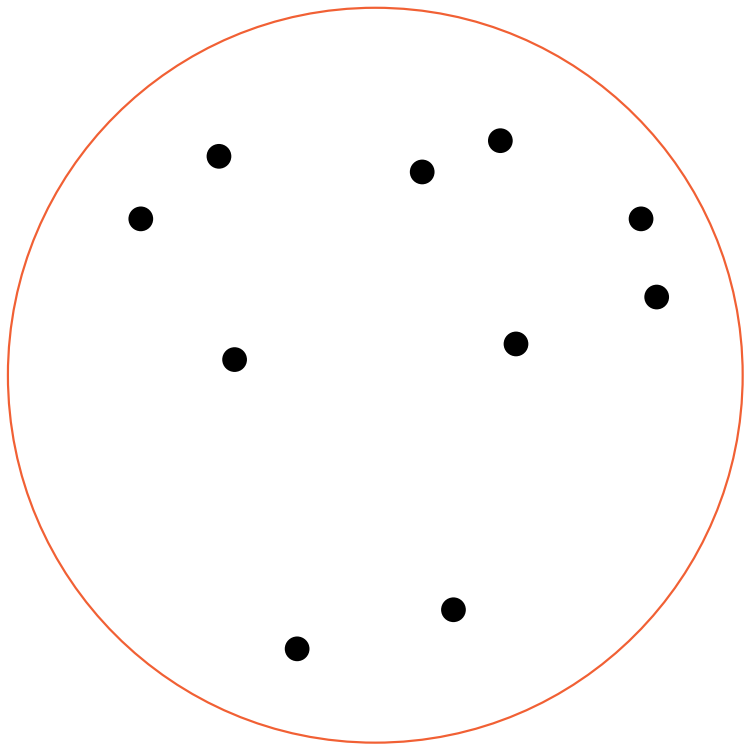
- **Individual names:**
  `ann`, `bob`, `w1`, ...
- **Concept names:**
  `Component`, `User`, `Window`, ...
- **Role names:**
  `LooksAt`, `FocusesOn`, `HasPart`, ...

**Symbols**

- **Individual names:**
  `ann`, `bob`, `w1`, ...
- **Concept names:**
  `Component`, `User`, `Window`, ...
- **Role names:**
  `LooksAt`, `FocusesOn`, `HasPart`, ...

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:

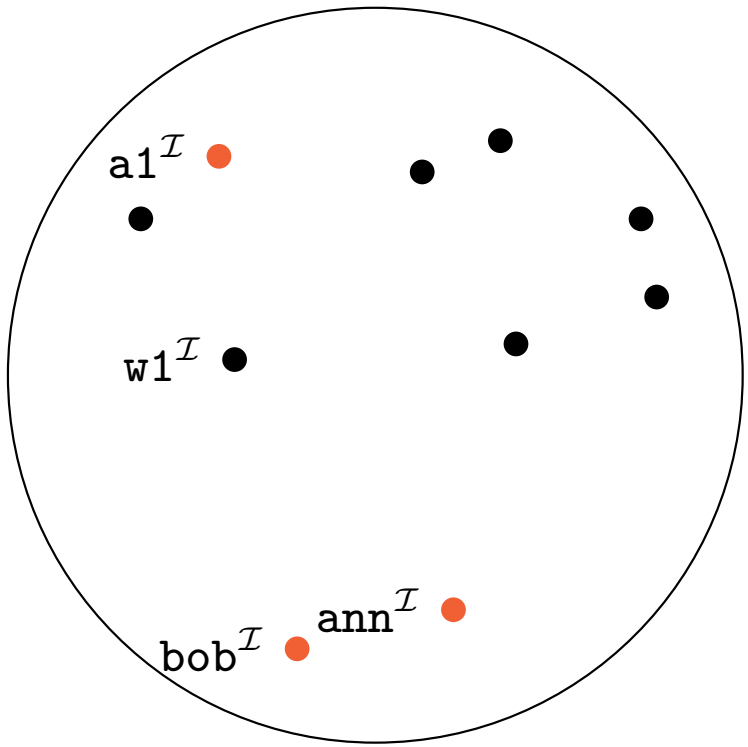


**Symbols**

- **Individual names:**
  ann, bob, w1, ...
- **Concept names:**
  Component, User, Window, ...
- **Role names:**
  LooksAt, FocusesOn, HasPart, ...

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:

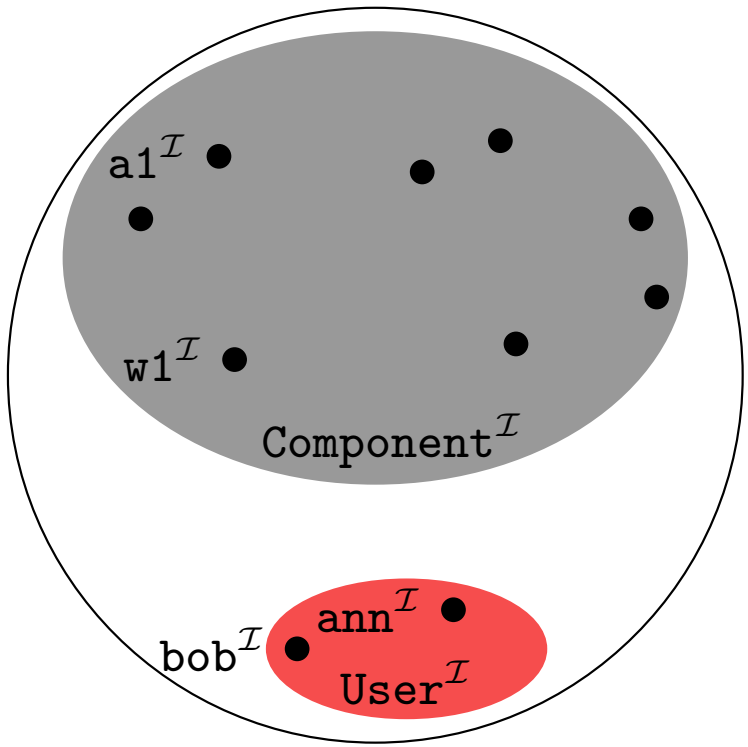


**Symbols**

- **Individual names:**
  ann, bob, w1, ...
- **Concept names:**
  Component, User, Window, ...
- **Role names:**
  LooksAt, FocusesOn, HasPart, ...

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:

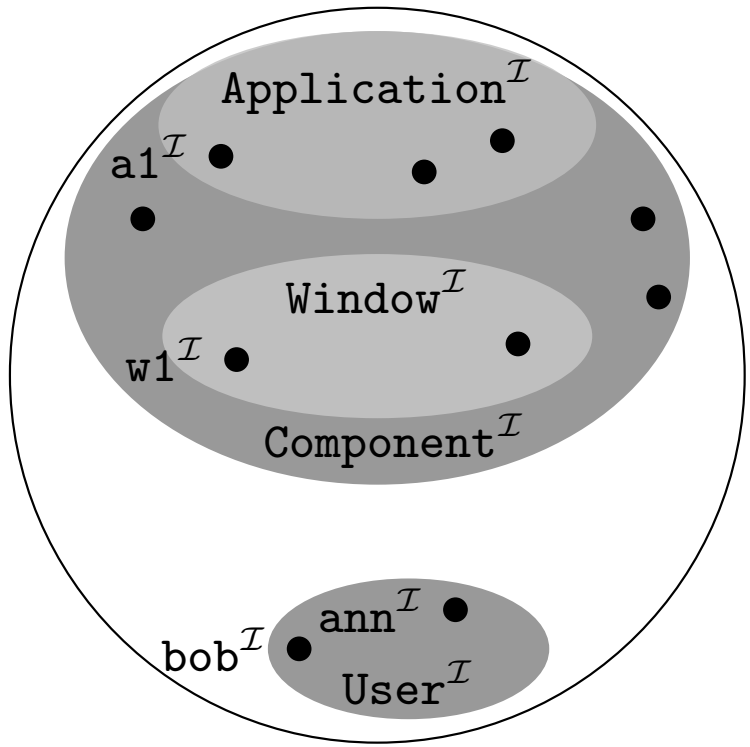


**Symbols**

- **Individual names:**
  ann, bob, w1, ...
- **Concept names:**
  Component, User, Window, ...
- **Role names:**
  LooksAt, FocusesOn, HasPart, ...

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:

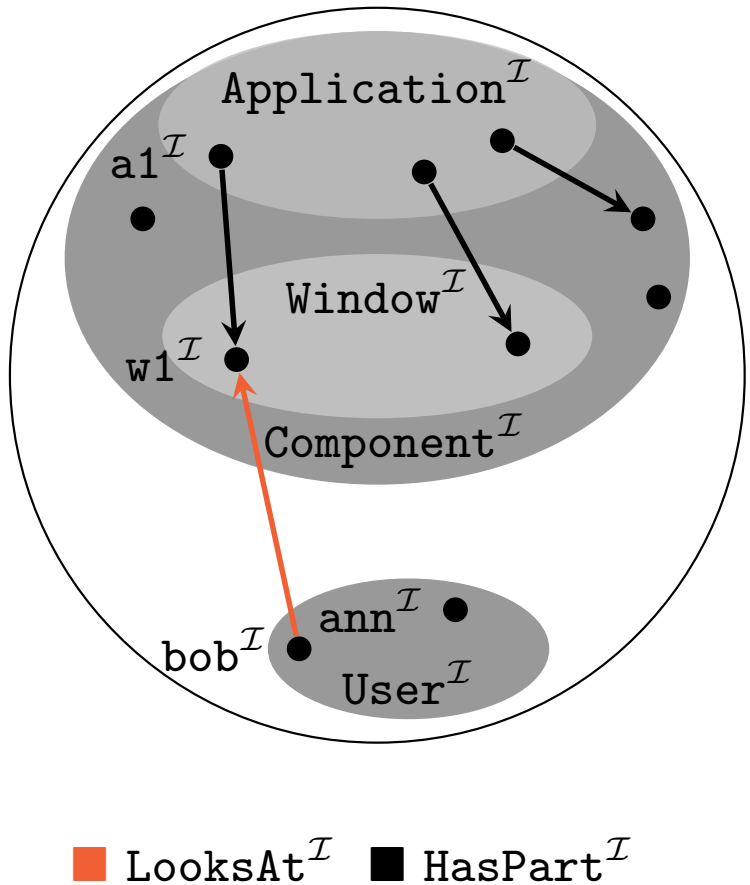


**Symbols**

- **Individual names:**
  ann, bob, w1, ...
- **Concept names:**
  Component, User, Window, ...
- **Role names:**
  LooksAt, FocusesOn, HasPart, ...

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:



**Symbols**

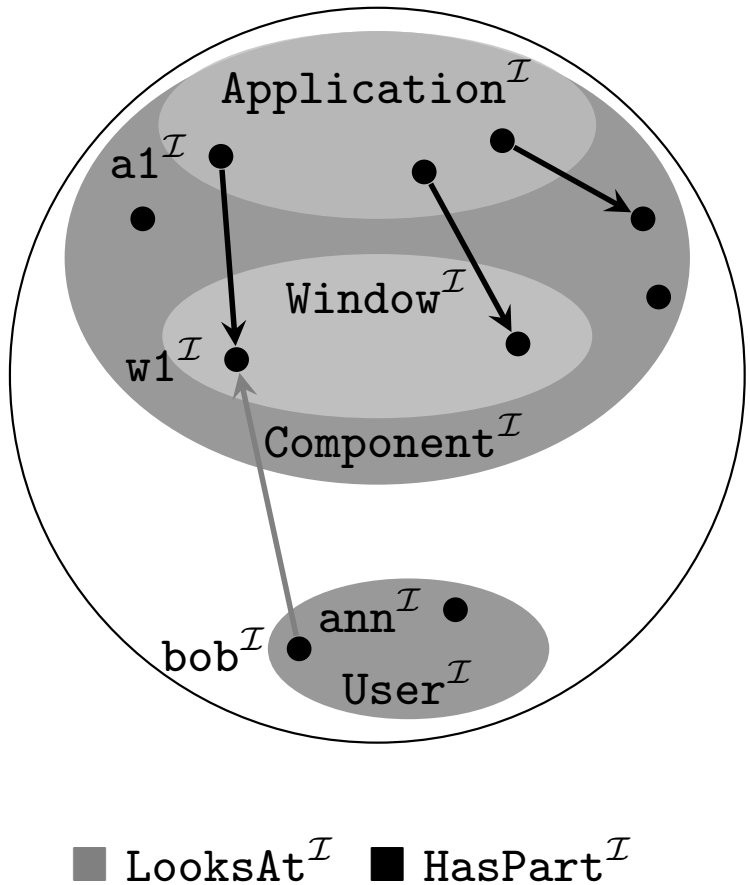
- **Individual names:**
  $\texttt{ann}, \texttt{bob}, \texttt{w1}, \ldots$
- **Concept names:**
  $\texttt{Component}, \texttt{User}, \texttt{Window}, \ldots$
- **Role names:**
  $\texttt{LooksAt}, \texttt{FocusesOn}, \texttt{HasPart}, \ldots$

**Fact base $\mathcal{F}$**

$\texttt{User}(\texttt{bob})$
$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:



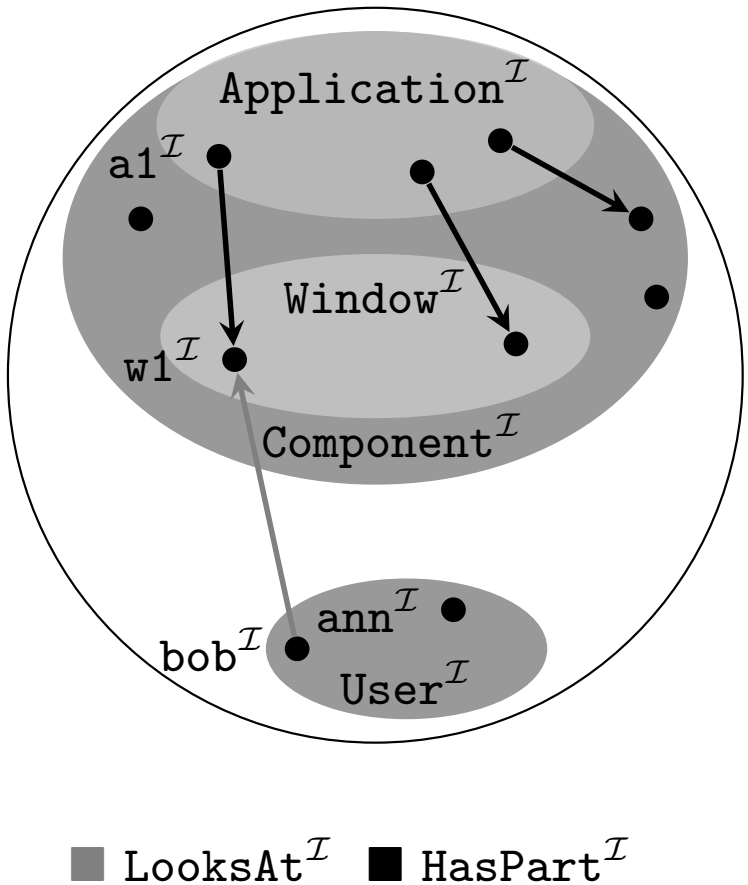
**Symbols**

- **Individual names:**
  $\texttt{ann}, \texttt{bob}, \texttt{w1}, \ldots$
- **Concept names:**
  $\texttt{Component}, \texttt{User}, \texttt{Window}, \ldots$
- **Role names:**
  $\texttt{LooksAt}, \texttt{FocusesOn}, \texttt{HasPart}, \ldots$

| **Fact base** $\mathcal{F}$ | $\mathcal{I} \models \mathcal{F}$ |
|---|---|
| $\texttt{User}(\texttt{bob})$ | $\texttt{bob}^{\mathcal{I}} \in \texttt{User}^{\mathcal{I}}$ |
| $\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$ | $(\texttt{bob}^{\mathcal{I}}, \texttt{w1}^{\mathcal{I}}) \in \texttt{LooksAt}^{\mathcal{I}}$ |

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:

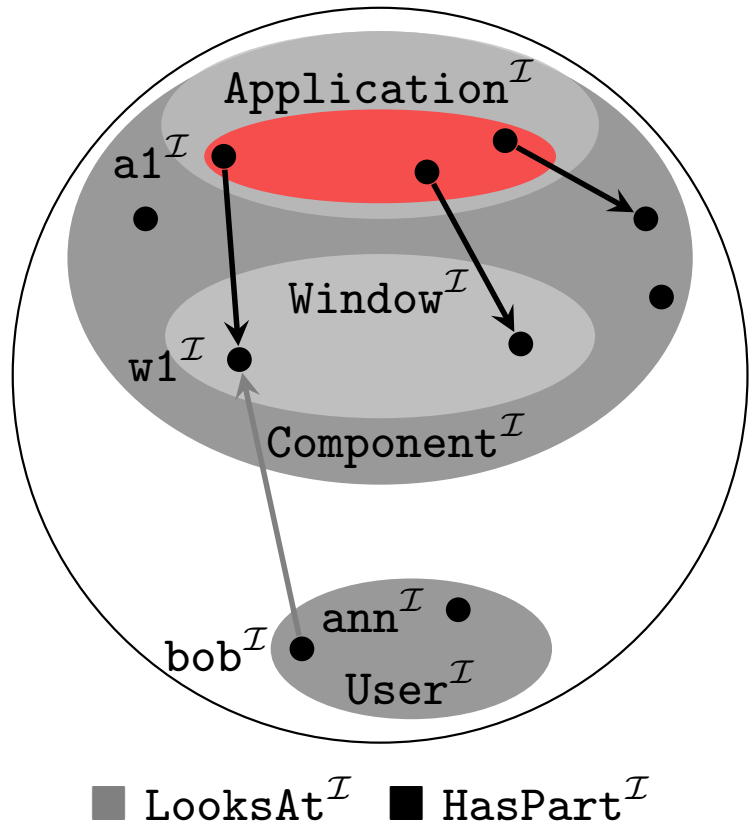


**Basic concepts**

- *DL-Lite*: $\texttt{User}$, $\exists\texttt{HasPart}$, $\exists\texttt{HasPart}^-$
- $\mathcal{EL}$: $\texttt{User}$, $\exists\texttt{HasPart}.\texttt{Window}$

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:

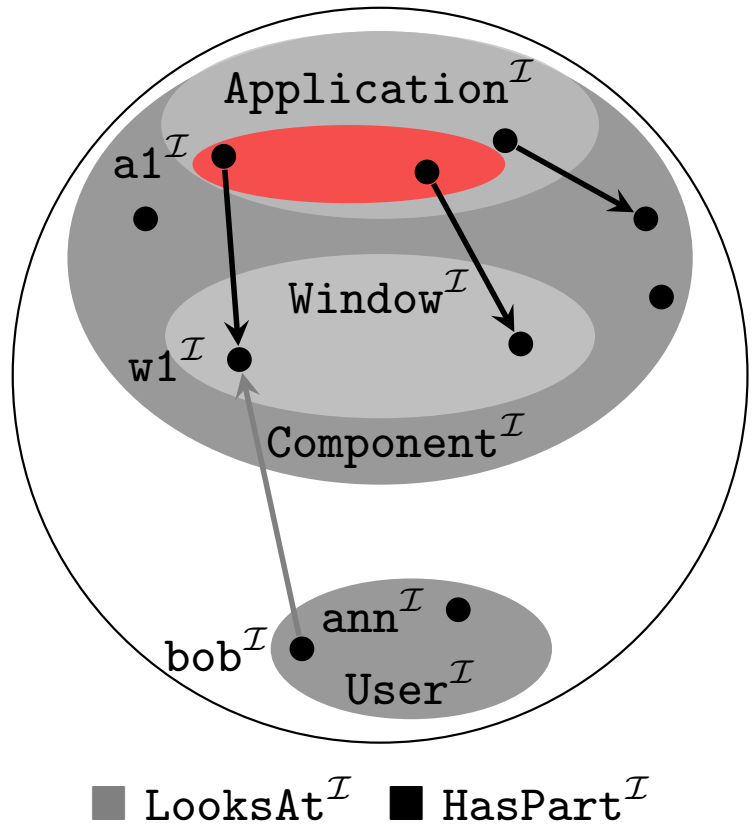


**Basic concepts**

- *DL-Lite*: $\texttt{User}, \exists\texttt{HasPart}, \exists\texttt{HasPart}^-$
- $\mathcal{EL}$: $\texttt{User}, \exists\texttt{HasPart.Window}$

**Semantics:** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Interpretation domain $\Delta^{\mathcal{I}}$ and function $\cdot^{\mathcal{I}}$:



**Basic concepts**

- *DL-Lite*: $\texttt{User}, \exists\texttt{HasPart}, \exists\texttt{HasPart}^-$
- $\mathcal{EL}$: $\texttt{User}, \exists\texttt{HasPart}.\texttt{Window}$

**Ontology $\mathcal{O}$**

**Concept inclusions**

$\texttt{Window} \sqsubseteq \texttt{Component}$

$\texttt{VideoPlayer} \sqsubseteq \neg\texttt{SystemCritical}$

**Role inclusions ($\cdot^{\mathcal{H}}$)**

$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn}$

**Basic concepts**

- *DL-Lite*: $\texttt{User}, \exists\texttt{HasPart}, \exists\texttt{HasPart}^-$
- $\mathcal{EL}$: $\texttt{User}, \exists\texttt{HasPart}.\texttt{Window}$

| | |
|---|---|
| **Ontology $\mathcal{O}$** | $\mathcal{I} \models \mathcal{O}$ |
| **Concept inclusions** | |
| $\texttt{Window} \sqsubseteq \texttt{Component}$ | $\texttt{Window}^{\mathcal{I}} \subseteq \texttt{Component}^{\mathcal{I}}$ |
| $\texttt{VideoPlayer} \sqsubseteq \neg\texttt{SystemCritical}$ | $\texttt{VideoPlayer}^{\mathcal{I}} \cap \texttt{SystemCritical}^{\mathcal{I}} = \emptyset$ |
| **Role inclusions ($\cdot^{\mathcal{H}}$)** | |
| $\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn}$ | $\texttt{LooksAt}^{\mathcal{I}} \subseteq \texttt{FocusesOn}^{\mathcal{I}}$ |

**Basic concepts**

- *DL-Lite*: $\texttt{User}, \exists\texttt{HasPart}, \exists\texttt{HasPart}^-$
- $\mathcal{EL}$: $\texttt{User}, \exists\texttt{HasPart}.\texttt{Window}$

| **Ontology** $\mathcal{O}$ | $\mathcal{I} \models \mathcal{O}$ |
|---|---|
| **Concept inclusions** | |
| $\texttt{Window} \sqsubseteq \texttt{Component}$ | $\texttt{Window}^{\mathcal{I}} \subseteq \texttt{Component}^{\mathcal{I}}$ |
| $\texttt{VideoPlayer} \sqsubseteq \neg\texttt{SystemCritical}$ | $\texttt{VideoPlayer}^{\mathcal{I}} \cap \texttt{SystemCritical}^{\mathcal{I}} = \emptyset$ |
| **Role inclusions** $(\cdot^{\mathcal{H}})$ | |
| $\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn}$ | $\texttt{LooksAt}^{\mathcal{I}} \subseteq \texttt{FocusesOn}^{\mathcal{I}}$ |

DLs we focus on: $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{core}$, $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{horn}$, $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{krom}$, $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{bool}$, $\mathcal{EL}$

**Basic concepts**

- *DL-Lite*: $\texttt{User}, \exists\texttt{HasPart}, \exists\texttt{HasPart}^-$
- $\mathcal{EL}$: $\texttt{User}, \exists\texttt{HasPart}.\texttt{Window}$

| **Ontology** $\mathcal{O}$ | $\mathcal{I} \models \mathcal{O}$ |
|---|---|
| **Concept inclusions** | |
| $\texttt{Window} \sqsubseteq \texttt{Component}$ | $\texttt{Window}^{\mathcal{I}} \subseteq \texttt{Component}^{\mathcal{I}}$ |
| $\texttt{VideoPlayer} \sqsubseteq \neg\texttt{SystemCritical}$ | $\texttt{VideoPlayer}^{\mathcal{I}} \cap \texttt{SystemCritical}^{\mathcal{I}} = \emptyset$ |
| **Role inclusions** $(\cdot^{\mathcal{H}})$ | |
| $\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn}$ | $\texttt{LooksAt}^{\mathcal{I}} \subseteq \texttt{FocusesOn}^{\mathcal{I}}$ |

DLs we focus on: *DL-Lite*$^{\mathcal{H}}_{core}$, *DL-Lite*$^{\mathcal{H}}_{horn}$, *DL-Lite*$^{\mathcal{H}}_{krom}$, *DL-Lite*$^{\mathcal{H}}_{bool}$, $\mathcal{EL}$

**Temporal knowledge base (TKB)** $\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \le i \le n} \rangle$

**Semantics:** $\mathfrak{I} = (\mathcal{I}_i)_{i \ge 0}$

**Basic concepts**

- *DL-Lite*: $\texttt{User}, \exists\texttt{HasPart}, \exists\texttt{HasPart}^-$
- $\mathcal{EL}$: $\texttt{User}, \exists\texttt{HasPart}.\texttt{Window}$

| **Ontology** $\mathcal{O}$ | $\mathcal{I} \models \mathcal{O}$ |
|---|---|
| **Concept inclusions** | |
| $\texttt{Window} \sqsubseteq \texttt{Component}$ | $\texttt{Window}^{\mathcal{I}} \subseteq \texttt{Component}^{\mathcal{I}}$ |
| $\texttt{VideoPlayer} \sqsubseteq \neg\texttt{SystemCritical}$ | $\texttt{VideoPlayer}^{\mathcal{I}} \cap \texttt{SystemCritical}^{\mathcal{I}} = \emptyset$ |
| **Role inclusions** $(\cdot^{\mathcal{H}})$ | |
| $\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn}$ | $\texttt{LooksAt}^{\mathcal{I}} \subseteq \texttt{FocusesOn}^{\mathcal{I}}$ |

DLs we focus on: $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{core}$, $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{horn}$, $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{krom}$, $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{bool}$, $\mathcal{EL}$

**Temporal knowledge base (TKB)**

$\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \le i \le n} \rangle$

**Semantics:** $\mathfrak{I} = (\mathcal{I}_i)_{i \ge 0}$ $\quad \mathfrak{I} \models \mathcal{K}$

$\mathcal{I}_i \models \mathcal{O}$ for all $i \ge 0$,
$\mathcal{I}_i \models \mathcal{F}_i$ for all $i \in [0, n]$,
and $\mathfrak{I}$ respects individual and rigid names

# Temporal Conjunctive Queries (TCQs)

Components in user focus in the past, and not visible anymore (now)?

$$\Phi_{\mathsf{Ex}}(x) := \Big(\Diamond_{\mathsf{P}} \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)\Big) \wedge \texttt{NotVisible}(x)$$

# Temporal Conjunctive Queries (TCQs)

Components in user focus in the past, and not visible anymore (now)?

$$\Phi_{\mathsf{Ex}}(x) := \Big(\Diamond_{\mathsf{P}} \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)\Big) \wedge \texttt{NotVisible}(x)$$

**TCQ** $\Phi, \Psi := \mathsf{CQ}\ \varphi \mid \neg\Phi \mid \Phi \wedge \Psi \mid \Phi \vee \Psi \mid$

# Temporal Conjunctive Queries (TCQs)

Components in user focus in the past, and not visible anymore (now)?

$$\Phi_{\mathsf{Ex}}(x) := \Big(\Diamond_{\mathsf{P}} \exists y.\mathtt{User}(y) \wedge \mathtt{FocusesOn}(y, x) \wedge \mathtt{Component}(x)\Big) \wedge \mathtt{NotVisible}(x)$$

**TCQ** $\Phi, \Psi := \mathsf{CQ}\ \varphi \mid \neg\Phi \mid \Phi \wedge \Psi \mid \Phi \vee \Psi \mid$

$\bigcirc_F \Phi$ (next) $\mid \bigcirc_P \Phi$ (previous) $\mid$

# Temporal Conjunctive Queries (TCQs)

Components in user focus in the past, and not visible anymore (now)?

$$\Phi_{\mathsf{Ex}}(x) := \Big(\Diamond_{\mathsf{P}} \exists y.\mathtt{User}(y) \wedge \mathtt{FocusesOn}(y, x) \wedge \mathtt{Component}(x)\Big) \wedge \mathtt{NotVisible}(x)$$

**TCQ** $\Phi, \Psi := \mathsf{CQ}\ \varphi \mid \neg\Phi \mid \Phi \wedge \Psi \mid \Phi \vee \Psi \mid$

$\bigcirc_F \Phi$ (next) $\mid \bigcirc_P \Phi$ (previous) $\mid \Phi\,\mathsf{U}\,\Psi$ (until) $\mid \Phi\,\mathsf{S}\,\Psi$ (since)

# Temporal Conjunctive Queries (TCQs)

Components in user focus in the past, and not visible anymore (now)?

$$\Phi_{\mathsf{Ex}}(x) := \Big(\Diamond_{\mathsf{P}} \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, x) \wedge \texttt{Component}(x)\Big) \wedge \texttt{NotVisible}(x)$$

**TCQ** $\Phi, \Psi := \mathsf{CQ}\ \varphi \mid \neg\Phi \mid \Phi \wedge \Psi \mid \Phi \vee \Psi \mid$

$\bigcirc_F \Phi$ (next) $\mid \bigcirc_P \Phi$ (previous) $\mid \Phi\,\mathsf{U}\,\Psi$ (until) $\mid \Phi\,\mathsf{S}\,\Psi$ (since)

$\rightarrow \Diamond_{\mathsf{P}}\varphi := \mathsf{true}\,\mathsf{S}\,\varphi$ (some time in the past)

# Temporal Conjunctive Queries (TCQs)

Components in user focus in the past, and not visible anymore (now)?

$$\Phi_{\mathsf{Ex}} := \Big(\Diamond_{\mathsf{P}} \exists y.\mathtt{User}(y) \wedge \mathtt{FocusesOn}(y, \mathtt{w1}) \wedge \mathtt{Component}(\mathtt{w1})\Big) \wedge \mathtt{NotVisible}(\mathtt{w1})$$

**TCQ** $\Phi, \Psi := \mathsf{CQ}\ \varphi \mid \neg\Phi \mid \Phi \wedge \Psi \mid \Phi \vee \Psi \mid$

$\bigcirc_F \Phi$ (next) $\mid \bigcirc_P \Phi$ (previous) $\mid \Phi\,\mathsf{U}\,\Psi$ (until) $\mid \Phi\,\mathsf{S}\,\Psi$ (since)

$\rightarrow \Diamond_{\mathsf{P}} \varphi := \mathsf{true}\,\mathsf{S}\,\varphi$ (some time in the past)

**Semantics:** sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations, Boolean queries

# Temporal Conjunctive Queries (TCQs)

Components in user focus in the past, and not visible anymore (now)?

$\Phi_{\text{Ex}} := \Big(\Diamond_P \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, \texttt{w1}) \wedge \texttt{Component}(\texttt{w1})\Big) \wedge \texttt{NotVisible}(\texttt{w1})$

**TCQ** $\Phi, \Psi := \text{CQ}\ \varphi \mid \neg\Phi \mid \Phi \wedge \Psi \mid \Phi \vee \Psi \mid$

$\bigcirc_F \Phi$ (next) $\mid \bigcirc_P \Phi$ (previous) $\mid \Phi\,\mathsf{U}\,\Psi$ (until) $\mid \Phi\,\mathsf{S}\,\Psi$ (since)

$\rightarrow \Diamond_P \varphi := \text{true}\,\mathsf{S}\,\varphi$ (some time in the past)

**Semantics:** sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations, Boolean queries

Example: $\mathfrak{I}, 2 \models \Phi_{\text{Ex}}$ if

- $\mathcal{I}_2 \models \texttt{NotVisible}(\texttt{w1})$

# Temporal Conjunctive Queries (TCQs)

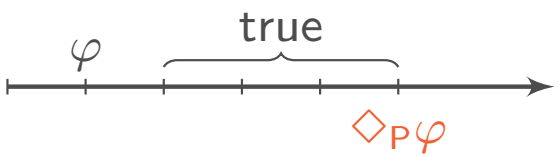
Components in user focus in the past, and not visible anymore (now)?

$$\Phi_{\text{Ex}} := \Big(\Diamond_P \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, \texttt{w1}) \wedge \texttt{Component}(\texttt{w1})\Big) \wedge \texttt{NotVisible}(\texttt{w1})$$

**TCQ** $\Phi, \Psi := \text{CQ}\ \varphi \mid \neg\Phi \mid \Phi \wedge \Psi \mid \Phi \vee \Psi \mid$

$\bigcirc_F \Phi$ (next) | $\bigcirc_P \Phi$ (previous) | $\Phi \,\mathsf{U}\, \Psi$ (until) | $\Phi \,\mathsf{S}\, \Psi$ (since)

$\rightarrow \Diamond_P \varphi := \text{true}\,\mathsf{S}\,\varphi$ (some time in the past)




**Semantics:** sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations, Boolean queries

Example: $\mathfrak{I}, 2 \models \Phi_{\text{Ex}}$ if

- $\mathcal{I}_2 \models \texttt{NotVisible}(\texttt{w1})$
- there is an $i \in [0, 2]$ such that
  $\mathcal{I}_i \models \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, \texttt{w1}) \wedge \texttt{Component}(\texttt{w1})$

# I Solving Satisfiability

- Given: Boolean TCQ $\Phi$ + TKB $\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$
- Sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations
- Complexity of TCQ entailment: $\mathfrak{I}, n \models \Phi$ for all $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$?
- Solve TCQ satisfiability: Is there an $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$ and $\mathfrak{I}, n \models \neg\Phi$?

# I Solving Satisfiability

## . . . good complexities for lightweight DLs and TCQs?

- Given: Boolean TCQ $\Phi$ + TKB $\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$
- Sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations
- Complexity of TCQ entailment: $\mathfrak{I}, n \models \Phi$ for all $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$?
- Solve TCQ satisfiability: Is there an $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$ and $\mathfrak{I}, n \models \neg\Phi$?

# I Solving Satisfiability

## . . . good complexities for lightweight DLs and TCQs?

- Given: Boolean TCQ $\Phi$ + TKB $\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$
- Sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations
- Complexity of TCQ entailment: $\mathfrak{I}, n \models \Phi$ for all $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$?
- Solve TCQ satisfiability: Is there an $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$ and $\mathfrak{I}, n \models \neg\Phi$?

| | Combined Complexity | | | Data Complexity | | |
|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (i) | (ii) | (iii) |
| $\textit{DL-Lite}^{[\ \mid\mathcal{H}]}_{[\textit{core}\mid\textit{horn}]}$ | $\geq$PSpace | ? | ? | ? | ? | ? |
| $\mathcal{EL}$ | $\geq$PSpace | ? | ? | $\geq$P | ? | ? |
| $\textit{DL-Lite}^{[\ \mid\mathcal{H}]}_{[\textit{krom}\mid\textit{bool}]}$ | $\geq$PSpace | ? | $\leq$2-ExpTime | $\geq$co-NP | ? | $\leq$ExpTime |
| $\mathcal{ALCHQ}$[1] | ExpTime | co-NExpTime | 2-ExpTime | co-NP | co-NP | $\leq$ExpTime |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

[1] (Baader et al. 2015)

# I Solving Satisfiability

## . . . good complexities for lightweight DLs and TCQs?

- Given: Boolean TCQ $\Phi$ + TKB $\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$
- Sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations
- Complexity of TCQ entailment: $\mathfrak{I}, n \models \Phi$ for all $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$?
- Solve TCQ satisfiability: Is there an $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$ and $\mathfrak{I}, n \models \neg\Phi$?

| | Combined Complexity | | | Data Complexity | | |
|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (i) | (ii) | (iii) |
| *DL-Lite*$^{[\ \mid\mathcal{H}]}_{[core\mid horn]}$ | $\geq$PSpace | ? | ? | ? | ? | ? |
| $\mathcal{EL}$ | $\geq$PSpace | ? | ? | $\geq$P | ? | ? |
| *DL-Lite*$^{[\ \mid\mathcal{H}]}_{[krom\mid bool]}$ | $\geq$PSpace | ? | **PSpace?** | $\geq$co-NP | ? | $\leq$ExpTime |
| $\mathcal{ALCHQ}$[1] | ExpTime | co-NExpTime | 2-ExpTime | co-NP | co-NP | $\leq$ExpTime |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

[1] (Baader et al. 2015)

# I Solving Satisfiability

## . . . good complexities for lightweight DLs and TCQs?

- Given: Boolean TCQ $\Phi$ + TKB $\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$
- Sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations
- Complexity of TCQ entailment: $\mathfrak{I}, n \models \Phi$ for all $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$?
- Solve TCQ satisfiability: Is there an $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$ and $\mathfrak{I}, n \models \neg\Phi$?

| | Combined Complexity | | | Data Complexity | | |
|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (i) | (ii) | (iii) |
| $\textit{DL-Lite}^{[\ \mid\mathcal{H}]}_{[\textit{core}\mid\textit{horn}]}$ | $\geq$PSpace | ? | ? | ? | ? | **FO rewritable?** |
| $\mathcal{EL}$ | $\geq$PSpace | ? | ? | $\geq$P | ? | ? |
| $\textit{DL-Lite}^{[\ \mid\mathcal{H}]}_{[\textit{krom}\mid\textit{bool}]}$ | $\geq$PSpace | ? | **PSpace?** | $\geq$co-NP | ? | $\leq$ExpTime |
| $\mathcal{ALCHQ}$[1] | ExpTime | co-NExpTime | 2-ExpTime | co-NP | co-NP | $\leq$ExpTime |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

[1] (Baader et al. 2015)

# I Solving Satisfiability

## . . . good complexities for lightweight DLs and TCQs?

- Given: Boolean TCQ $\Phi$ + TKB $\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$
- Sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations
- Complexity of TCQ entailment: $\mathfrak{I}, n \models \Phi$ for all $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$?
- Solve TCQ satisfiability: Is there an $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$ and $\mathfrak{I}, n \models \neg\Phi$?

| | Combined Complexity | | | Data Complexity | | |
|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (i) | (ii) | (iii) |
| $\textit{DL-Lite}^{[\ \mid\mathcal{H}]}_{[\textit{core}\mid\textit{horn}]}$ | $\geq$PSPACE | ? | ? | ? | ? | **FO rewritable?** |
| $\mathcal{EL}$ | $\geq$PSPACE | ? | ? | $\geq$P | ? | **Tractable?** |
| $\textit{DL-Lite}^{[\ \mid\mathcal{H}]}_{[\textit{krom}\mid\textit{bool}]}$ | $\geq$PSPACE | ? | **PSpace?** | $\geq$co-NP | ? | $\leq$EXPTIME |
| $\mathcal{ALCHQ}$[1] | EXPTIME | co-NEXPTIME | 2-EXPTIME | co-NP | co-NP | $\leq$EXPTIME |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

[1] (Baader et al. 2015)

# I Solving Satisfiability

## . . . good complexities for lightweight DLs and TCQs?

- Given: Boolean TCQ $\Phi$ + TKB $\mathcal{K} = \langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$
- Sequences $\mathfrak{I} = (\mathcal{I}_i)_{i \geq 0}$ of interpretations
- Complexity of TCQ entailment: $\mathfrak{I}, n \models \Phi$ for all $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$?
- Solve TCQ satisfiability: Is there an $\mathfrak{I}$ such that $\mathfrak{I} \models \mathcal{K}$ and $\mathfrak{I}, n \models \neg\Phi$?

| | Combined Complexity | | | Data Complexity | | |
|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (i) | (ii) | (iii) |
| *DL-Lite*$^{[\ \mid\mathcal{H}]}_{[core\mid horn]}$ | $\geq$PSpace | ? | ? | ? | ? | **FO rewritable?** |
| $\mathcal{EL}$ | $\geq$PSpace | ? | ? | $\geq$P | ? | **Tractable?** |
| *DL-Lite*$^{[\ \mid\mathcal{H}]}_{[krom\mid bool]}$ | $\geq$PSpace | ? | **PSpace?** | $\geq$co-NP | ? | **co-NP?** |
| $\mathcal{ALCHQ}$[1] | ExpTime | co-NExpTime | 2-ExpTime | co-NP | co-NP | $\leq$ExpTime |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

[1] (Baader et al. 2015)

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

**Satisfiability of** $\neg\Phi$ w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle \rightarrow (\mathcal{I}_i)_{i \geq 0}$ ?

$\Phi_{\mathsf{Ex}} = (\Diamond_{\mathsf{P}} \varphi_1) \wedge \varphi_2$

$\varphi_1 := \exists \mathtt{y}.\mathtt{User}(\mathtt{y}) \wedge \mathtt{FocusesOn}(\mathtt{y}, \mathtt{w1}) \wedge \mathtt{Component}(\mathtt{w1})$
$\varphi_2 := \mathtt{NotVisible}(\mathtt{w1})$

$\neg\Phi_{\mathsf{Ex}} = (\neg\Diamond_{\mathsf{P}} \varphi_1) \vee \neg\varphi_2$

**Satisfiability of** $\neg\Phi$ w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \le i \le n} \rangle \rightarrow (\mathcal{I}_i)_{i \ge 0}$ ?

$\Phi_{\mathsf{Ex}} = (\Diamond_{\mathsf{P}} \varphi_1) \wedge \varphi_2$

$\varphi_1 := \exists \mathtt{y}.\mathtt{User}(\mathtt{y}) \wedge \mathtt{FocusesOn}(\mathtt{y}, \mathtt{w1}) \wedge \mathtt{Component}(\mathtt{w1})$
$\varphi_2 := \mathtt{NotVisible}(\mathtt{w1})$

$\neg\Phi_{\mathsf{Ex}} = (\neg\Diamond_{\mathsf{P}} \varphi_1) \vee \neg\varphi_2$

1. **Replace CQs** $\varphi_1, \varphi_2$ by propositional variables $p_1, p_2$

**Satisfiability of** $\neg\Phi$ w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \le i \le n} \rangle \rightarrow (\mathcal{I}_i)_{i \ge 0}$ ?

$\Phi_{\mathsf{Ex}} = (\Diamond_{\mathsf{P}} \varphi_1) \wedge \varphi_2$

$\varphi_1 := \exists \mathtt{y}.\mathtt{User}(\mathtt{y}) \wedge \mathtt{FocusesOn}(\mathtt{y}, \mathtt{w1}) \wedge \mathtt{Component}(\mathtt{w1})$
$\varphi_2 := \mathtt{NotVisible}(\mathtt{w1})$

$\neg\Phi_{\mathsf{Ex}} = (\neg\Diamond_{\mathsf{P}} \varphi_1) \vee \neg\varphi_2$

1. **Replace CQs** $\varphi_1, \varphi_2$ by propositional variables $p_1, p_2$

   $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

**Satisfiability of** $\neg\Phi$ w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \le i \le n} \rangle \rightarrow (\mathcal{I}_i)_{i \ge 0}$ ?

$\Phi_{\mathsf{Ex}} = (\Diamond_{\mathsf{P}} \varphi_1) \wedge \varphi_2$

$\varphi_1 := \exists \mathtt{y}.\mathtt{User}(\mathtt{y}) \wedge \mathtt{FocusesOn}(\mathtt{y}, \mathtt{w1}) \wedge \mathtt{Component}(\mathtt{w1})$
$\varphi_2 := \mathtt{NotVisible}(\mathtt{w1})$

$\neg\Phi_{\mathsf{Ex}} = (\neg\Diamond_{\mathsf{P}} \varphi_1) \vee \neg\varphi_2$

1. **Replace CQs** $\varphi_1, \varphi_2$ by propositional variables $p_1, p_2$

   $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$

2. **LTL satisfiability problem:**

   Look for an LTL structure $(w_i)_{i \ge 0}$ that satisfies the formula at time point $n$
   $w_i$ : propositions true at $i$

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

**Satisfiability of** $\neg\Phi$ w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \le i \le n} \rangle \rightarrow (\mathcal{I}_i)_{i \ge 0}$ ?

$\Phi_{\mathsf{Ex}} = (\Diamond_{\mathsf{P}} \varphi_1) \wedge \varphi_2$

$\varphi_1 := \exists \mathtt{y}.\mathtt{User}(\mathtt{y}) \wedge \mathtt{FocusesOn}(\mathtt{y}, \mathtt{w1}) \wedge \mathtt{Component}(\mathtt{w1})$
$\varphi_2 := \mathtt{NotVisible}(\mathtt{w1})$

$\neg\Phi_{\mathsf{Ex}} = (\neg\Diamond_{\mathsf{P}} \varphi_1) \vee \neg\varphi_2$

1. **Replace CQs** $\varphi_1, \varphi_2$ by propositional variables $p_1, p_2$

   $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$

2. **LTL satisfiability problem:**

   Look for an LTL structure $(w_i)_{i \ge 0}$ that satisfies the formula at time point $n$
   $w_i$ : propositions true at $i$

   Possible LTL model: $(w_i)_{i \ge 0} = \emptyset, \emptyset, \{p_2\}, \emptyset \ldots$ (for $n = 2$)

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

**Satisfiability of** $\neg\Phi$ w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \le i \le n} \rangle \rightarrow (\mathcal{I}_i)_{i \ge 0}$ ?

$\Phi_{\mathsf{Ex}} = (\Diamond_{\mathsf{P}} \varphi_1) \wedge \varphi_2$

$\varphi_1 := \exists \mathtt{y}.\mathtt{User}(\mathtt{y}) \wedge \mathtt{FocusesOn}(\mathtt{y}, \mathtt{w1}) \wedge \mathtt{Component}(\mathtt{w1})$
$\varphi_2 := \mathtt{NotVisible}(\mathtt{w1})$

$\neg\Phi_{\mathsf{Ex}} = (\neg\Diamond_{\mathsf{P}} \varphi_1) \vee \neg\varphi_2$

1. **Replace CQs** $\varphi_1, \varphi_2$ by propositional variables $p_1, p_2$

   $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$

2. **LTL satisfiability problem:**

   Look for an LTL structure $(w_i)_{i \ge 0}$ that satisfies the formula at time point $n$
   $w_i$ : propositions true at $i$

   Possible LTL model: $(w_i)_{i \ge 0} = \emptyset, \emptyset, \{p_2\}, \emptyset \ldots$ (for $n = 2$)

3. **DL satisfiability problems (atemporal):**
   Look for DL interpretations $(\mathcal{I}_i)_{i \ge 0}$ such that each $\mathcal{I}_i$ satisfies
   - $\langle \mathcal{O}, \mathcal{F}_i \rangle$
   - the CQs according to $w_i$: $\mathcal{I}_i \models \varphi_j$ iff $p_j \in w_i$

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

- $\mathcal{I}_i \models \langle \mathcal{O}, \mathcal{F}_i \rangle$

$\varphi_1 := \exists y.\texttt{User}(y) \wedge \texttt{FocusesOn}(y, \texttt{w1}) \wedge \texttt{Component}(\texttt{w1})$
$\varphi_2 := \texttt{NotVisible}(\texttt{w1})$

$$
\begin{aligned}
\texttt{VideoPlayer} &\sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical} \\
\texttt{Window} &\sqsubseteq \texttt{Component} \qquad\qquad \texttt{NotVisible} \\
\texttt{LooksAt} &\sqsubseteq \texttt{FocusesOn} \qquad\qquad \texttt{User}
\end{aligned}
$$

$\mathcal{F}_0$
$\texttt{User(bob)}$
$\texttt{VideoPlayer(a1)}$
$\texttt{Window(w1)}$
$\texttt{HasPart(a1, w1)}$

$\mathcal{F}_1$
$\texttt{LooksAt(bob, w1)}$

$\mathcal{F}_2$
$\texttt{NotVisible(w1)}$

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

- $\mathcal{I}_i \models \langle \mathcal{O}, \mathcal{F}_i \rangle$

$\varphi_1 := \exists y.\texttt{User}(y) \sqcap \texttt{FocusesOn}(y, \texttt{w1}) \sqcap \texttt{Component}(\texttt{w1})$
$\varphi_2 := \texttt{NotVisible}(\texttt{w1})$

$$
\begin{aligned}
\texttt{VideoPlayer} &\sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical} \\
\texttt{Window} &\sqsubseteq \texttt{Component} \qquad\qquad \texttt{NotVisible} \\
\texttt{LooksAt} &\sqsubseteq \texttt{FocusesOn} \qquad\qquad \texttt{User}
\end{aligned}
$$

$\mathcal{I}_0$
$\texttt{User}(\texttt{bob})$
$\texttt{VideoPlayer}(\texttt{a1})$
$\texttt{Window}(\texttt{w1})$
$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$
$\texttt{Component}(\texttt{w1})$

$\mathcal{I}_1$
$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$
$\texttt{FocusesOn}(\texttt{bob}, \texttt{w1})$

$\mathcal{I}_2$
$\texttt{NotVisible}(\texttt{w1})$

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

- $\mathcal{I}_i \models \langle \mathcal{O}, \mathcal{F}_i \rangle$
- $(w_i)_{i \geq 0} = \emptyset, \emptyset, \{p_2\}, \ldots : \mathcal{I}_0, \mathcal{I}_1$ must not satisfy $\varphi_1$, $\varphi_2$

$\varphi_1 := \exists y.\texttt{User}(y) \sqcap \texttt{FocusesOn}(y, \texttt{w1}) \sqcap \texttt{Component}(\texttt{w1})$
$\varphi_2 := \texttt{NotVisible}(\texttt{w1})$

$$\begin{aligned} \texttt{VideoPlayer} &\sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical} \\ \texttt{Window} &\sqsubseteq \texttt{Component} \qquad\qquad \texttt{NotVisible} \\ \texttt{LooksAt} &\sqsubseteq \texttt{FocusesOn} \qquad\qquad \texttt{User} \end{aligned}$$

$\mathcal{I}_0$
$\texttt{User}(\texttt{bob})$
$\texttt{VideoPlayer}(\texttt{a1})$
$\texttt{Window}(\texttt{w1})$
$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$
$\texttt{Component}(\texttt{w1})$

$\mathcal{I}_1$
$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$
$\texttt{FocusesOn}(\texttt{bob}, \texttt{w1})$

$\mathcal{I}_2$
$\texttt{NotVisible}(\texttt{w1})$

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

- $\mathcal{I}_i \models \langle \mathcal{O}, \mathcal{F}_i \rangle$
- $(w_i)_{i \geq 0} = \emptyset, \emptyset, \{p_2\}, \ldots : \mathcal{I}_0, \mathcal{I}_1$ must not satisfy $\varphi_1, \varphi_2$

$\varphi_1 := \exists y.\texttt{User}(y) \sqcap \texttt{FocusesOn}(y, \texttt{w1}) \sqcap \texttt{Component}(\texttt{w1})$
$\varphi_2 := \texttt{NotVisible}(\texttt{w1})$

$\texttt{VideoPlayer} \sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical}$
$\texttt{Window} \sqsubseteq \texttt{Component}$
$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn}$
$\texttt{NotVisible}$
$\texttt{User}$

$\mathcal{I}_0$
$\texttt{User}(\texttt{bob})$
$\texttt{VideoPlayer}(\texttt{a1})$
$\texttt{Window}(\texttt{w1})$
$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$
$\texttt{Component}(\texttt{w1})$

$\mathcal{I}_1$
$\texttt{User}(\texttt{bob})$
$\texttt{Component}(\texttt{w1})$
$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$
$\texttt{FocusesOn}(\texttt{bob}, \texttt{w1})$

$\mathcal{I}_2$
$\texttt{User}(\texttt{bob})$
$\texttt{Component}(\texttt{w1})$
$\texttt{NotVisible}(\texttt{w1})$

# I Solving Satisfiability: A General Algorithm (Baader et al. 2012, 2015)

- $\mathcal{I}_i \models \langle \mathcal{O}, \mathcal{F}_i \rangle$
- $(w_i)_{i \geq 0} = \emptyset, \emptyset, \{p_2\}, \ldots : \mathcal{I}_0, \mathcal{I}_1$ must not satisfy $\varphi_1$, $\varphi_2$

$\varphi_1 := \exists y.\texttt{User}(y) \sqcap \texttt{FocusesOn}(y, \texttt{w1}) \sqcap \texttt{Component}(\texttt{w1})$
$\varphi_2 := \texttt{NotVisible}(\texttt{w1})$

$\texttt{VideoPlayer} \sqsubseteq \texttt{Application} \sqcap \texttt{EnergyIntensive} \sqcap \neg\texttt{SystemCritical}$
$\texttt{Window} \sqsubseteq \texttt{Component}$
$\texttt{LooksAt} \sqsubseteq \texttt{FocusesOn}$
$\texttt{NotVisible}$
$\texttt{User}$

$\mathcal{I}_0$
$\texttt{User}(\texttt{bob})$
$\texttt{VideoPlayer}(\texttt{a1})$
$\texttt{Window}(\texttt{w1})$
$\texttt{HasPart}(\texttt{a1}, \texttt{w1})$
$\texttt{Component}(\texttt{w1})$

$\mathcal{I}_1$
$\texttt{User}(\texttt{bob})$
$\texttt{Component}(\texttt{w1})$
$\texttt{LooksAt}(\texttt{bob}, \texttt{w1})$
$\texttt{FocusesOn}(\texttt{bob}, \texttt{w1})$

$\mathcal{I}_2$
$\texttt{User}(\texttt{bob})$
$\texttt{Component}(\texttt{w1})$
$\texttt{NotVisible}(\texttt{w1})$

# I Solving Satisfiability: First Results for TCQ Entailment

| | Combined Complexity | | | Data Complexity | | |
|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (i) | (ii) | (iii) |
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{core}\mid\mathit{horn}]}$ | $\geq$PSPACE | ? | $\leq$co-NEXPTIME | ? | ? | $\leq$co-NP |
| $\mathcal{EL}$ | $\geq$PSPACE | ? | $\leq$co-NEXPTIME | $\geq$P | ? | $\leq$co-NP |
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{krom}\mid\mathit{bool}]}$ | $\geq$PSPACE | ? | $\leq$2-EXPTIME | $\geq$co-NP | ? | $\leq$EXPTIME |
| $\mathcal{ALCHQ}$[1] | EXPTIME | co-NEXPTIME | 2-EXPTIME | co-NP | co-NP | $\leq$EXPTIME |

(i) no rigid concept or role names

(ii) rigid concept names

(iii) rigid role names (and rigid concept names)

[1] (Baader et al. 2015)

**LTL satisfiability algorithm**
(Sistla and Clarke 1985): If LTL model exists, then there is a periodic one

**LTL satisfiability algorithm** Model $(w_i)_{i\geq 0}$ for $(\neg \Diamond_{\mathsf{P}} p_1) \vee \neg p_2$?
(Sistla and Clarke 1985): If LTL model exists, then there is a periodic one

**LTL satisfiability algorithm** Model $(w_i)_{i\geq 0}$ for $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$?
(Sistla and Clarke 1985): If LTL model exists, then there is a periodic one

- **Guess** start $s$ and end $e$ of the period
- **Memory**: LTL formula sets $\mathcal{W}_{i-1}$, $\mathcal{W}_i$, $\mathcal{W}_s$ representing $w_{i-1}, w_i, w_s$

**LTL satisfiability algorithm** Model $(w_i)_{i\geq 0}$ for $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$?
(Sistla and Clarke 1985): If LTL model exists, then there is a periodic one

- **Guess** start $s$ and end $e$ of the period
- **Memory**: LTL formula sets $\mathcal{W}_{i-1}$, $\mathcal{W}_i$, $\mathcal{W}_s$ representing $w_{i-1}, w_i, w_s$
- **Iterate over time t** and always
  - $\mathcal{W}_{i-1} := \mathcal{W}_i$
    $\mathcal{W}_i :=$ **guess** a set of subformulas
  - **Check** if $\mathcal{W}_i$ may follow after $\mathcal{W}_{i-1}$ $\qquad p_1 \in \mathcal{W}_{i-1} \Rightarrow \Diamond_{\mathsf{P}} p_1 \in \mathcal{W}_i$
  - At $s$: $\mathcal{W}_s := \mathcal{W}_i$
  - At $e$: **check** if $\mathcal{W}_s$ may follow after $\mathcal{W}_i$

**LTL satisfiability algorithm** Model $(w_i)_{i\geq 0}$ for $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$? $\rightarrow (\mathcal{I}_t)_{t\geq 0}$ ?
(Sistla and Clarke 1985): If LTL model exists, then there is a periodic one

- **Guess** start $s$ and end $e$ of the period
- **Memory**: LTL formula sets $\mathcal{W}_{i-1}$, $\mathcal{W}_i$, $\mathcal{W}_s$ representing $w_{i-1}, w_i, w_s$
- **Iterate over time t** and always
  - $\mathcal{W}_{i-1} := \mathcal{W}_i$
    $\mathcal{W}_i :=$ **guess** a set of subformulas
  - **Check** if $\mathcal{W}_i$ may follow after $\mathcal{W}_{i-1}$ $\quad p_1 \in \mathcal{W}_{i-1} \Rightarrow \Diamond_{\mathsf{P}} p_1 \in \mathcal{W}_i$
  - At $s$: $\mathcal{W}_s := \mathcal{W}_i$
  - At $e$: **check** if $\mathcal{W}_s$ may follow after $\mathcal{W}_i$

**LTL satisfiability algorithm** Model $(w_i)_{i\geq 0}$ for $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$? $\rightarrow (\mathcal{I}_t)_{t\geq 0}$ ?
(Sistla and Clarke 1985): If LTL model exists, then there is a periodic one

- **Guess** start $s$ and end $e$ of the period
- **Memory**: LTL formula sets $\mathcal{W}_{i-1}$, $\mathcal{W}_i$, $\mathcal{W}_s$ representing $w_{i-1}, w_i, w_s$
- **Iterate over time t** and always
  - $\mathcal{W}_{i-1} := \mathcal{W}_i$
    $\mathcal{W}_i :=$ **guess** a set of subformulas
  - **Check** if $\mathcal{W}_i$ may follow after $\mathcal{W}_{i-1}$ $\quad p_1 \in \mathcal{W}_{i-1} \Rightarrow \Diamond_{\mathsf{P}} p_1 \in \mathcal{W}_i$
  - **DL satisfiability testing on the fly**:
    Look for $\mathcal{I}_t$ such that
    - $\mathcal{I}_t \models \langle \mathcal{O}, \mathcal{F}_t \rangle$
    - $\mathcal{I}_t \models \varphi_j$ iff $p_j \in w_t$ (given by $\mathcal{W}_i$)
  - At $s$: $\mathcal{W}_s := \mathcal{W}_i$
  - At $e$: **check** if $\mathcal{W}_s$ may follow after $\mathcal{W}_i$

## with Rigid Names

**LTL satisfiability algorithm** Model $(w_i)_{i\geq 0}$ for $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$? $\rightarrow (\mathcal{I}_t)_{t\geq 0}$ ?
(Sistla and Clarke 1985): If LTL model exists, then there is a periodic one

- **Guess** start $s$ and end $e$ of the period
- **Memory**: LTL formula sets $\mathcal{W}_{i-1}$, $\mathcal{W}_i$, $\mathcal{W}_s$ representing $w_{i-1}, w_i, w_s$
- **Iterate over time t** and always
  - $\mathcal{W}_{i-1} := \mathcal{W}_i$
    $\mathcal{W}_i :=$ **guess** a set of subformulas
  - **Check** if $\mathcal{W}_i$ may follow after $\mathcal{W}_{i-1}$ $\quad p_1 \in \mathcal{W}_{i-1} \Rightarrow \Diamond_{\mathsf{P}} p_1 \in \mathcal{W}_i$
  - **DL satisfiability testing on the fly**:
    Look for $\mathcal{I}_t$ such that
    - $\mathcal{I}_t \models \langle \mathcal{O}, \mathcal{F}_t \rangle$
    - $\mathcal{I}_t \models \varphi_j$ iff $p_j \in w_t$ (given by $\mathcal{W}_i$)
  - At $s$: $\mathcal{W}_s := \mathcal{W}_i$
  - At $e$: **check** if $\mathcal{W}_s$ may follow after $\mathcal{W}_i$

# I Solving Satisfiability: PSPACE Combined Complexity

## with Rigid Names

**LTL satisfiability algorithm** Model $(w_i)_{i\geq 0}$ for $(\neg\Diamond_{\mathsf{P}} p_1) \vee \neg p_2$? $\rightarrow (\mathcal{I}_t)_{t\geq 0}$ ?
(Sistla and Clarke 1985): If LTL model exists, then there is a periodic one

- Guess a polynomial amount of data $\mathcal{D}$ `User(bob)`,...
- **Guess** start $s$ and end $e$ of the period
- **Memory**: LTL formula sets $\mathcal{W}_{i-1}$, $\mathcal{W}_i$, $\mathcal{W}_s$ representing $w_{i-1}, w_i, w_s$
- **Iterate over time t** and always
  - $\mathcal{W}_{i-1} := \mathcal{W}_i$
    $\mathcal{W}_i :=$ **guess** a set of subformulas
  - **Check** if $\mathcal{W}_i$ may follow after $\mathcal{W}_{i-1}$ $p_1 \in \mathcal{W}_{i-1} \Rightarrow \Diamond_{\mathsf{P}} p_1 \in \mathcal{W}_i$
  - **DL satisfiability testing on the fly**:
    Look for $\mathcal{I}_t$ such that
    - $\mathcal{I}_t \models \langle \mathcal{O}, \mathcal{F}_t \rangle$
    - $\mathcal{I}_t \models \varphi_j$ iff $p_j \in w_t$ (given by $\mathcal{W}_i$)

    + additional tests w.r.t. $\mathcal{D}$ $\mathcal{I}_i \models$ `User(bob)`...?
  - At $s$: $\mathcal{W}_s := \mathcal{W}_i$
  - At $e$: **check** if $\mathcal{W}_s$ may follow after $\mathcal{W}_i$

# I Solving Satisfiability: Results for TCQ Entailment

## Combined Complexity

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $DL\text{-}Lite^{[\ \mid\mathcal{H}]}_{[core\mid horn]}$ | PSPACE | PSPACE | PSPACE |
| $\mathcal{EL}$ | PSPACE | PSPACE | $\geq$co-NEXPTIME |
| $DL\text{-}Lite_{[krom\mid bool]}$ | $\geq$PSPACE | ? | $\leq$2-EXPTIME |
| $DL\text{-}Lite^{\mathcal{H}}_{[krom\mid bool]}$ | $\geq$PSPACE | ? | $\leq$2-EXPTIME |
| $\mathcal{ALCHQ}$[1] | EXPTIME | co-NEXPTIME | 2-EXPTIME |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

[1] (Baader et al. 2015)

# I Solving Satisfiability: Results for TCQ Entailment

## Combined Complexity

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{core}\mid\mathit{horn}]}$ | PSPACE | PSPACE | PSPACE |
| $\mathcal{EL}$ | PSPACE | PSPACE | co-NEXPTIME |
| $\mathit{DL\text{-}Lite}_{[\mathit{krom}\mid\mathit{bool}]}$ | $\geq$PSPACE | ? | $\leq$2-EXPTIME |
| $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{[\mathit{krom}\mid\mathit{bool}]}$ | $\geq$PSPACE | ? | $\leq$2-EXPTIME |
| $\mathcal{ALCHQ}$[1] | EXPTIME | co-NEXPTIME | 2-EXPTIME |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

- PSPACE: rigid roles critical if DL powerful enough

[1] (Baader et al. 2015)

# I Solving Satisfiability: Results for TCQ Entailment

## Combined Complexity

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{core}\mid\mathit{horn}]}$ | PSPACE | PSPACE | PSPACE |
| $\mathcal{EL}$ | PSPACE | PSPACE | co-NEXPTIME |
| $\mathit{DL\text{-}Lite}_{[\mathit{krom}\mid\mathit{bool}]}$ | EXPTIME | ? | $\leq$2-EXPTIME |
| $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{[\mathit{krom}\mid\mathit{bool}]}$ | 2-EXPTIME | 2-EXPTIME | 2-EXPTIME |
| $\mathcal{ALCHQ}$[1] | EXPTIME | co-NEXPTIME | 2-EXPTIME |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

- PSPACE: rigid roles critical if DL powerful enough
- TCQ satisfiability in $\mathit{DL\text{-}Lite}_{\mathit{bool}}$ reducible to $\mathit{DL\text{-}Lite}_{\mathit{krom}}$

$\mathtt{User} \sqsubseteq \mathtt{Male} \sqcup \mathtt{Female}$

**CIs** $\top \sqsubseteq \mathtt{Male} \sqcup \overline{\mathtt{Male}},\ \mathtt{Male} \sqsubseteq \neg\overline{\mathtt{Male}},\ \ldots$

**TCQ** $\neg\exists x.\mathtt{User}(x) \wedge \overline{\mathtt{Male}}(x) \wedge \overline{\mathtt{Female}}(x)$

[1] (Baader et al. 2015)

# I Solving Satisfiability: Results for TCQ Entailment

## Combined Complexity

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{core}\mid\mathit{horn}]}$ | PSPACE | PSPACE | PSPACE |
| $\mathcal{EL}$ | PSPACE | PSPACE | co-NEXPTIME |
| $\mathit{DL\text{-}Lite}_{[\mathit{krom}\mid\mathit{bool}]}$ | EXPTIME | co-NEXPTIME | 2-EXPTIME |
| $\mathit{DL\text{-}Lite}^{\mathcal{H}}_{[\mathit{krom}\mid\mathit{bool}]}$ | 2-EXPTIME | 2-EXPTIME | 2-EXPTIME |
| $\mathcal{ALCHQ}$[1] | EXPTIME | co-NEXPTIME | 2-EXPTIME |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

- PSPACE: rigid roles critical if DL powerful enough
- TCQ satisfiability in $\mathit{DL\text{-}Lite}_{\mathit{bool}}$ reducible to $\mathit{DL\text{-}Lite}_{\mathit{krom}}$

$\mathtt{User} \sqsubseteq \mathtt{Male} \sqcup \mathtt{Female}$

**CIs** $\top \sqsubseteq \mathtt{Male} \sqcup \overline{\mathtt{Male}},\ \mathtt{Male} \sqsubseteq \neg\overline{\mathtt{Male}},\ \ldots$

**TCQ** $\neg\exists x.\mathtt{User}(x) \wedge \overline{\mathtt{Male}}(x) \wedge \overline{\mathtt{Female}}(x)$

[1] (Baader et al. 2015)

# I Solving Satisfiability: Reduction of SAT

$(\neg x \vee \neg y \vee z) \wedge \ldots$ satisfiable iff $\Phi$ is satisfiable w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$

Represent formula in the fact bases, three per clause

$\mathcal{F}_0$

$\mathsf{C}(c)$

$\mathsf{R}(\overline{x}, c)$

$\mathcal{F}_1$

$\mathsf{R}(\overline{y}, c)$

$\mathcal{F}_2$

$\mathsf{R}(z, c)$

# I Solving Satisfiability: Reduction of SAT

$(\neg x \vee \neg y \vee z) \wedge \ldots$ satisfiable iff $\Phi$ is satisfiable w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$

Select a literal in the TCQ

$$\Box_{\mathsf{P}} \left( \mathsf{C}(c) \rightarrow \left( \mathsf{L}(c) \vee \bigcirc_F \mathsf{L}(c) \vee \bigcirc_F \bigcirc_F \mathsf{L}(c) \right) \right)$$

Represent formula in the fact bases, three per clause

$\mathcal{F}_0$

$\mathsf{C}(c)$

$\mathsf{R}(\overline{x}, c)$

$\mathcal{F}_1$

$\mathsf{R}(\overline{y}, c)$

$\mathcal{F}_2$

$\mathsf{R}(z, c)$

# I Solving Satisfiability: Reduction of SAT

$(\neg x \vee \neg y \vee z) \wedge \ldots$ satisfiable iff $\Phi$ is satisfiable w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$

Select a literal in the TCQ

$$\Box_{\mathsf{P}} \Big( \mathsf{C}(c) \rightarrow \Big( \mathsf{L}(c) \vee \bigcirc_F \mathsf{L}(c) \vee \bigcirc_F \bigcirc_F \mathsf{L}(c) \Big) \Big)$$

Transfer choice of TCQ to literal individuals. A to express assignment

$$\exists \mathsf{R}.\mathsf{L} \sqsubseteq \mathsf{A}$$

Represent formula in the fact bases, three per clause

$\mathcal{F}_0$

$\mathsf{C}(c)$

$\mathsf{R}(\overline{x}, c)$

$\mathcal{F}_1$

$\mathsf{R}(\overline{y}, c)$

$\mathcal{F}_2$

$\mathsf{R}(z, c)$

# I Solving Satisfiability: Reduction of SAT

$(\neg x \vee \neg y \vee z) \wedge \ldots$ satisfiable iff $\Phi$ is satisfiable w.r.t. $\langle \mathcal{O}, (\mathcal{F}_i)_{0 \leq i \leq n} \rangle$

Select a literal in the TCQ, and ensure valid assignments: $\mathsf{A}(\overline{x})$ iff $\neg \mathsf{A}(x)$

$$\Box_{\mathsf{P}} \left( \mathsf{C}(c) \rightarrow \left( \mathsf{L}(c) \vee \bigcirc_F \mathsf{L}(c) \vee \bigcirc_F \bigcirc_F \mathsf{L}(c) \right) \right) \wedge$$
$$\Box_{\mathsf{P}} \neg \exists u, v . \mathsf{S}(u, v) \wedge \mathsf{A}(u) \wedge \mathsf{A}(v)$$

Transfer choice of TCQ to literal individuals. $\mathsf{A}$ to express assignment

$$\exists \mathsf{R}.\mathsf{L} \sqsubseteq \mathsf{A}$$

Represent formula in the fact bases, three per clause

$\mathcal{F}_0$

$\mathsf{C}(c)$

$\mathsf{R}(\overline{x}, c)$

$\mathsf{S}(\overline{x}, x)$

$\mathcal{F}_1$

$\mathsf{R}(\overline{y}, c)$

$\mathsf{S}(\overline{y}, y)$

$\mathcal{F}_2$

$\mathsf{R}(z, c)$

$\mathsf{S}(z, \overline{z})$

# I Solving Satisfiability: Results for TCQ Entailment

## Data Complexity

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{core}\mid\mathit{horn}]}$ | ALogTime | ALogTime | ALogTime |
| $\mathcal{EL}$ | $\geq$P | co-NP | co-NP |
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{krom}\mid\mathit{bool}]}$ | $\geq$co-NP | ? | $\leq$ExpTime |
| $\mathcal{ALCHQ}$[1] | co-NP | co-NP | $\leq$ExpTime |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

- *DL-Lite*: no FO rewritability
- ALogTime: efficient parallel algorithms exist!

[1] (Baader et al. 2015)

# I Solving Satisfiability: Results for TCQ Entailment

## Data Complexity

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{core}\mid\mathit{horn}]}$ | ALogTime | ALogTime | ALogTime |
| $\mathcal{EL}$ | P | co-NP | co-NP |
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{krom}\mid\mathit{bool}]}$ | $\geq$co-NP | ? | $\leq$ExpTime |
| $\mathcal{ALCHQ}$[1] | co-NP | co-NP | $\leq$ExpTime |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

- *DL-Lite*: no FO rewritability
- ALogTime: efficient parallel algorithms exist!
- $\mathcal{EL}$: best result possible if no rigid symbols, but already rigid concepts critical

[1] (Baader et al. 2015)

# I Solving Satisfiability: Results for TCQ Entailment

## Data Complexity

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{core}\mid\mathit{horn}]}$ | ALogTime | ALogTime | ALogTime |
| $\mathcal{EL}$ | P | co-NP | co-NP |
| $\mathit{DL\text{-}Lite}^{[\ \mid\mathcal{H}]}_{[\mathit{krom}\mid\mathit{bool}]}$ | co-NP | co-NP | $\leq$ExpTime |
| $\mathcal{ALCHQ}$[1] | co-NP | co-NP | $\leq$ExpTime |

(i) no rigid names
(ii) rigid concept names
(iii) rigid role names (and rigid concept names)

- *DL-Lite*: no FO rewritability
- ALogTime: efficient parallel algorithms exist!
- $\mathcal{EL}$: best result possible if no rigid symbols, but already rigid concepts critical
- Upper bounds: apply general approach

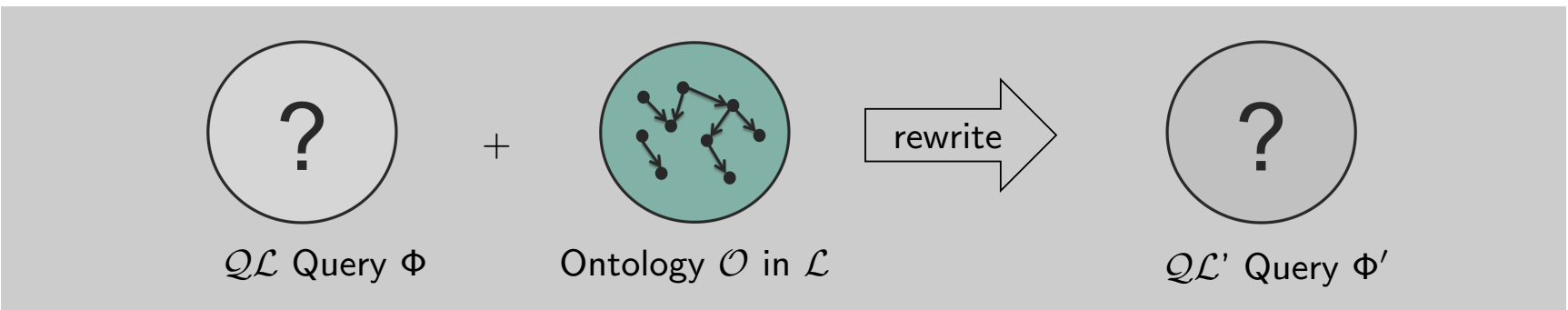
[1] (Baader et al. 2015)

# II Rewritability of Temporal Query Answering

- Positive Temporal $\mathcal{QL}$ queries: LTL without negation + $\mathcal{QL}$ queries
- Temporal KB with ontology in some lightweight logic $\mathcal{L}$
- $\mathcal{QL}$ and $\mathcal{L}$ must satisfy certain requirements
  $\rightarrow$ Rewritability of $\mathcal{QL}$ queries w.r.t. KBs in $\mathcal{L}$

# II Rewritability of Temporal Query Answering

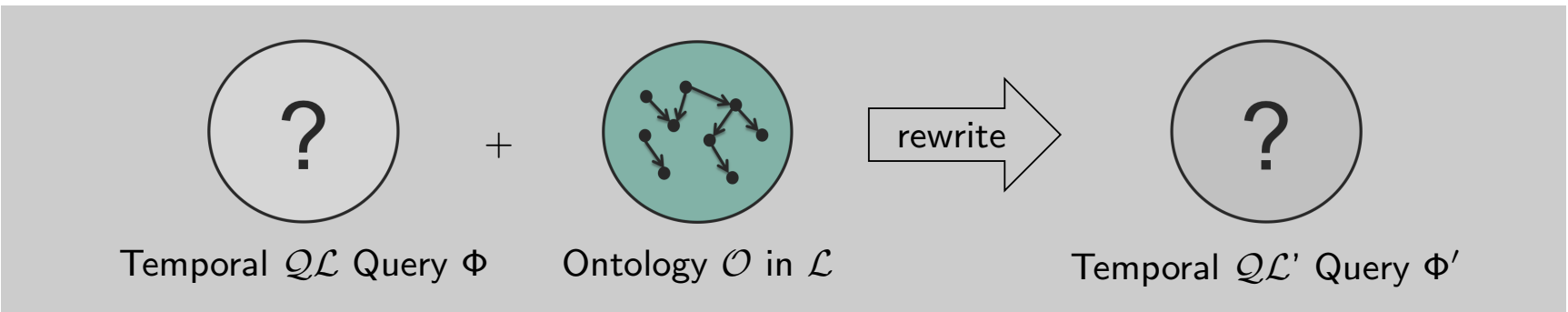
- Positive Temporal $\mathcal{QL}$ queries: LTL without negation + $\mathcal{QL}$ queries
- Temporal KB with ontology in some lightweight logic $\mathcal{L}$
- $\mathcal{QL}$ and $\mathcal{L}$ must satisfy certain requirements
  $\rightarrow$ Rewritability of $\mathcal{QL}$ queries w.r.t. KBs in $\mathcal{L}$




Answers to $\Phi'$ over $\mathcal{F}$ $=$
Answers to $\Phi$ w.r.t. $\langle \mathcal{O}, \mathcal{F} \rangle$

# II Rewritability of Temporal Query Answering

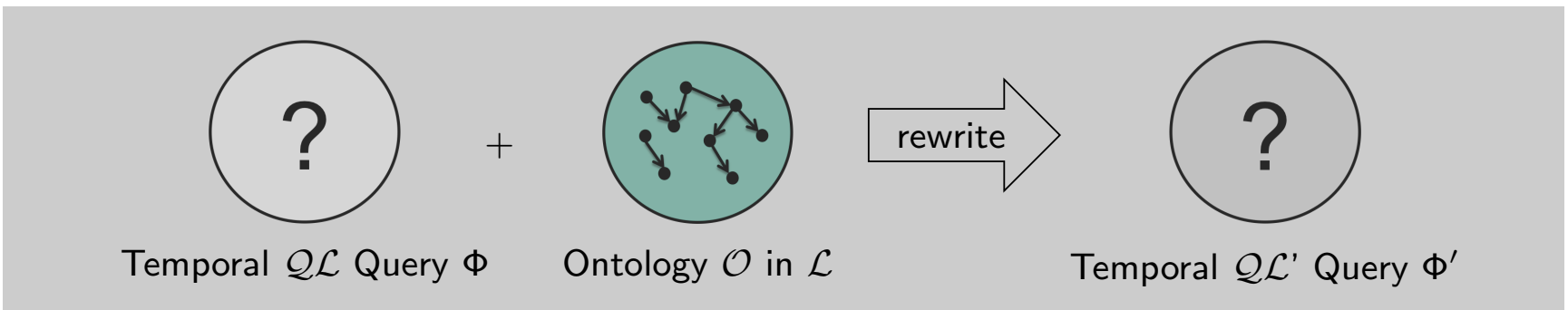
- Positive Temporal $\mathcal{QL}$ queries: LTL without negation + $\mathcal{QL}$ queries
- Temporal KB with ontology in some lightweight logic $\mathcal{L}$
- $\mathcal{QL}$ and $\mathcal{L}$ must satisfy certain requirements
  $\rightarrow$ Rewritability of $\mathcal{QL}$ queries w.r.t. KBs in $\mathcal{L}$
- Generic rewritability result for PTQ answering




Answers to $\Phi'$ over $(\mathcal{F})_{0 \leq i \leq n}$ =
Answers to $\Phi$ w.r.t. $\langle \mathcal{O}, (\mathcal{F})_{0 \leq i \leq n} \rangle$

# II Rewritability of Temporal Query Answering

- Positive Temporal $\mathcal{QL}$ queries: LTL without negation + $\mathcal{QL}$ queries
- Temporal KB with ontology in some lightweight logic $\mathcal{L}$
- $\mathcal{QL}$ and $\mathcal{L}$ must satisfy certain requirements
  $\rightarrow$ Rewritability of $\mathcal{QL}$ queries w.r.t. KBs in $\mathcal{L}$
- Generic rewritability result for PTQ answering
- Many formalisms satisfy our requirements
  $\rightarrow$ Tools for answering $\mathcal{QL}$ queries often exist




Answers to $\Phi'$ over $(\mathcal{F})_{0 \leq i \leq n}$ =
Answers to $\Phi$ w.r.t. $\langle \mathcal{O}, (\mathcal{F})_{0 \leq i \leq n} \rangle$

| $\mathcal{L}$ | $\mathcal{QL}$ | $\mathcal{QL}'$ |
|---|---|---|
| $\mathcal{EL}^{++}$ | subs. | subs. |
| *DL-Lite*$_{\mathcal{R}}$ | CQ | UCQ |
| $\mathcal{ELH}^{dr}_{\bot}$ | CQ | FO$_{=}$ |
| *DL-Lite*$^{\mathcal{N}}_{horn}$ | CQ | FO$_{=}$ |
| *DL-Lite*$_{\mathcal{R}}$ | UCQ | PEQ |
| *DL-Lite* | CQ | UCQ |
| $\mathcal{ELHI}^{\neg}$ | CQ | Datalog |
| *DL-Lite*$_{\mathcal{R}}$ | CQ | UCQ |
| *DL-Lite*$^{+}$ | CQ | UCQ$^{+}$ |
| Horn-$\mathcal{ALCHIQ}$ | CQ | UCQ |
| $\mathcal{LDL}^{+}$ | IQ | IQ |
| $\mathcal{SROEL}(\sqcap, \times)$ | IQ | IQ |
| Datalog$^{\pm}$ family | CQ | UCQ |

# Summary & Outlook

- Ontology-based data access: common domain terminology and knowledge
- We need extensions for recognizing complex contexts
- Temporal query answering w.r.t. ontologies in lightweight logics

# Summary & Outlook

- Ontology-based data access: common domain terminology and knowledge
- We need extensions for recognizing complex contexts
- Temporal query answering w.r.t. ontologies in lightweight logics

# Summary & Outlook

- Ontology-based data access: common domain terminology and knowledge
- We need extensions for recognizing complex contexts
- Temporal query answering w.r.t. ontologies in lightweight logics

**Combined and data complexity of TCQ satisfiability**

**Rewritability of TQ answering**

# Summary & Outlook

- Ontology-based data access: common domain terminology and knowledge
- We need extensions for recognizing complex contexts
- Temporal query answering w.r.t. ontologies in lightweight logics

**Combined and data complexity of TCQ satisfiability**

- Description logics *DL-Lite* and $\mathcal{EL}$
- Solutions inherently exponential
- New algorithms: PSPACE combined complexity in many cases
- Feasible data complexity for $\mathit{DL\text{-}Lite}_{\mathit{horn}}^{\mathcal{H}}$
- Similar results for TQs where $\mathcal{QL}$ = DL axioms (not in this talk)

**Rewritability of TQ answering**

# Summary & Outlook

- Ontology-based data access: common domain terminology and knowledge
- We need extensions for recognizing complex contexts
- Temporal query answering w.r.t. ontologies in lightweight logics

**Combined and data complexity of TCQ satisfiability**

- Description logics *DL-Lite* and $\mathcal{EL}$
- Solutions inherently exponential
- New algorithms: $\mathrm{PSPACE}$ combined complexity in many cases
- Feasible data complexity for $\textit{DL-Lite}_{horn}^{\mathcal{H}}$
- Similar results for TQs where $\mathcal{QL}$ = DL axioms (not in this talk)

**Rewritability of TQ answering**

- Generic rewritability result for positive TQs
- Conditions are satisfied by many existing formalisms
- Hints at implementations

# Summary & Outlook

- Ontology-based data access: common domain terminology and knowledge
- We need extensions for recognizing complex contexts
- Temporal query answering w.r.t. ontologies in lightweight logics

Metric temporal logic operators? Other DLs?

**Combined and data complexity of TCQ satisfiability**

- Description logics *DL-Lite* and $\mathcal{EL}$
- Solutions inherently exponential
- New algorithms: PSPACE combined complexity in many cases
- Feasible data complexity for $\mathit{DL\text{-}Lite}_{horn}^{\mathcal{H}}$ The CO-NP/EXPTIME gap?
- Similar results for TQs where $\mathcal{QL}$ = DL axioms (not in this talk)

**Rewritability of TQ answering** Implementations? Use cases?

- Generic rewritability result for positive TQs Other restrictions?
- Conditions are satisfied by many existing formalisms
- Hints at implementations

- S. Borgwardt, M. Lippmann, and T.:
  **Temporalizing Rewritable Query Languages over Knowledge Bases**.
  Journal of Web Semantics, 2015.

- S. Borgwardt and T.:
  **Temporal Query Answering in DL-Lite with Negation**.
  In Proc. of GCAI, EasyChair, 2015.
  **Temporal Query Answering in the Description Logic $\mathcal{EL}$**.
  In Proc. of IJCAI, AAAI Press, 2015.

- S. Borgwardt, M. Lippmann, and T.:
  **Temporal Query Answering in the Description Logic DL-Lite**.
  In Proc. of FroCoS, LNCS, 2013.

- T. and E. Zenker:
  **Temporal Query Answering in a Fuzzy World**.
  In Proc. of the Posters and Demos Track of SEMANTiCS, CEUR WS, 2015.

- S. Borgwardt and T.:
  **Temporal Query Answering in the Description Logic $\mathcal{EL}$** (ext. abstract).
  In Proc. of DL, CEUR WS, 2015.

- T., J. Holste, and Ö. Özçep:
  **On Implementing Temporal Query Answering in DL-Lite** (ext. abstract).
  In Proc. of DL, CEUR WS, 2015.

- S. Borgwardt, M. Lippmann, and T.:
  **Temporal Query Answering in DL-Lite** (best student paper).
  In Proc. of DL, CEUR WS, 2013.

- F. Baader, S. Borgwardt, P. Koopmann, A. Ozaki, and T.:
  **Metric Temporal Description Logics with Interval-Rigid Names**.
  In Proc. of **FroCoS**, LNCS, 2017.
- F. Baader, S. Borgwardt, P. Koopmann, A. Ozaki, and T.:
  **Metric Temporal Description Logics with Interval-Rigid Names** (ext. abstract).
  In Proc. of **DL**, CEUR WS, 2017.

# Thank you!

Stefan Borgwardt
Franz Baader
Marcel Lippmann
Markus Krötzsch
Anni-Yasmin Turhan
Ana Ozaki
Kerstin Achtruth
Carsten Lutz

My family and friends

(Baader et al. 2015)
F. Baader, S. Borgwardt, and M. Lippmann:
**Temporal Query Entailments in the Description Logic $\mathcal{SHQ}$.**
Journal of Web Semantics, 2015.

(Baader et al. 2012)
F. Baader, S. Ghilardi, and C. Lutz:
**LTL over Description Logic Axioms**.
ACM Transactions on Computational Logic, 2012.

(Sistla and Clarke 1985)
A. P. Sistla and E. M. Clarke:
**The Complexity of Propositional Linear Temporal Logics**,
Journal of the ACM, 1985.